CHAPTER 4

Further Expansion and Factorisation of Algebraic Expressions

⟩⟩⟩ Key Notes

4.1 Special Algebraic Identities

We can expand and factorise algebraic expressions using the algebraic identities as shown below:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$

4.2 Further Expansion of Algebraic Expressions

There are two methods to expand algebraic expressions with the following formats.

Completing the square method

For example,
Expand $(2x - 3)^2$. ⟵

$$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$$
$$= 4x^2 - 12x + 9$$

Note:
We must use the algebraic identities to expand algebraic expressions.

For example,
$(2x - 3)^2 \neq (2x)^2 - (3)^2$

Difference between two squares method

For example,
Expand $(x + 2)(x - 2)$.

$$(x + 2)(x - 2) = (x)^2 - (2)^2$$
$$= x^2 - 4$$

4.3 Further Factorisation of Algebraic Expressions

There are two methods to factorise algebraic expressions with the following formats.

Completing the square method

For example,
Factorise $9x^2 + 30x + 25$.

$$9x^2 + 30x + 25 = (3x)^2 + 2(3x)(5) + (5)^2$$
$$= (3x + 5)^2$$

Chapter 4 • *Further Expansion and Factorisation of Algebraic Expressions*

<u>Difference between two squares method</u>

For example,
Factorise $4x^2 - 25$.

$$4x^2 - 25 = (2x)^2 - (5)^2$$
$$= (2x - 5)(2x + 5)$$

4.4 Factorisation by Grouping
Recall what we have learnt in Secondary 1, we can factorise algebraic expressions by grouping terms with common factors.

<u>Grouping 2 by 2 method</u>

For example,
Factorise $2ac + 4bc - 2ad - 4bd$.

$$2ac + 4bc - 2ad - 4bd = 2ac - 2ad + 4bc - 4bd$$
$$= 2a(c - d) + 4b(c - d)$$
$$= (2a + 4b)(c - d)$$
$$= 2(a + 2b)(c - d)$$

Note:
- Rearrange the terms, if necessary
- Identify common factors

<u>Grouping 3 by 1 method</u>
For example,
Factorise $4x^2 - 12x + 9 - y^2$.

$$4x^2 - 12x + 9 - y^2 = (4x^2 - 12x + 9) - y^2$$
$$= [(2x)^2 - 2(2x)(3) + (3)^2] - y^2$$
$$= (2x - 3)^2 - y^2$$
$$= (2x - 3 - y)(2x - 3 + y)$$

Note:
We group the first three terms together and complete the square.

Note:
We can still factorise the expression using the difference between two squares method.

<u>Grouping 1 by 3 method:</u>

For example,
Factorise $25y^2 - x^2 - 6x - 9$.

$$25y^2 - x^2 - 6x - 9 = 25y^2 - (x^2 + 6x + 9)$$
$$= 25y^2 - [x^2 + 2(x)(3) + (3)^2]$$
$$= 25y^2 - (x + 3)^2$$
$$= (5y)^2 - (x + 3)^2$$
$$= [5y - (x + 3)](5y + x + 3)$$
$$= (5y - x - 3)(5y + x + 3)$$

Note:
We group the last three terms together and complete the square.

Note:
We have to multiply the minus sign outside the bracket with $(x + 3)$.