4 ALGEBRAIC EXPRESSIONS AND FORMULAE

LEARNING OBJECTIVES

In this topic, we will learn to:
- understand the concept of using letters to represent numerical values
- perform the four basic operations $(+, -, \times$ and $\div)$ on algebra
- evaluate algebraic expressions and formulae
- translate simple real-world situations into algebraic expressions
- add and subtract simple linear expressions

4.1 EVALUATING ALGEBRAIC EXPRESSIONS AND FORMULAE

WORKED EXAMPLE 1

If $a = 3, b = 2, c = -1$ and $d = 5$, evaluate the following.

(a) $3a + 2b$

(b) $5c - 2d$

(c) $ab - 7$

(d) ab^2

(e) $b(d - 3c)$

(f) $ac - 3bd$

Worked Solution:

(a) $3a + 2b = 3(3) + 2(2)$
$= 9 + 4$
$= \mathbf{13}$

(b) $5c - 2d = 5(-1) - 2(5)$
$= -5 - 10$
$= \mathbf{-15}$

(c) $ab - 7 = (3)(2) - 7$
$= 6 - 7$
$= \mathbf{-1}$

(d) $ab^2 = (3)(2)^2$
$= (3)(4)$
$= \mathbf{12}$

(e) $b(d - 3c) = (2)[(5) - 3(-1)]$
$= (2)[5 + 3]$
$= (2)(8)$
$= \mathbf{16}$

(f) $ac - 3bd = (3)(-1) - 3(2)(5)$
$= (-3) - 30$
$= \mathbf{-33}$

4.2 FOUR OPERATIONS OF ALGEBRA

WORKED EXAMPLE 2

Add the following algebras.

(a) $3a + 2$ (b) $x + 3x + 4x$ (c) $x + y + 3x + y$

(d) $2x^2 + 2x + x + 3x^2$ (e) $(-2x) + (-2x)$ (f) $(-4y^2) + 3y + (-2y^2)$

Worked Solution:

> **Note:**
> x^2 and x are considered as two different algebraic terms. Hence, they cannot be added together.

(a) $3a + 2 = \mathbf{3a + 2}$ (b) $x + 3x + 4x = \mathbf{8x}$

(c) $x + y + 3x + y = \mathbf{4x + 2y}$ (d) $2x^2 + 2x + x + 3x^2 = \mathbf{5x^2 + 3x}$

(e) $(-2x) + (-2x) = (-2x) - 2x$
$\qquad\qquad\qquad = \mathbf{-4x}$

(f) $(-4y^2) + 3y + (-2y^2) = (-4y^2) + 3y - 2y^2$
$\qquad\qquad\qquad\qquad\qquad = \mathbf{3y - 6y^2}$

WORKED EXAMPLE 3

Subtract the following algebras.

(a) $3x - 1$ (b) $15x - 4x$ (c) $\dfrac{2}{5}x + 2y - \dfrac{1}{3}x - y$

(d) $7x^2 + 4x - x^2 - 5x$ (e) $3x - (-2x)$ (f) $(-3x^2) - (-4x) - (-5x^2)$

Worked Solution:

(a) $3x - 1 = \mathbf{3x - 1}$

(b) $15x - 4x = \mathbf{11x}$

(c) $\dfrac{2}{5}x + 2y - \dfrac{1}{3}x - y = \dfrac{2x}{5} + 2y - \dfrac{x}{3} - y$

$\qquad\qquad\qquad\qquad = \dfrac{2x \times 3}{5 \times 3} + 2y - \dfrac{x \times 5}{3 \times 5} - y$

$\qquad\qquad\qquad\qquad = \dfrac{6x}{15} + 2y - \dfrac{5x}{15} - y$

$\qquad\qquad\qquad\qquad = \dfrac{x}{15} + y$

> **Note:**
> $\dfrac{2}{5}x$ is the same as $\dfrac{2x}{5}$.
>
> Likewise, $\dfrac{1}{3}x$ is the same as $\dfrac{x}{3}$.

(d) $7x^2 + 4x - x^2 - 5x = 7x^2 - x^2 + 4x - 5x$
$\qquad\qquad\qquad\qquad = \mathbf{6x^2 - x}$

(e) $3x - (-2x) = 3x + 2x$
$= \mathbf{5x}$

(f) $(-3x^2) - (-4x) - (-5x^2) = (-3x^2) + 4x + 5x^2$
$= \mathbf{2x^2 + 4x}$

WORKED EXAMPLE 4

Multiply the following algebras.

(a) $2a \times a$ 　　　　(b) $a^2 \times a^2$ 　　　　(c) $4x \times 2y^2$

(d) $\dfrac{1}{2}x^2 \times 6x$ 　　　(e) $\dfrac{4}{5x} \times \dfrac{15x}{24}$

Worked Solution:

(a) $2a \times a = \mathbf{2a^2}$ 　　　　　　　　　(b) $a^2 \times a^2 = \mathbf{a^4}$

(c) $4x \times 2y^2 = \mathbf{8xy^2}$ 　　　　　　(d) $\dfrac{1}{\cancel{2}_1}x^2 \times \cancel{6}^3 x = \dfrac{1}{1}x^2 \times 3x$
$= \mathbf{3x^3}$

(e) $\dfrac{\cancel{4}^{\,1}}{{}_1\cancel{5x}^1} \times \dfrac{\cancel{15}^{3}\cancel{x}^1}{\cancel{24}6} = \dfrac{1}{1} \times \dfrac{3}{6}$
$= \mathbf{\dfrac{1}{2}}$

> **Note:**
> To simplify algebra, eliminate like terms before multiplying or dividing the expression.
> Example: $\dfrac{\cancel{2}^1}{\cancel{8x}_1} \times \dfrac{\cancel{15}x^3}{\cancel{8}_4} = \dfrac{1}{1} \times \dfrac{3}{4}$
> $= \dfrac{3}{4}$

WORKED EXAMPLE 5

Divide the following algebras.

(a) $10x \div 5$ 　　　　(b) $\dfrac{4x}{5} \div 2$ 　　　　(c) $8 \div \dfrac{6x}{7}$

(d) $15n^2 \div 5$ 　　　(e) $\dfrac{3}{5x} \div \left(-\dfrac{x}{6}\right)$

Worked Solution:

(a) $10x \div 5 = \dfrac{\cancel{10}^2 x}{\cancel{5}_1}$
$= \mathbf{2x}$

(b) $\dfrac{4x}{5} \div 2 = \dfrac{4x}{5} \div \dfrac{2}{1}$
$= \dfrac{{}^2\cancel{4}x}{5} \times \dfrac{1}{\cancel{2}_1}$
$= \dfrac{2x}{5} \times \dfrac{1}{1}$
$= \mathbf{\dfrac{2x}{5}}$

(c) $8 \div \dfrac{6x}{7} = 8 \times \dfrac{7}{6x}$

$= \dfrac{\overset{4}{8}}{1} \times 3 \dfrac{7}{6x}$

$= \dfrac{4}{1} \times \dfrac{7}{3x}$

$= \dfrac{28}{3x}$

(d) $15n^2 \div 5 = \dfrac{\overset{3}{\cancel{15}}n^2}{\underset{1}{\cancel{5}}}$

$= \dfrac{3n^2}{1}$

$= 3n^2$

(e) $\dfrac{3}{5x} \div \left(-\dfrac{x}{6}\right) = \dfrac{3}{5x} \times \left(-\dfrac{6}{x}\right)$

$= -\dfrac{18}{5x^2}$

4.3   SIMPLIFICATION OF LINEAR ALGEBRAIC EXPRESSIONS

WORKED EXAMPLE 6

Multiply the following algebraic expressions.

(a) $3(3x + 5y)$ (b) $-(4x - y)$ (c) $a(3a - b)$

(d) $-\dfrac{1}{2}(4m - 6n)$ (e) $7(3h - 2k + h)$

Worked Solution:

$= 28h - 14k$

(a) $3(3x + 5y) = 3(3x + 5y)$

$= 9x + 15y$

(b) $-(4x - y) = -(4x - y)$

$= -4x + y$

$= y - 4x$

(c) $a(3a - b) = a(3a - b)$

$= 3a^2 - ab$

(d) $-\dfrac{1}{2}(4m - 6n) = -\dfrac{1}{2}(4m - 6n)$

$= -2m + 3n$

$= 3n - 2m$

(e) $7(3h - 2k + h) = 7(3h - 2k + h)$

$= 21h - 14k + 7h$

PRACTICE QUESTIONS

1. Given that $a = 4, b = -3, c = 5, d = 2$ and $e = 0$, evaluate the following algebraic expressions.

    (a) $4a - 3e$ (b) $a + 5c$
    (c) $2ad$ (d) $14 - 4b + 5a$
    (e) $a + b - 2c - (-d)$ (f) $8a - 5de$
    (g) b^3 (h) $2a(-b) + 4dc$
    (i) $ac - ad$ (j) c^d
    (k) $\dfrac{3a - 2d}{c}$ (l) $\dfrac{a^2 - 5c}{4d}$

2. Given that $p = -2, q = 3, r = -1$ and $s = 0.5$, evaluate the following algebraic expressions.

    (a) $3p + 2r$ (b) $4s + 3q$
    (c) $p + 2q - 3r$ (d) $p(2q - 3r)$
    (e) $rs - 2p$ (f) $5q + 7ps$
    (g) p^2q^2

3. The speed of a particle can be calculated using the formula, $v = u + at$. Find the value of v if the values of $u, a,$ and t are 20, 10 and 5 respectively.

4. The distance of a moving object can be calculated using the formula, $s = ut + \dfrac{1}{2}at^2$. Find the value of s if $u = 25, t = 5$, and $a = -10$.

5. The distance travelled by a moving particle can be calculated using the formula, $s = \dfrac{v^2 - u^2}{2a}$. Find the value of s if $v = 50, u = 26$, and $a = 4$.

6. Add the following algebras.

    (a) $4y + 9$ (b) $11y + 3y$
    (c) $13g + 8g + 5g$ (d) $2 + 9x + 12$
    (e) $2x + 4 + 8x$ (f) $4m + 4mn + 3mn$
    (g) $\dfrac{1}{2}x + \dfrac{3}{4} + \dfrac{1}{5}x$ (h) $\dfrac{1}{3}m + 3 + \dfrac{2}{5}m$
    (i) $0.5 + 1.2x + 3.9x$ (j) $3p + 3 + 11$
    (k) $\dfrac{1}{3}y + 0.25y + 13$ (l) $6.2 + 1.3x + \dfrac{11}{10}x$

7. Subtract the following algebras.

    (a) $9x - 2$ (b) $3y - 7y$
    (c) $(-4t) - 2t$ (d) $(-g) - 5g$
    (e) $12 - 3n - 2n$ (f) $29 - 4h - 11h$
    (g) $5p - 12p - 7p$ (h) $4d - 9d - 19d$
    (i) $12 - 6f - 20$ (j) $j - 9j - 7$
    (k) $(-5) - 4y - 9y$ (l) $(-1) - 4g - 17g$

8. Evaluate the following algebras.

(a) $4x + (-9x) - 13x$

(b) $5b - 8b + (-3b)$

(c) $5v - (-4v) - 20v$

(d) $(-2s) - (-5s) - 16s$

(e) $[2x - (-2x)] + 10x$

(f) $3u - [(-4u) - (-9u)]$

(g) $\left[3\frac{1}{2} - (-5x)\right] + 2\frac{1}{3}$

(h) $5\frac{1}{3} - \left(-\frac{7}{8}x\right) + \frac{3}{4}x$

(i) $4m - 3m - 9n + (-4n)$

(j) $(-7p) + 2pq - (-3pq) + (-10p)$

(k) $3\frac{1}{4} - 2q - 8\frac{1}{2} + (-7q)$

(l) $5xy - 3y - (-5xy) - (-3y)$

9. Multiply the following algebras.

(a) $4a \times 7$

(b) $\frac{3}{4} \times 8b$

(c) $3a \times 2b$

(d) $5p \times 2q$

(e) $6a \times 2a$

(f) $3p \times 9t$

(g) $p \times p \times 2p$

(h) $6m^3 \times 2m$

(i) $3m \times 2n \times \frac{1}{2}n$

(j) $\frac{2}{3} \times \frac{8}{9p}$

(k) $\frac{9}{4x} \times \frac{x}{3}$

(l) $\frac{6}{x} \times \frac{2x}{3}$

10. Divide the following algebras.

(a) $2a \div 3$

(b) $15b \div 9$

(c) $20p \div 4$

(d) $24t \div 3$

(e) $5t^2 \div t$

(f) $18m^2 \div 3m$

(g) $54 \div 6n$

(h) $108 \div 12p$

(i) $4m \div \frac{2}{3}$

(j) $27y^2 \div \frac{9}{10}$

(k) $\frac{4}{5} \div \frac{2}{y}$

(l) $\frac{3}{5m} \div \frac{9}{15}$

11. Evaluate the following algebras.

    (a) $(4a \times 3) \div 6$ (b) $(6m \times 4m) \div 12$

    (c) $8p \times (8q \div 4)$ (d) $5m \times [15n \div (-5)]$

    (e) $[(-3m) \times (-m)] \div 9$ (f) $12 \div [(-2x) \times (-3x)]$

    (g) $(4xy \div 2y) \times 7$ (h) $(10m \times 3) \div 5m$

    (i) $3 \times \dfrac{4}{5p} \div \dfrac{16}{15p}$ (j) $(-2x) \times 1\dfrac{1}{5} \div \left(-\dfrac{3}{4}x\right)$

    (k) $p \times p \div \left(\dfrac{1}{2} \times 2q\right)$ (l) $\left(3 \div \dfrac{3}{4}x\right) \times (-2x)$

12. John has \$$3x$ in his savings account. His sister, Jolin, has thrice as much savings as him. What is the sum of money the siblings have in all?

13. Luke spent \$$(5x + 3)$ last weekend. His friend, Kevin spent \$$4x$ more. Find the amount Kevin spent last weekend.

14. In a recent Mathematics test, Sandy scored a total of $(5x - 9)$ marks, whereas her good friend, Nancy, scored 8 marks fewer. Write an expression, in terms of x, for Nancy's marks.

15. The population of School A is $8y$. The population of School B is $\frac{3}{4}$ times the population of School A. Write an expression, in terms of y, for the total population of the two schools.

16. The total cost of 2 cars is $\$7x$. If one car is $\$x$ more expensive than the other, find the cost of the cheaper car, in terms of x.

17. The total cost of buying a toaster and an electric oven is $\$(4y + 30)$. If the cost of the electric oven is $\$150$ more than the toaster, find the cost of the toaster, in terms of y.

18. The average speed of an aircraft is $8y$ km/h. If it has travelled for $2x$ hours, find the distance travelled, in terms of x and y.

19. The total time taken for a car to travel $(20x + 4)$ km is $2x$ hours. Find the average speed of the car, in terms of x.

20. The average speed of a car is $4x$ km/h. Find the time, in terms of x, taken by the car to travel $(x + 10)^2$ km.

21. Simplify the following algebraic expressions.

   (a) $2(x + y)$
   (b) $3(2x - y)$
   (c) $x(5 + x)$
   (d) $y(y - 3)$
   (e) $3x(2 + y)$
   (f) $2m(m + 4)$
   (g) $p(p + q + 4)$
   (h) $x(x + y - 3)$
   (i) $5(x - 4y + 3x)$
   (j) $7(2x + 4y - 5x)$
   (k) $4(p + 3q - 2p)$
   (l) $6(7a - 3b + 2b)$

22. Simplify the following algebraic expressions.

   (a) $-(2 + x)$
   (b) $-(2x + 9)$
   (c) $-2(p - q)$
   (d) $-7(3 - y)$
   (e) $-3(3x + 3y)$
   (f) $-10(-x - y)$
   (g) $-8\left(\dfrac{1}{2} - \dfrac{1}{4}y\right)$
   (h) $-9\left(-\dfrac{1}{3} + \dfrac{2}{3}d\right)$
   (i) $-\dfrac{2}{3}(6 - 6h)$
   (j) $-\dfrac{4}{5}(-15 - 10k)$
   (k) $-5\left(4x + 2y - \dfrac{1}{5}x\right)$
   (l) $-8\left(\dfrac{1}{2}x + \dfrac{1}{4}y - \dfrac{1}{8}x\right)$

23. Evaluate the following algebraic expressions.

   (a) $3(x + 1) + 4(x + 2)$
   (b) $5(p + 3) + 2(2 + 3p)$
   (c) $7(x + y) + 6(2x + y)$
   (d) $4(2 + 3g) + 3(3g + 5)$
   (e) $11(x - 1) + 6(2x + 3)$
   (f) $3(h + 2k) + 5(3h - 2k)$
   (g) $-(a - 4) + 5(4 - a)$
   (h) $-3x(1 + y) - (x + 2y)$
   (i) $4m(3 - m) + 4(2m - m^2)$
   (j) $5(3x + 4x^2) + x(-3 - 2x)$
   (k) $\dfrac{3}{2}(4x - 2) + \dfrac{2}{3}(3 - 3x)$
   (l) $\dfrac{7}{4}(2x + 1) + \dfrac{3}{4}(2 + 6x)$