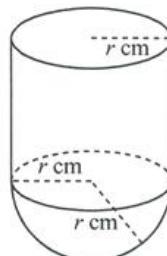




New Trend Modelling Questions

1. A chemical plant wants to produce a container to store toxic chemicals. The container consists of a closed cylindrical container with a hemispherical base as shown in the diagram below.



The cost of each cm^2 of the circular top costs $\$P$ and the cost of each cm^2 of the curved surface and hemispherical base is $\$3P$. The total cost of the container is a fixed amount, $\$C$.

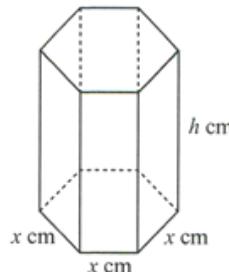
- Given that the radius of the hemisphere is r cm, show that the volume V of the container can be expressed as $V = \frac{Cr}{6P} - \frac{\pi r^3}{2}$.

Given further that cost is fixed at $\$250$, $P = 2$, and $r = 2.5$,

- find the volume of 1 such container.
- Suppose that there is another option to manufacture the container such that the volume of 1 such container is 30 cm^3 and the radius is 5 cm, what will be the total cost of the container? Which option is more cost saving to manufacture?

New Trend Modelling Questions

2. Candy is being stored in a closed container in the form of a regular hexagonal prism with sides x cm and height h cm, as shown in the figure below.



(a) Show that the base area is given by $\frac{3\sqrt{3}x^2}{2}$ cm². [Hint: $\tan 30^\circ = \frac{1}{\sqrt{3}}$]

(b) Given that the container has a volume of 972 cm³, find an expression in terms of x for the total surface area, A cm², of the container.

(c) It is required that $x = 5$ and the cost of the material for the container is \$0.85 per 100 cm² as quoted by manufacturer X . To ensure that costs are minimised, the management obtained a quotation from manufacturer Y which offers to manufacture containers for them at a cost of \$500 for every 100 containers. Which manufacturer, X or Y , is more cost saving for the management?

3. In a simplified version, a crane can be represented by a rectangle $ABCD$ of height 11 m and a jib of length 20 m. The jib can be rotated in a vertical plane about B . A vertical cable RS , carries a load S . Figures 1 and 2 below show the two possible positions of the jib, cable and load.

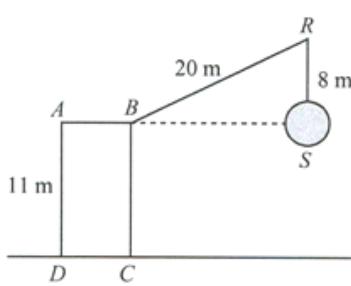


Figure 1

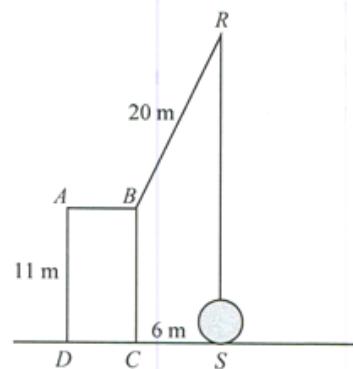


Figure 2

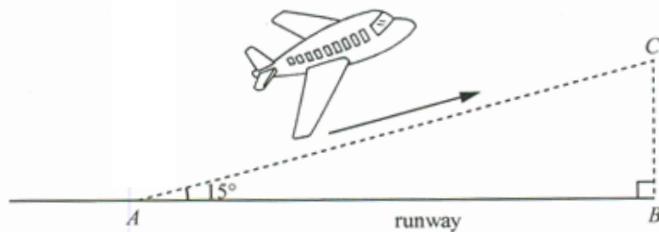
(a) Figure 1 shows the case when BS is horizontal and $RS = 8$ m. Calculate the distance BS .

(b) Figure 2 shows the case when the jib, BR , has been rotated and the length RS increases. The load, S , is on the ground at 6 m from C . Calculate the length of RS .

(c) What is the angle that the jib has rotated from figure 1 to figure 2?

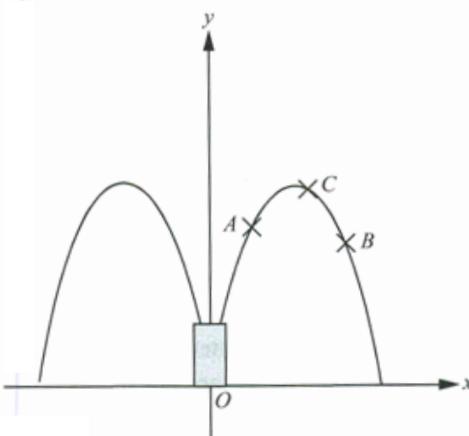
New Trend Modelling Questions

4. The authorities are designing the layout of a new airport and its neighbouring buildings and structures. In a simplified plan, an aeroplane takes off at an angle of 15° from a runway at point A . The average takeoff speed of the aeroplane is 78 m/s and C is a point vertically above point B on the horizontal ground. The aeroplane takes half a minute to climb from point A to point C .



(a) Find the distance AC in km.
 (b) Find the height BC in m.
 (c) For safety purposes, buildings and structures near the airport should not exceed 1 km in height. Assuming that the maximum takeoff angle of the aeroplane is 15° and takeoff time is half a minute, find the maximum distance of AC , giving your answer in km. Hence, find the maximum takeoff speed of the aeroplane, giving your answer in m/s.

5. The diagram below shows the trajectory of a water fountain modelled after a Cartesian plane.

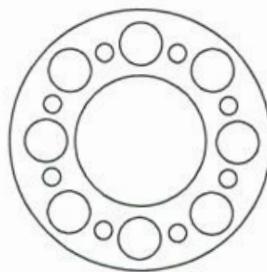


(a) Given that the equation of the water path follows a quadratic graph of the form $y = ax^2 + bx + c$ and the graph passes through the points $A(2, 18)$, $B(6, 14)$ and $C(4, 20)$, find the values of the constants, a , b and c .
 (b) Find the coordinates of the point where the curve cuts the x -axis for $x \geq 0$.
 (c) Suppose the circular enclosure around the water fountain needs to be designed such that the water does not spill out of the enclosure. If it is required that the inner edge of the enclosure should be 0.5 unit away from the point where the water first hits the horizontal, find the minimum area of the circular enclosure.

New Trend Modelling Questions

6. A manufacturer of iron ball bearings stated that the cost of making ball bearing is directly proportional to the cube of the radius of the ball. It is given that the cost of making a ball bearing of radius 4 cm is \$160.

- Find the cost of making a ball bearing of radius 5 cm.
- A ball bearing of radius 4 cm is sold for \$200. A ball bearing of radius 5 cm is sold for \$375. Which ball bearing has a higher percentage profit on its cost?
- In order to make the gear as shown in the figure below, the manufacturer needs to use 8 4-cm ball bearings and 8 2-cm ball bearings. The alloy used to make the plate to hold the ball bearings costs \$15 per cm^3 and the gear has a thickness of 4 cm, outer radius of 15 cm and inner radius of 4 cm. Find the cost of making 1 such gear, giving your answer correct to the nearest whole number.

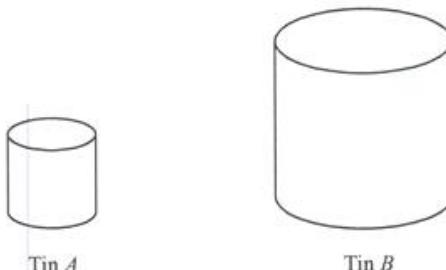


7. Linear graphs are often seen in the study of economies, in particular, the demand and supply of the market. The demand relationship curve illustrates the negative relationship between the price and the quantity demanded. The higher the price of a good, the lower the quantity demanded and the lower the price, the more the good will be in demand. For supply relationship, the higher the price, the higher the quantity supplied. Producers supply more at a higher price because selling a higher quantity at a higher price increases revenue. Demand and supply relationships are often represented using linear graphs.

- For a particular brand of mobile phone, the demand relationship is given by $y = 17 - 3x$, and the supply relationship is represented by $y = x + 5$. On the same diagram, sketch the graphs of $y = 17 - 3x$ and $y = x + 5$.
- State the coordinates of the point of intersection of the two graphs.
- Explain briefly the significance of the answer in (b).

New Trend Modelling Questions

8. Two tins of condensed milk shown below are geometrically similar to each other. Their heights are 10 cm and 18 cm respectively.



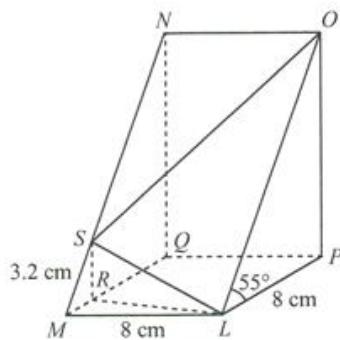
(a) Given that the radius of tin A is 12 cm, find the radius of tin B .

(b) The cost of a tin A is \$1.25. Find the cost of a tin B , assuming that there is no difference in the cost per unit of milk for both tins, giving your answer correct to the nearest 10 cents.

[Hint: $\frac{\text{Volume of tin } A}{\text{Volume of tin } B} = \left(\frac{\text{Height of tin } A}{\text{Height of tin } B}\right)^3$]

(c) Mrs Li has used up $\frac{3}{4}$ of the milk in tin B that she bought. Find the area of the tin in contact with the leftover milk, leaving your answer correct to 2 decimal places.

9. The diagram below shows a model of a ramp, built to a scale of 1: 50. $LMQP$ is a horizontal square plane of side 8 cm. $LMNO$ is a rectangular plane inclined at 55° to $LMQP$. S is a point on MN such that $MS = 3.2$ cm.



(a) Find

- LS ,
- $\angle RLS$,
- OL ,
- OS ,
- $\angle OLS$,
- the area of $\triangle LOS$.

(b) If the ramp is to be painted and each tin of paint, costing \$8, can cover an area of 5 m^2 , how much will it cost to paint the ramp? Leave your answer correct to the nearest dollar.

New Trend Modelling Questions

10. Figure 1 shows a test tube made up of a hollow cylindrical part of height h cm and a hemispherical part of radius r cm. $42.55\pi \text{ cm}^3$ of water is needed to fill up an empty test tube to the brim. The hemispherical part of the test tube contains $\frac{1}{15}$ of the volume of the water in the cylindrical part of the test tube.

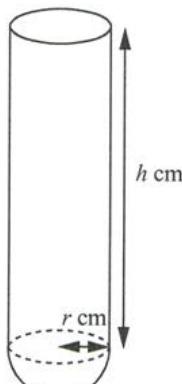


Figure 1

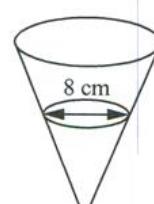


Figure 2

- (a) Find the values of h and r .
- (b) The water in the filled test tube is then poured into a right circular cone as shown in figure 2. If the diameter of the surface area of the water level is 8 cm, find
 - (i) the height of the water level in the cone,
 - (ii) the surface area of the cone that is in contact with the water.
- (c) Suppose that there is a leak in the cone at the vertex and water is dripping out at a rate of $2.5 \text{ cm}^3/\text{s}$. How long will it take for all the water to drip out completely, giving your answer correct to the nearest second?