11 MENSURATION

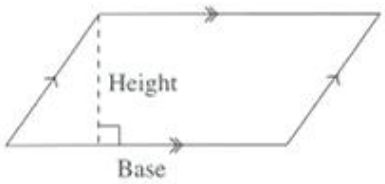
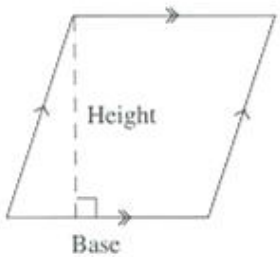
LEARNING OBJECTIVES

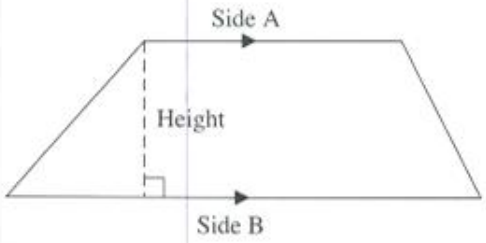
In this topic, we will learn to:

- compute the area of parallelogram and trapezium
- solve problems involving perimeter and area of composite plane figures
- compute the volume and surface area of the cube, cuboid, prism and cylinder
- convert between units such as m^3 to cm^3 and m^2 to cm^2 and vice versa
- solve problems involving volume and surface area of composite solids
- visualise and draw sketches of 3D shapes from different views

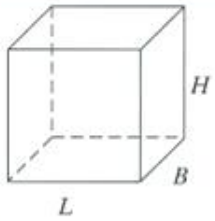
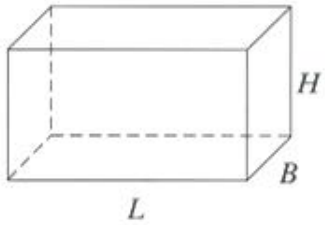
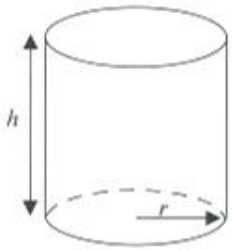
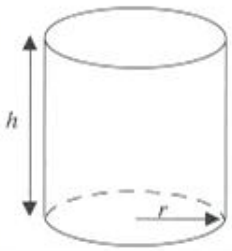
11.1 FORMULAE

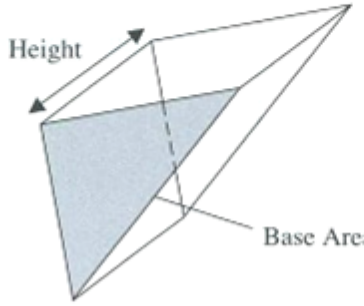
For Plane Figures:

Shapes	Area
Parallelogram 	Base × Height
Rhombus 	Base × Height

Trapezium	
	$\frac{1}{2}$ × (Sum of parallel sides) × Height

For Geometric Solids:

Shapes		Volume	Surface Area
Cube		$L \times B \times H$	$6 \times L \times B$
Cuboid		$L \times B \times H$	$2 \times L \times B +$ $2 \times L \times H +$ $2 \times B \times H$
Solid Cylinder		$\pi r^2 h$	$2\pi rh + 2\pi r^2$
Open Cylinder		$\pi r^2 h$	$2\pi rh + \pi r^2$

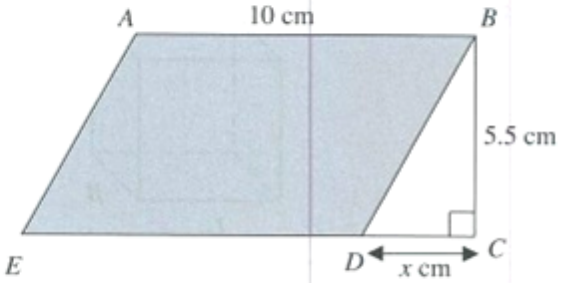
Triangular Prism		Base Area × Height	Calculate the area of the net diagram of the triangular prism.

Worked Example 1

In the diagram, $ABDE$ forms a parallelogram where $ABCE$ forms a trapezium.
Given that $AB = 10$ cm, $BC = 5.5$ cm and $\angle BCD = 90°$, find

(a) the area of parallelogram $ABDE$, and

(b) the value of x if the area of trapezium is 66 cm².



Worked Solution:

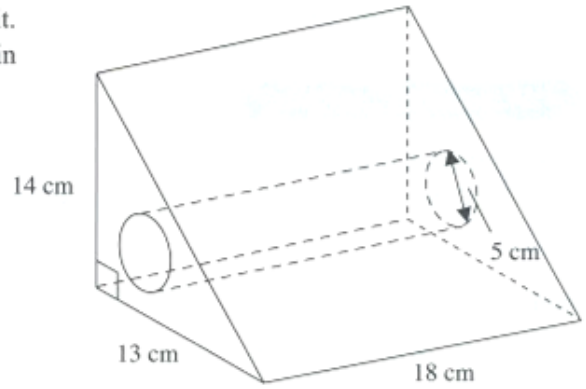
(a) Area of parallelogram $ABDE$ = Base × Height
$= 10 \times 5.5$
$= \mathbf{55\ cm^2}$

(b) Area of trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) × Height

$$66 = \frac{1}{2} \times (10 + 10 + x) \times 5.5$$
$$66 = \frac{1}{2} \times (20 + x) \times 5.5$$
$$132 = (20 + x) \times 5.5$$
$$20 + x = 132 \div 5.5$$
$$= 24$$
$$x = \mathbf{4}$$

WORKED EXAMPLE 2

A wooden prism has a cylindrical hole bored into it. Find the volume of the composite figure as shown in the diagram. Take π to be 3.142.



Worked Solution:

Volume of prism
= Base Area × Height
= Area of triangle × Height
$= \frac{1}{2} \times (13 \times 14) \times 18$
= 1638 cm³

Volume of cylinder
$= \pi r^2 h$
$= 3.142 \times \left(\frac{5}{2}\right)^2 \times 18$
= 353.475

Volume of the composite figure = 1638 – 353.475
= 1284.525 cm³
= **1280 cm³** (3 sig.fig.)

11.2 CONVERSION OF UNITS

It is known that 1 metre is equivalent to 100 centimetres and 1 kilometre is equivalent 1000 metres. However, in area and volume, the conversion is not as direct.

$1\text{ m} = 100\text{ cm}$
$1\text{ m}^2 = 10\,000\text{ cm}^2$
$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$

WORKED EXAMPLE 3

Convert the following from m^2 to cm^2.
(a) 2.3 m^2 (b) 0.7 m^2 (c) 0.045 m^2

Worked Solution:

(a) $1\text{ m}^2 = 10\,000\text{ cm}^2$
$2.3\text{ m}^2 = 2.3 \times 10\,000$
$= \mathbf{23\,000\text{ cm}^2}$

(b) $1\text{ m}^2 = 10\,000\text{ cm}^2$
$0.7\text{ m}^2 = 0.7 \times 10\,000$
$= \mathbf{7000\text{ cm}^2}$

(c) $1\ m^2 = 10\ 000\ cm^2$

$0.045\ m^2 = 0.045 \times 10\ 000$

$= \mathbf{450\ cm^2}$

Worked Example 4

Convert the following from cm^2 to m^2.

(a) $2300\ cm^2$ (b) $140\ cm^2$ (c) $80\ cm^2$

Worked Solution:

(a) $1\ m^2 = 10\ 000\ cm^2$

$1\ cm^2 = \frac{1}{10\ 000}\ m^2$

$2300\ cm^2 = \frac{1}{10\ 000} \times 2300\ m^2$

$= \mathbf{0.23\ m^2}$

(b) $1\ cm^2 = \frac{1}{10\ 000}\ m^2$

$140\ cm^2 = \frac{1}{10\ 000} \times 140\ m^2$

$= \mathbf{0.014\ m^2}$

(c) $1\ cm^2 = \frac{1}{10\ 000}\ m^2$

$80\ cm^2 = \frac{1}{10\ 000} \times 80\ m^2$

$= \mathbf{0.008\ m^2}$

Worked Example 5

Convert the following from m^3 to cm^3.

(a) $6.2\ m^3$ (b) $1.07\ m^3$ (c) $0.057\ m^3$

Worked Solution:

(a) $1\ m^3 = 1\ 000\ 000\ cm^3$

$6.2\ m^3 = 6.2 \times 1\ 000\ 000$

$= \mathbf{6\ 200\ 000\ cm^3}$

(b) $1\ m^3 = 1\ 000\ 000\ cm^3$

$1.07\ m^3 = 1.07 \times 1\ 000\ 000$

$= \mathbf{1\ 070\ 000\ cm^3}$

(c) $1\ m^3 = 1\ 000\ 000\ cm^3$

$0.057\ m^3 = 0.057 \times 1\ 000\ 000$

$= \mathbf{57\ 000\ cm^3}$

WORKED EXAMPLE 6

Convert the following from cm^3 to m^3.

(a) 28 000 cm^3 (b) 9200 cm^3 (c) 1940 m^3

Worked Solution:

(a) $1\ m^3 = 1\ 000\ 000\ cm^3$

$1\ cm^3 = \frac{1}{1\ 000\ 000}\ m^3$

$28\ 000\ cm^3 = \frac{1}{1\ 000\ 000} \times 28\ 000\ m^3$

$= \mathbf{0.028\ m^3}$

(b) $1\ cm^3 = \frac{1}{1\ 000\ 000}\ m^3$

$9200\ cm^3 = \frac{1}{1\ 000\ 000} \times 9200\ m^3$

$= \mathbf{0.009\ 2\ m^3}$

(c) $1\ cm^3 = \frac{1}{1\ 000\ 000}\ m^3$

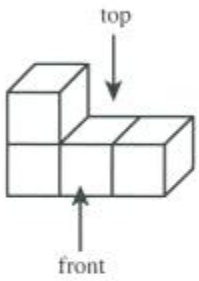
$1940\ cm^3 = \frac{1}{1\ 000\ 000} \times 1940\ m^3$

$= \mathbf{0.001\ 94\ m^3}$

11.3 VIEW AND NETS OF 3D SOLIDS

WORKED EXAMPLE 7

The diagram below shows an object formed by wooden cubes of the same size.

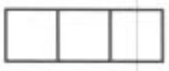


(a) When viewed from the top, how would the object look like?

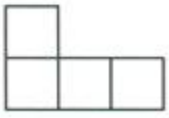
(b) When viewed from the front, how would the object look like?

Worked Solution:

(a)



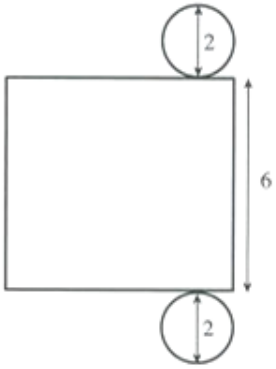
(b)



WORKED EXAMPLE 8

The figure below shows a cylinder, where the unit of length is centimetres. Draw a net of the cylinder.

Worked Solution:



PRACTICE QUESTIONS

1. Express the following in cm^2.

(a)	$0.07\ m^2$	(b)	$0.15\ m^2$
(c)	$1.05\ m^2$	(d)	$1.67\ m^2$
(e)	$2.5\ m^2$	(f)	$3.8\ m^2$
(g)	$8.85\ m^2$	(h)	$10.04\ m^2$
(i)	$15.25\ m^2$	(j)	$20.005\ m^2$

2. Express the following in m^2.

(a)	$100\ cm^2$	(b)	$1500\ cm^2$
(c)	$18\ 000\ cm^2$	(d)	$29\ 000\ cm^2$
(e)	$35\ 000\ cm^2$	(f)	$40\ 050\ cm^2$
(g)	$100\ 100\ cm^2$	(h)	$125\ 000\ cm^2$
(i)	$250\ 800\ cm^2$	(j)	$500\ 000\ cm^2$

3. Express the following in cm^3.

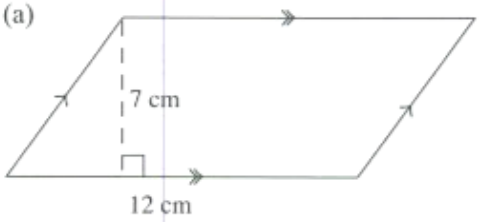
(a) $0.05\ m^3$ (b) $0.08\ m^3$
(c) $1.75\ m^3$ (d) $2.44\ m^3$
(e) $3.03\ m^3$ (f) $5.91\ m^3$
(g) $6.68\ m^3$ (h) $10.012\ m^3$
(i) $15.009\ m^3$ (j) $20.107\ m^3$

4. Express the following in m^3.

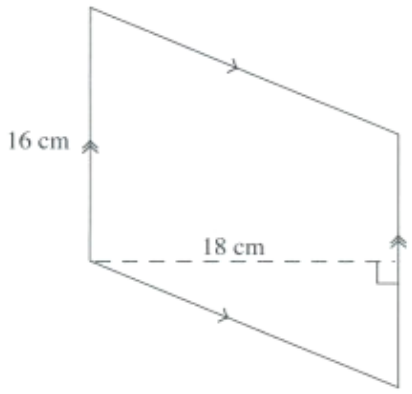
(a) $150\ cm^3$ (b) $380\ cm^3$
(c) $800\ cm^3$ (d) $1450\ cm^3$
(e) $5050\ cm^3$ (f) $9005\ cm^3$
(g) $10\ 030\ cm^3$ (h) $15\ 500\ cm^3$
(i) $25\ 000\ cm^3$ (j) $30\ 700\ cm^3$

5. Find the area of the following plane figures.

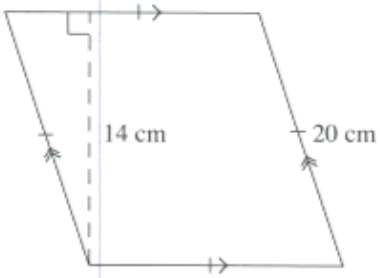
(a)



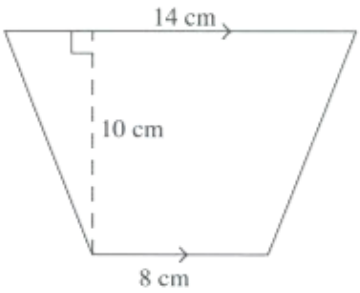
(b)



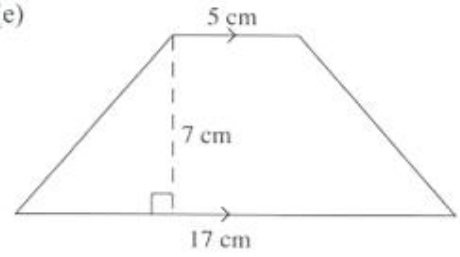
(c)



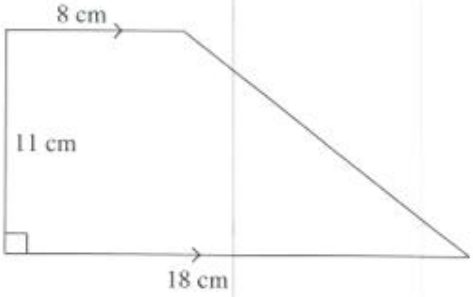
(d)



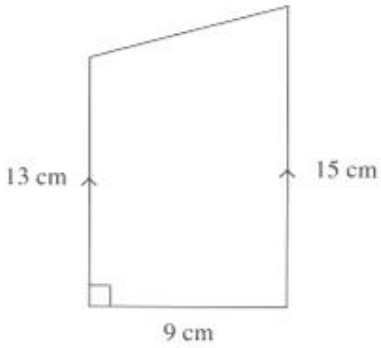
(e)


5 cm
7 cm
17 cm

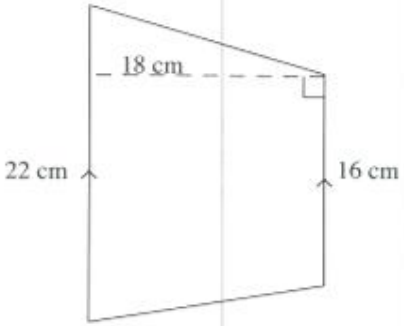
(f)


8 cm
11 cm
18 cm

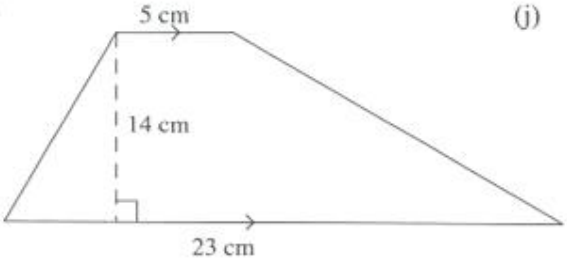
(g)


13 cm
15 cm
9 cm

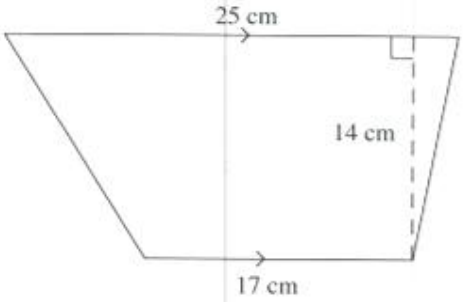
(h)


18 cm
22 cm
16 cm

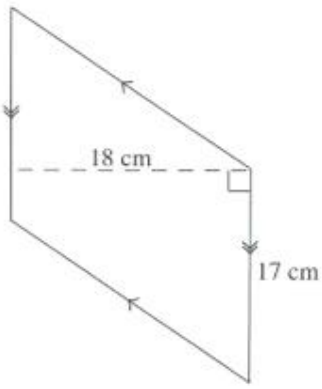
(i)


5 cm
14 cm
23 cm

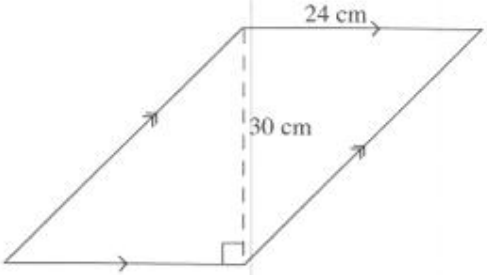
(j)


25 cm
14 cm
17 cm

(k)

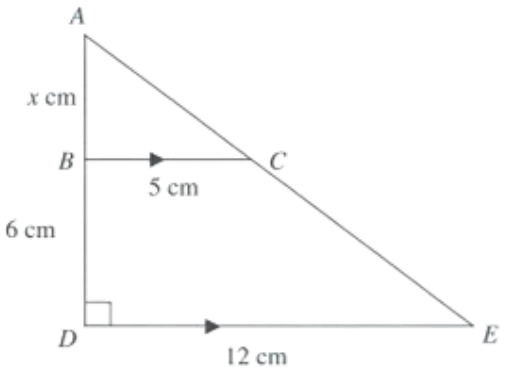

18 cm
17 cm

(l)


24 cm
30 cm

6. In the diagram, ΔABC is a right-angled triangle. Given that the area of ΔABC is $\frac{1}{4}$ the area of trapezium $BCED$,

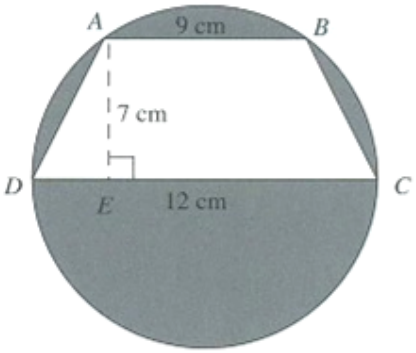
(a) calculate the area of trapezium $BCED$, and
(b) the value of length x.



7. In the diagram, a trapezium is inscribed inside a circle. AB is parallel to DC and DC is the diameter of the circle. Find

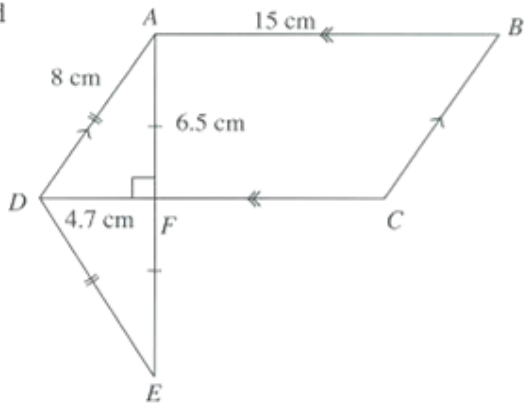
(a) the area of the circle,
(b) the area of the trapezium, and
(c) the shaded area.

Take π to be 3.142.

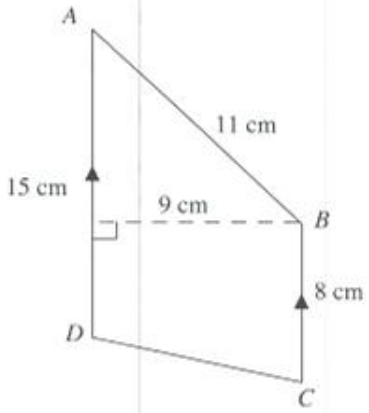


8. In the diagram, $ABCD$ is a parallelogram and ΔADE is an isosceles triangle. Find

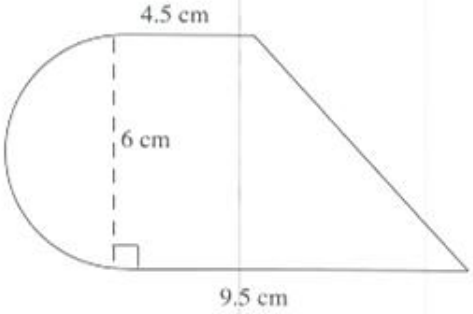
(a) the area of the parallelogram,
(b) the area of the isosceles triangle, and
(c) the perimeter of the figure.



9. In the diagram, $ABCD$ is a trapezium. Find the area of the trapezium.

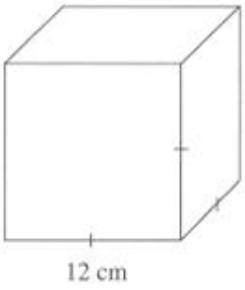


10. The figure shows a semicircle and a trapezium. Find the area of the figure. Take π to be 3.142.

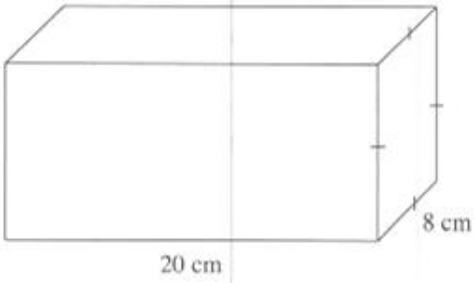


11. Find the surface area and volume of the following solids. Take π to be 3.142.

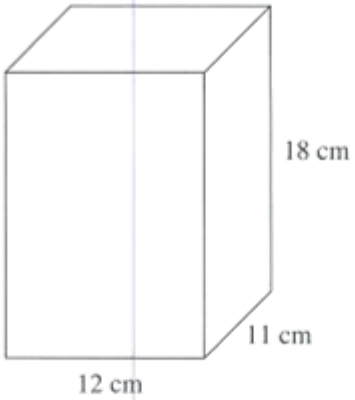
(a)



(b)



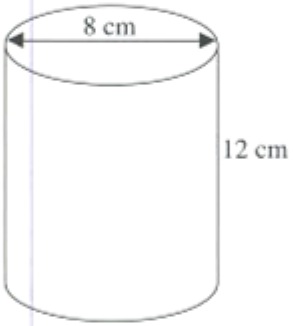
(c)


18 cm
11 cm
12 cm

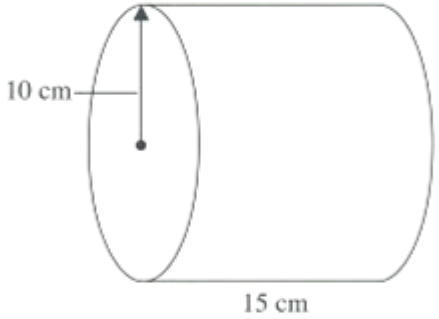
(d)


35 cm
3 cm
4 cm

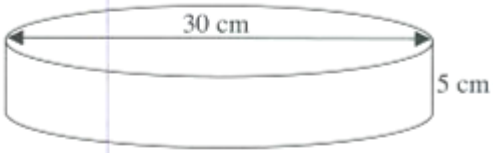
(e)


8 cm
12 cm

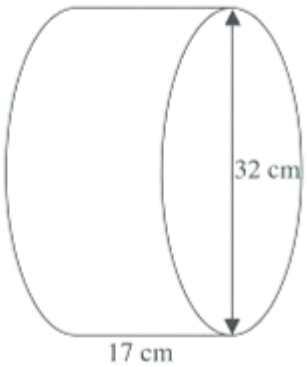
(f)


10 cm
15 cm

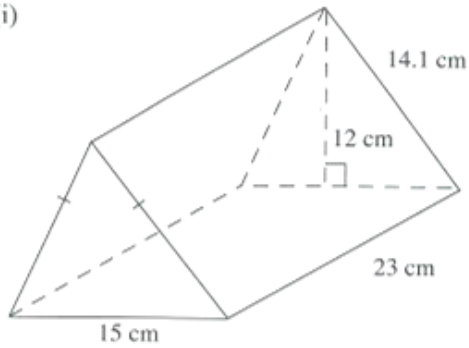
(g)


30 cm
5 cm

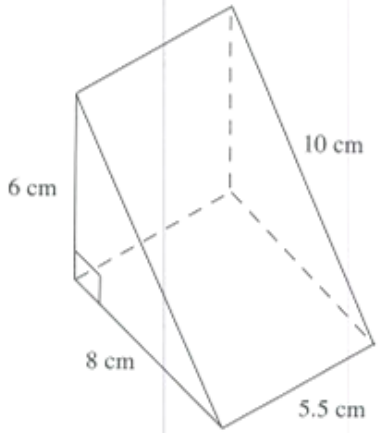
(h)


32 cm
17 cm

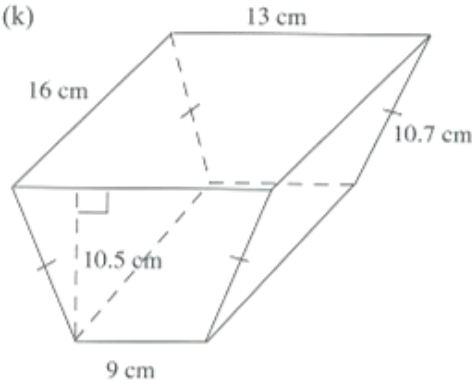
(i)



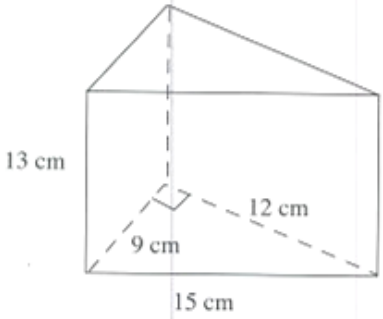
(j)



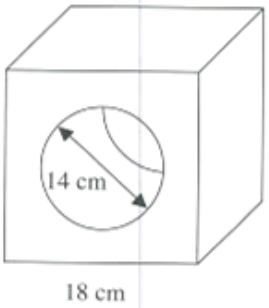
(k)



(l)

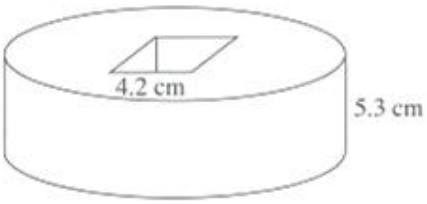


12. A hole of diameter 14 cm is bored through a cube of side 18 cm. Find
    (a) the volume of the composite solid, and
    (b) the surface area of the composite solid.
    Take π to be 3.142.

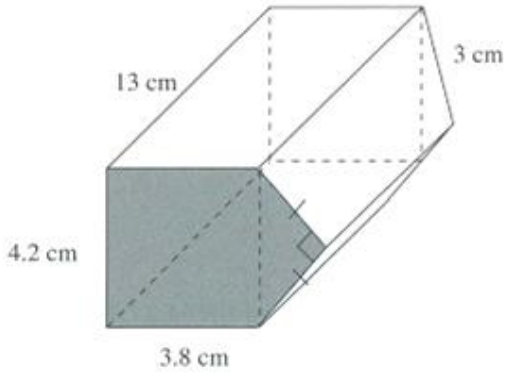


13. A square hole was removed from a cylindrical block. Given that the radius of the cylinder is 10 cm with height of 5.3 cm and the length of the square hole is 4.2 cm, find
    (a) the volume of the cylinder before the hole was removed, and
    (b) the volume of the cuboid.

Take π to be 3.142.

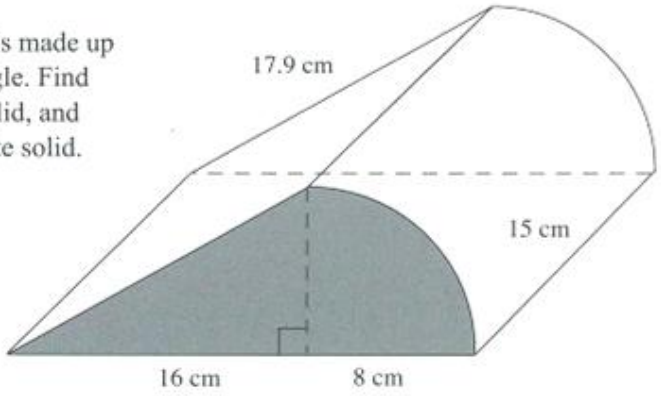


14. The cross-sectional area of the prism is made up of a rectangle and an isosceles triangle. Find
    (a) the volume of the composite solid, and
    (b) the surface area of the composite solid.

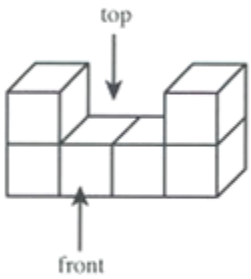


15. The cross-sectional area of the figure is made up of a quadrant and a right-angled triangle. Find
    (a) the volume of the composite solid, and
    (b) the surface area of the composite solid.

Take π to be 3.142.



16. The diagram below shows an object formed by wooden cubes of the same size.



(a) When viewed from the top, how would the object look like?
(b) When viewed from the front, how would the object look like?

17. The figure below shows a cuboid, where the unit of length is centimetres. Draw a net of the cuboid.