CHAPTER 6

Algebraic Fractions and Formulae

Key Notes

6.1 Algebraic Fractions

When the numerator and denominator of a fraction is multiplied or divided by the same non-zero expression, the value of the fraction is unchanged.

$$\frac{x}{y} = \frac{ax}{ay} = \frac{x \div a}{y \div a}$$

6.2 Multiplication and Division of Algebraic Fractions

$$\frac{x}{y} \times \frac{a}{b} = \frac{ax}{by}$$

$$\frac{x}{y} \div \frac{a}{b} = \frac{x}{y} \times \frac{b}{a} = \frac{bx}{ay}$$

6.3 Addition and Subtraction of Algebraic Fractions

Similar to adding and subtracting numerical fractions, the denominators of algebraic fractions should be equal in order to add and subtract them.

For example,

$$\frac{x+y}{2} + \frac{x}{5} = \frac{5(x+y)+2x}{10} = \frac{5x+5y+2x}{10} = \frac{7x+5y}{10}$$

← Common denominator $= 2 \times 5 = 10$

$$\frac{x}{a} - \frac{2x}{b} = \frac{bx-2ax}{ab}$$

← Common denominator $= ab$

$$\frac{3}{x+1} + \frac{4x}{x-1} = \frac{3(x-1)+4x(x+1)}{(x+1)(x-1)} = \frac{3x-3+4x^2+4x}{(x+1)(x-1)} = \frac{4x^2+7x-3}{(x+1)(x-1)}$$

← Common denominator $= (x+1)(x-1)$

6.4 Manipulation of Algebraic Formulae

1. Changing the subject of a formula

For example, in $p = \frac{2+q}{4r} - 1$, make q the subject.

$$p = \frac{2+q}{4r} - 1$$
$$p + 1 = \frac{2+q}{4r}$$
$$4r(p+1) = 2 + q$$
$$q = 4pr + 4r - 2$$

2. Solving equations involving algebraic fractions

For example, solve for x in $\frac{2}{x+1} + \frac{3x}{4} = 2$.

$$\frac{2}{x+1} + \frac{3x}{4} = 2$$
$$\frac{4 \times 2 - 3x(x+1)}{4(x+1)} = 2$$ ← Common denominator = $4(x + 1)$
$$\frac{8 - 3x^2 - 3x}{4(x+1)} = 2$$
$$8 - 3x^2 - 3x = 8(x+1)$$
$$8 - 3x^2 - 3x = 8x + 8$$
$$3x^2 + 8x + 3x + 8 - 8 = 0$$
$$3x^2 + 11x = 0$$ ← To solve a quadratic equation, the expression must equate to zero.
$$x(3x + 11) = 0$$
$$x = 0 \quad \text{or} \quad 3x + 11 = 0$$
$$3x = -11$$
$$x = -\frac{11}{3}$$