CHAPTER 2

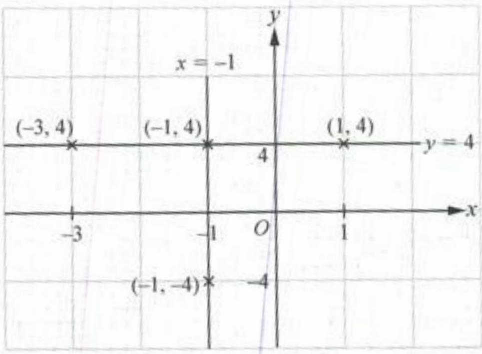
Linear Graphs and Simultaneous Linear Equations

Key Notes

2.1 Linear Graphs

Let us recall that the equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.

<u>Horizontal line ($y = c$) and vertical line ($x = b$)</u>

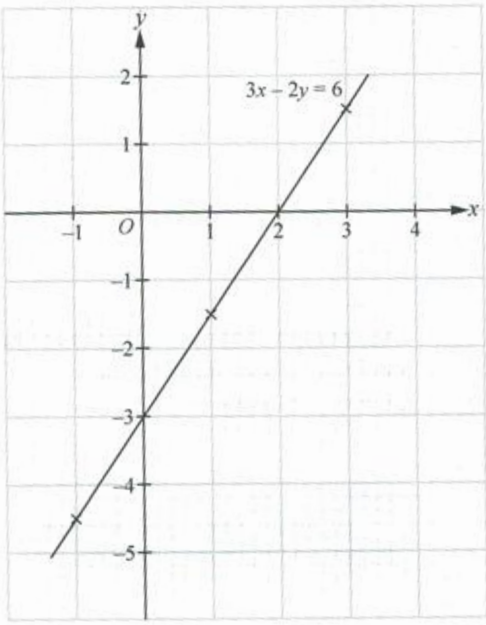


Note:
Every point on the horizontal line has the y-coordinate, c, and every point on the vertical line has the x-coordinate, b.

<u>Linear graph ($ax + by = k$)</u>

Let us consider the equation of the line $3x - 2y = 6$.

x	–1	1	3
y	–4.5	–1.5	1.5



2.2 Solving Simultaneous Linear Equations

Simultaneous linear equations involve **2 different linear equations**, each involving the same **2 variables**.

Let us consider the set of simultaneous equations $2x + 5y = 7$ and $-4x - y = 4$.

To find the **solution** for these simultaneous equations, we need to **solve for x and y**.

Graphical method

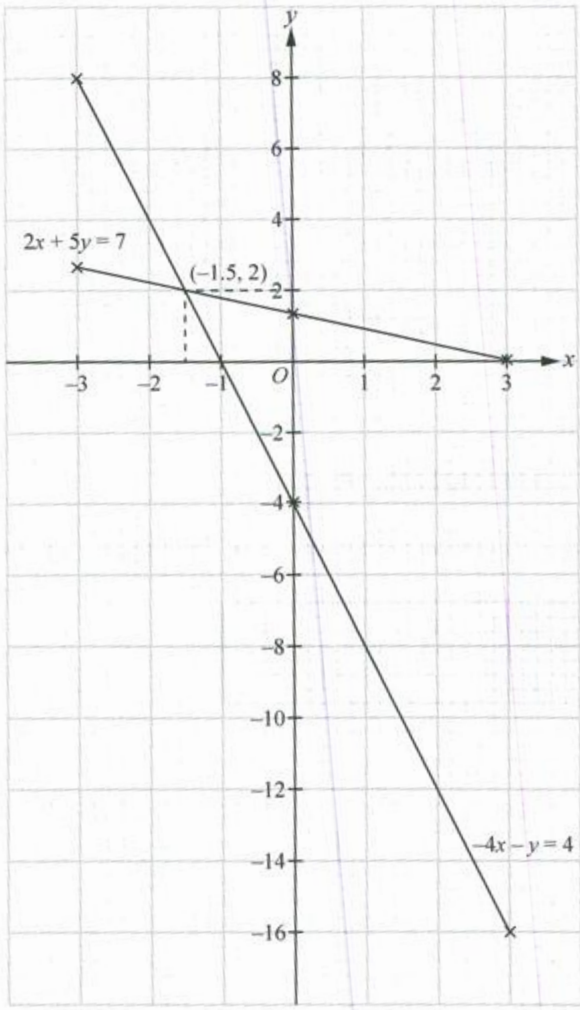
On the same axes, draw the graphs for both equations, using an appropriate scale for both axes.

$2x + 5y = 7$

x	-3	0	3
y	$2\frac{3}{5}$	$1\frac{2}{5}$	$\frac{1}{5}$

$-4x - y = 4$

x	-3	0	3
y	8	-4	-16



Note:
Label both graphs clearly.

The solution is the coordinates of the point where both lines intersect, i.e. $x = -1.5$ and $y = 2$.

Note:
The graphical solution is an estimation and may not be completely accurate.

Elimination method

The elimination method involves eliminating one of the variables from the equations and hence, solving for the other variable.

$2x + 5y = 7$ .... (1)

$-4x - y = 4$................. (2)

Note:
First, label both equations (1) and (2) respectively.

The elimination method can be used when the absolute value of the coefficients of one variable in both equations are the same.

Let us multiply equation (1) by 2.

(1) × 2: $4x + 10y = 14$ ....(3)

The coefficient of x in (3) is equal to the absolute value of the coefficient of x in (2). So, we add (2) and (3) in order to eliminate x.

(2) + (3) : $(-4x - y) + (4x + 10y) = 4 + 14$

$$-4x - y + 4x + 10y = 18$$
$$9y = 18$$
$$y = \frac{18}{9}$$
$$= 2$$

Note:
Add the left-hand sides (LHS) of both equations together and the right-hand sides (RHS) of both equations together.

Substitute $y = 2$ into (1):

$$2x + 5(2) = 7$$
$$2x + 10 = 7$$
$$2x = 7 - 10$$
$$= -3$$
$$x = \frac{-3}{2}$$
$$= -1.5$$

Substitution method

The substitution method involves using one equation to find one variable in terms of the other. We then substitute this expression into the other equation to find the solution.

$2x + 5y = 7$ ....(1)

$-4x - y = 4$ ....(2)

Note:
Do not substitute (3) into (2) as (3) is derived from (2).

From (2): $y = -4x - 4$ ....(3)

Substitute (3) into (1):

$$\begin{aligned} 2x + 5(-4x - 4) &= 7 \\ 2x - 20x - 20 &= 7 \\ -18x &= 7 + 20 \\ &= 27 \\ x &= \frac{27}{-18} \\ &= -1.5 \end{aligned}$$

Substitute $x = -1.5$ into (3):

$$\begin{aligned} y &= -4(-1.5) - 4 \\ &= 2 \end{aligned}$$

Remember to check your solutions by substituting the values obtained into the original equations.

Substitute $x = -1.5$ and $y = 2$ into (1):

$$\begin{aligned} \text{LHS} &= 2(-1.5) + 5(2) \\ &= 7 \\ &= \text{RHS } (\checkmark) \end{aligned}$$

Substitute $x = -1.5$ and $y = 2$ into (2):

$$\begin{aligned} \text{LHS} &= -4(-1.5) - 2 \\ &= 4 \\ &= \text{RHS } (\checkmark) \end{aligned}$$