Unit 1

Number and Algebra

63

1.1 Numbers and the Four Operations

Answer **all** questions. Show your workings clearly in the space provided.

1. Given that p, q and r are prime numbers,
   (a) find the highest common factor (HCF) of p^2q^2r and p^4q,
   (b) find the cube root of the product of p^3q^4r and q^2r^8.

Answer: (a) _____ [1]

(b) _____ [1]

2. When expressed as the product of their prime factors,

$$336 = 2^4 \times 3 \times 7,$$
$$700 = 2^2 \times 5^2 \times 7.$$

   (a) Find the greatest number which divides 336 and 700 exactly.
   (b) Find the smallest integer n such that $\sqrt{\frac{336 \times 700}{n}}$ is an integer.

Answer: (a) _____ [1]

(b) _____ [1]

3. (a) Express 5.74% as a fraction.
   (b) Convert 8.6 m/s to km/h.

Answer: (a) _____ [1]

(b) _____ [2]

4. Simplify each of the following, leaving your answer in positive index form.
   (a) $-2y \div 4x^{-5}$
   (b) $\left(\frac{p^2}{2r^3}\right)^{-2} \div \sqrt[4]{(r^6p^4)^3} \times (5p)^0$

Answer: (a) _____ [1]

(b) _____ [2]

5. Given that $5^x = 7$ and $5^y = 3$, find the value of
   (a) 5^{3y+x},
   (b) $25^{3+x-\frac{1}{2}y}$.

Answer: (a) _____ [1]

(b) _____ [2]

6. (a) Simplify $1 \div \left(\frac{7p^2}{4p^{-1}q^2}\right)^{-2} \times (8p^2q^3)^0$, leaving your answer in positive index form.
   (b) Write down the last digit of 3^{2018}.

Answer: (a) _____ [2]

(b) _____ [1]

7. Write down the last digit of
   (a) 2^{9999},
   (b) $2(5^{323}) + 7$.

Answer: (a) _____ [1]

(b) _____ [2]

8. (a) Solve the following equation.

$$\sqrt{3^x} \times 27^{x+1} \div 9^{3x-4} = \frac{1}{81}$$

   (b) Given that $(3x)^{2a} \times 5^{2a} = (xy)^a$, express y in terms of x.

Answer: (a) _____ [2]

(b) _____ [2]

9. (a) Simplify $\left(\frac{-2x^2y^{-3}}{5x^{-4}y^5}\right)^2 \div \left(\frac{3x^{-5}y^{-1}}{x^2y^6}\right)^{-3}$, leaving your answer in positive index form.

(b) Solve the equation $\left(\frac{1}{2}\right)^{x+1} \div 32^{x+2} \times 72 = 4^{2x} \times 3^2$.

Answer: (a) _____ [2]

(b) _____ [2]

10. (a) Simplify $\frac{(-2a^2b)^2}{5c} \div \left(\frac{3ac}{b^3}\right)^{-2} \times \frac{c^5b}{(8a)^0}$, leaving your answer in positive index form.

(b) Given that $\frac{\sqrt[3]{7} \times 343}{49^{\frac{2}{3}}} = \frac{1}{7^y}$, find the value of y.

Answer: (a) _____ [2]

(b) _____ [2]

11. (a) Simplify $\left(\frac{\sqrt[3]{x^4} \times 7y^0}{2x^{-1}y^{\frac{1}{3}}}\right)^{-2}$, leaving your answer in positive index form.

(b) Solve the following equation.

$$\frac{8^{x-1} \times 32^x}{4^{2-3x}} = \sqrt{2}$$

Answer: (a) _____ [2]

(b) _____ [2]

12. (a) Solve the following equation.

$$2 \times 9^{x-1} \div \sqrt[3]{3^x} = 54$$

(b) Given that $5^{2y-1} = \frac{1}{25}$, find the value of $7^{y+\frac{1}{2}}$.

Answer: (a) _____ [2]

(b) _____ [3]

13. A circular organism has a radius of (1.9×10^{-5}) metres.
    (a) (1.9×10^{-5}) metres can be written as k micrometres. Find the value of k.
    (b) 156 of these circular organisms are placed side by side along a straight line. What will be the total length formed by these organisms? Give your answer in centimetres and in standard form.

Answer: (a) _____ [1]

(b) _____ [2]

14. (a) Evaluate $\frac{\sqrt[3]{3.5826} \times (-2.5512)^2}{12.95}$, giving your answer correct to 4 significant figures.
    (b) The speed of light is about 300 million m/s. Convert this speed to km/h, leaving your answer in standard form.

Answer: (a) _____ [1]

(b) _____ [2]

15. (a) 140 000 000 can be written as k million. Find the value of k.

(b) Carmen bought a harddisk drive that has a storage size of 1 terabyte (TB). She used it to store 28 movies with an average size of 4.3 gigabytes (GB) each. She also stored 572 songs with an average size of 3.6 megabytes (MB) each. How much space did she have left in her hard disk drive? Give your answer in GB and in standard form.

Answer: (a) _____ [1]

(b) _____ [3]

16. It is given that $x = 2.21 \times 10^n$ and $y = 470 \times 10^n$. Find the value of each of the followings, expressing your answers in standard form and in terms of n.

(a) $2x + y$

(b) x^2y

Answer: (a) _____ [2]

17. (a) Convert 825 000 milligrams to gigagrams. Leave your answer in standard form.

(b) The distance between the Sun and Earth is about 149.6 million kilometres.

(i) Express this distance in metres, giving your answer in standard form.

(ii) Given that light is able to travel 300 Mm (megametre) in one second, find the time taken for light from the Sun to reach Earth after it has been emitted from the Sun's surface. Give your answer in minutes and seconds, rounded off to the nearest second.

Answer: (a) _____ [1]

(b)(i) _____ [1]

(ii) _____ [2]

18. The table below shows the estimated population and area of Iceland and Australia in 2018.

	Iceland	Australia
Population	337 780	24 769 768
Area (km^2)	1.03×10^5	7.6823×10^6

(a) The population of Iceland can be written as k million. Find the value of k.

(b) The population in Iceland has grown by 0.82% this year. Find the population of Iceland in 2017. Round off your answer to the nearest whole number.

(c) Which country has a higher average number of people per square kilometre? Show your working clearly.

Answer: (a) _____ [1]

(b) _____ [1]

(c) _____ [3]