8 RATE AND SPEED

LEARNING OBJECTIVES

In this topic, we will learn to:

- understand the concept of average rate
- solve problems related to rate
- understand the relationship between distance, time and speed
- write speeds in various units such as km/h, m/min, m/s and cm/s
- convert from one unit to another
- compute speed, distance or time given two of the three variables
- understand the concept of speed, average speed and uniform speed
- solve problems related to speed

8.1 RATE AND AVERAGE RATE

A rate compares two quantities of different kinds. Rate is usually expressed as one quantity per unit of another.

The formula for average rate is:

$$\text{Average rate} = \frac{\text{Sum of quantity A}}{\text{Sum of quantity B}}$$

WORKED EXAMPLE 1

The table below shows the rates of charges for water consumption in a month.

Usage	First 50 m^3	Every additional m^3
Rate	\$1.62/m^3	\$2.12/m^3

A household uses 64 m^3 of water in a month. Calculate the average water consumption rate of this household.

Worked Solution:

Cost of the first 50 m^3 = 50 × \$1.62
= \$81
Cost of the next 14 m^3 = 14 × \$2.12
= \$29.68

$$\text{Average water consumption} = \frac{\text{Total cost}}{\text{Total volume}}$$
$$= \frac{\$81 + \$29.68}{64\ m^3}$$
$$= \frac{\$110.68}{64\ m^3}$$
$$= \mathbf{\$1.73/m^3} \text{ (nearest cent)}$$

WORKED EXAMPLE 2

A lorry travelling 420 km from Town A to Town B on the North-South Highway has to pay a toll charge of S\$57.12. What is the rate of the toll charge in RM/km if S\$1 = RM2.40?

Worked Solution:

$$\text{Rate of toll charge} = \frac{\text{Total cost}}{\text{Total distance}}$$
$$= \frac{S\$57.12}{420\ km}$$
$$= S\$0.136/km$$
$$= 0.136 \times 2.40$$
$$= \mathbf{RM0.3264/km}$$

8.2 SPEED AND AVERAGE SPEED

To calculate speed, we need the following information:

(a) Distance — how far the object moved
(b) Time — time taken for the object to move

The formula for speed is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The formula for average speed is:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

8.3 CONVERSION OF UNITS

WORKED EXAMPLE 3

Convert the following km/h to m/s.

(a) 30.6 km/h (b) 51 km/h (c) 120 km/h

Worked Solution:

(a) 30.6 km/h:

1 h = 30.6 km
1 h = 30 600 m
60 min = 30 600 m
1 min = 30 600 ÷ 60
= 510 m
60 s = 510 m
1 s = 510 ÷ 60
= 8.5 m
30.6 km/h = **8.5 m/s**

(b) 51 km/h:

1 h = 51 km
1 h = 51 000 m
60 min = 51 000 m
1 min = 51 000 ÷ 60
= 850 m
60 s = 850 m
1 s = 850 ÷ 60
≈ 14.167
51 km/h = **14.2 m/s** (3 sig.fig.)

(c) 120 km/h:

1 h = 120 km
1 h = 120 000 m
60 min = 120 000 m
1 min = 120 000 ÷ 60
= 2000 m
60 s = 2000 m
1 s = 2000 ÷ 60
≈ 33.33 m/s
120 km/h = **33.3 m/s** (3 sig.fig.)

WORKED EXAMPLE 4

Convert the following m/s to km/h.

(a) 5 m/s (b) 18 m/s (c) 23 m/s

Worked Solution:

(a) 5 m/s:

$$\begin{aligned} 1 \text{ s} &= 5 \text{ m} \\ 3600 \text{ s} &= 5 \times 3600 \\ &= 18\,000 \text{ m} \\ &= 18 \text{ km} \\ 1 \text{ h} &= 18 \text{ km} \\ 5 \text{ m/s} &= \mathbf{18 \text{ km/h}} \end{aligned}$$

(b) 18 m/s:

$$\begin{aligned} 1 \text{ s} &= 18 \text{ m} \\ 3600 \text{ s} &= 18 \times 3600 \\ &= 64\,800 \text{ m} \\ &= 64.8 \text{ km} \\ 1 \text{ h} &= 64.8 \text{ km} \\ 18 \text{ m/s} &= \mathbf{64.8 \text{ km/h}} \end{aligned}$$

(c) 23 m/s:

$$\begin{aligned} 1 \text{ s} &= 23 \text{ m} \\ 3600 \text{ s} &= 23 \times 3600 \\ &= 82\,800 \text{ m} \\ &= 82.8 \text{ km} \\ 1 \text{ h} &= 82.8 \text{ km} \\ 23 \text{ m/s} &= \mathbf{82.8 \text{ km/h}} \end{aligned}$$

8.4 PROBLEMS INVOLVING SPEED

WORKED EXAMPLE 5

An aeroplane travels at 2000 km/h from City X to City Y which is 3000 km apart.

(a) Find out the time it takes for the plane to travel from City X to City Y.

(b) If the speed of the plane increased by 25%, how much time did it save to travel between the two cities?

Worked Solution:

(a) $\text{Time taken} = \dfrac{\text{Distance}}{\text{Speed}}$

$$= \frac{3000}{2000}$$

$$= \frac{3}{2}$$

$$= \mathbf{1.5 \text{ hours}}$$

(b) New speed = 125% of original speed

$$= \frac{125}{100} \times 2000$$

$$= 2500 \text{ km/h}$$

$$\text{New time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{3000}{2500}$$

$$= \frac{6}{5}$$

$$= 1.2 \text{ hours}$$

Time saved = 1.5 – 1.2
= 0.3 hours
= **18 minutes**

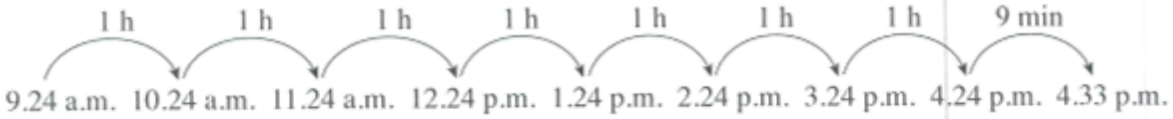
Worked Example 6

Raymond set out on a journey at 9.24 a.m. He travelled on a train for 286 km at an average speed of 40 km/h. At what time did he reach his destination?

Worked Solution:

$$\text{Time taken} = \frac{286}{40}$$

= 7.15 hours
= 7 hours and 9 minutes



He reached his destination at **4.33 p.m.**

WORKED EXAMPLE 7

A tour coach travelled from Singapore to Kuala Lumpur. During the first three hours, it travelled at an average speed of 67 km/h. During the next four hours, it travelled at an average speed of 73 km/h. Calculate the distance travelled.

Worked Solution:

Distance travelled in first 3 hours = Speed × Time
= 67 × 3
= 201 km

Distance travelled in next 4 hours = Speed × Time
= 73 × 4
= 292 km

Total distance = 201 + 292
= **493 km**

PRACTICE QUESTIONS

1. A tap can fill up a tank with a capacity of 200 *l* in 50 minutes. Calculate the rate of flow of water, in *l*/min, of the tap.

2. Peter typed 54 words in the first minute, 47 words in the second minute and 64 words in the third minute. What is his average rate of typing?

3. Geraldine read 145 pages on Monday, 45 pages on Tuesday and 245 pages on Wednesday. What is her average rate of reading per day for the three days?

4. The basic taxi fare rate in the year 2012 is shown in the table below.

Flag-down (inclusive of first km or less)	$3.30
Every 225 metres thereafter or less up to 10 km	$0.15
Every 200 metres thereafter (after 10 km)	$0.10

Calculate the average rate in dollars per kilometre ($/km) for a 25-km journey.

5. The table below shows the rates of charges for water consumption in a month.

Usage	First 50 m^3	Every additional m^3
Rate	\$1.65/m^3	\$2.20/m^3

A family uses 100 m^3 of water in a month. Calculate the average water consumption rate of this family.

6. Given the distances and times, calculate the speeds. Express your answers in the units as shown in the brackets.

(a) 120 km, 3 h (km/h)
(b) 250 km, 10 h (km/h)
(c) 180 km, 4.5 h (km/h)
(d) 80 km, 0.8 h (km/h)
(e) 3 km, $\frac{1}{2}$ h (km/h)
(f) 15 km, 20 min (km/h)
(g) 18 km, 40 min (km/h)
(h) 33 km, 36 min (km/h)
(i) 0.42 km, 12 min (m/min)
(j) 3.6 km, 1.2 h (m/min)
(k) 6 km, 1 h 15 min (m/min)
(l) 8.4 km, 70 min (m/min)
(m) 15 m, 2 s (m/s)
(n) 0.4 km, 50 s (m/s)
(o) 6.2 km, 5 min 10 s (m/s)
(p) 72 km, 1 h (m/s)

7. Given the speeds and times, find the distances travelled. Express your answers in the units as shown in the brackets.

(a) 20 km/h, 4 h (km)
(b) 80 km/h, 1.5 h (km)
(c) 320 km/h, $\frac{1}{2}$ h (km)
(d) 156 km/h, 3 h (km)
(e) 200 m/min, 1.5 h (km)
(f) 10 m/min, 70 min (m)
(g) 1.5 m/min, 1 h 10 min (m)
(h) 12 m/min, 2.3 h (m)
(i) 12 m/s, 4 min (km)
(j) 36 m/s, $\frac{1}{2}$ h (km)

8. Given the distances and speeds, find the times taken. Express your answers in the units as shown in the brackets.

(a)	30 km, 15 km/h (h)	(b)	45 km, 18 km/h (h)
(c)	50 km, 30 km/h (h and min)	(d)	62 km, 40 km/h (h and min)
(e)	1500 m, 10 km/h (min)	(f)	2000 m, 25 km/h (min)
(g)	300 m, 20 m/s (s)	(h)	1350 m, 75 m/s (s)
(i)	4.2 km, 120 m/s (min)	(j)	2 km 880 m, 24 m/s (min)
(k)	2 km, 25 m/s (min and s)	(l)	4275 m, 45 m/s (min and s)

9. The distance between Town A and Town B is 405 km. Peter took 2 hours to drive the first 185 km and 2.5 hours to complete the remaining journey. Find the average speed.

10. Michael took $\frac{3}{4}$ hours to drive from Jurong to Changi which is 45 km apart. On his return journey, he took 1 hour when travelling on the same route. What was his average speed?

11. Jacky took 3 hours to cover 305 km and 2.5 hours to finish the remaining 300 km. What was his average speed?

12. During a 42-km marathon race, Patrick took 2 hours 15 minutes to run the first half. He then finished the remaining half in 1 hour 45 minutes. What was his average speed in km/h?

13. During the All Women 12-km Cross Country Run, Sharon took 35 minutes to finish the first 4 km and 55 minutes for the remaining distance. What was her average speed?

14. Susan travelled at 100 km/h for 1 hour before she slowed down to 88 km/h for $\frac{1}{2}$ hour. What was her average speed?

15. Roger was travelling at 120 km/h for 45 minutes before reducing his speed to 90 km/h for the next half hour. Find his average speed.

16. A bullet train travels at 200 km/h for $\frac{3}{4}$ hour from Station X to Station Y. It then continues its journey to Station Z, which is 300 km away, in 1 hour 15 minutes. Find its average speed.

17. A long journey bus was travelling at 85 km/h for the first 2 hours before decreasing its speed to 65 km/h for the next 3 hours. What was the average speed?

18. Express the following speeds in m/s.

(a) 18 km/h
(b) 27 km/h
(c) 36 km/h
(d) 54 km/h
(e) 72 km/h
(f) 108 km/h
(g) 126 km/h
(h) 162 km/h

19. Express the following speeds in km/h.

(a) 10 m/s
(b) 12 m/s
(c) 25 m/s
(d) 32 m/s
(e) 40 m/s
(f) 50 m/s
(g) 75 m/s
(h) 85 m/s

20. The distance between two towns is 230 km. Helen drove at an average speed of 75 km/h. If she left one town at 11 a.m., what time did she arrive at the other town?

21. When Larry left home for the airport, the time was 13 10. His house is 36 km away from the airport. He took a taxi and travelled at an average speed of 80 km/h. What time did he arrive at the airport?

22. Travelling at an average speed of 94 km/h, Violet arrived at her destination at 1809 hours. What time did she start her journey if the distance she travelled was 211.5 km?

23. Nancy drove at an average speed of 70 km/h for 3 hours. In the first part of the journey, she travelled 60 km/h for 1.25 hours and the second part at x km/h for 1 hour and 30 minutes. Find the value of x.

24. Bobby travelled at an average speed of 87 km/h for 3 hours 20 minutes. In the first part of the journey, he travelled at a constant speed of 75 km for 1.5 hours. For the remaining journey, he travelled at x km/h for 2.5 hours. What is the value of x?

25. Linda cycles to school every day. Her school is 12 km away from her house. On a particular day, she cycled constantly at a speed of 300 m/min for 25 minutes before reducing her speed to x m/min for the rest of the journey. She spent 45 minutes in total. What is the value of x?