Unit 1

Number and Algebra

62

1.3 Functions and Graphs

Answer **all** questions. Show your workings clearly in the space provided.

1. In each of the diagrams below, the point (0, 1) is marked. On each of these diagrams, sketch the graph of

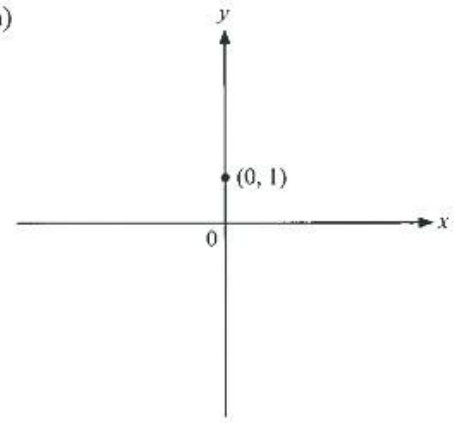
(a) $y = 2^x$ [1]

(b) $x^2y = 2$ [1]

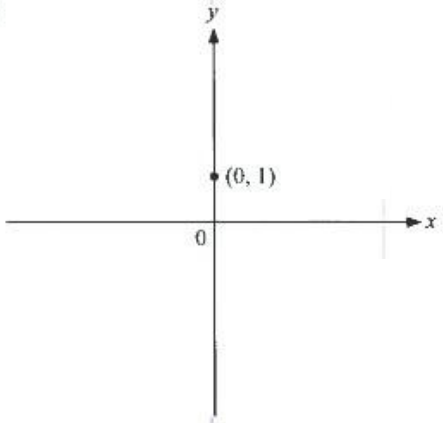
(c) $y = -\frac{1}{x}$ [1]

(d) $y = 1 - x^3$ [1]

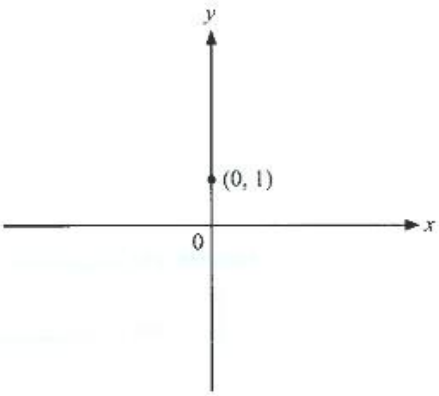
(a)



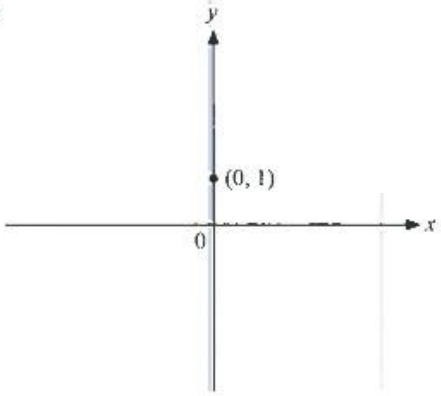
(b)



(c)



(d)



2. In each of the diagrams below, the point (1, 1) is marked with '•'. Using the diagrams, sketch the graph of

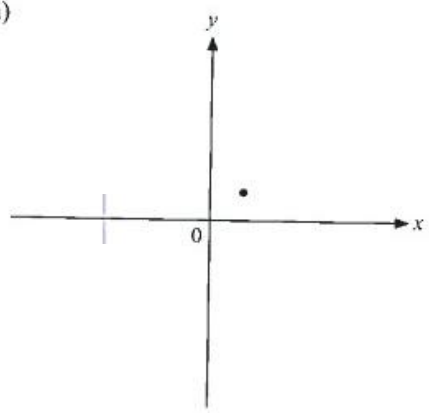
(a) $3y - 2x = 1$ [1]

(b) $y = \frac{2}{x}$ [1]

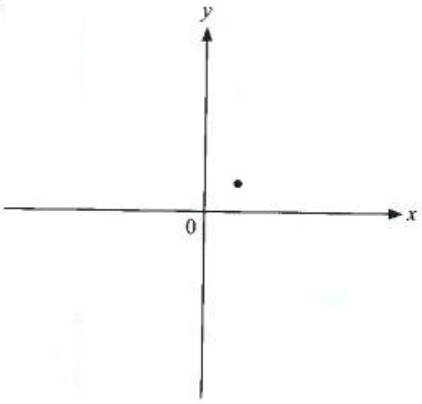
(c) $\frac{y}{x^3} = 1$ [1]

(d) $y = (0.5)^x$ [1]

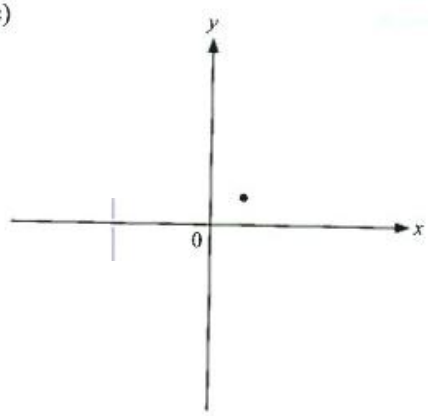
(a)



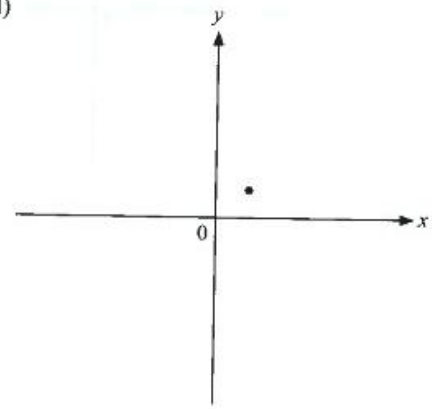
(b)



(c)

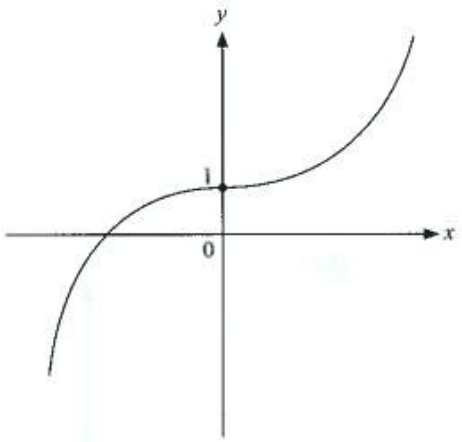


(d)

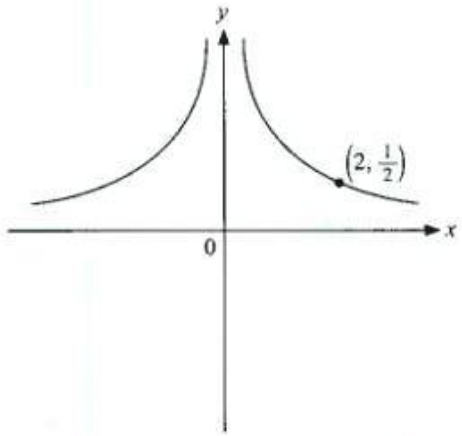


3. Write down a possible equation for each of the graphs shown below.

(a)



(b)

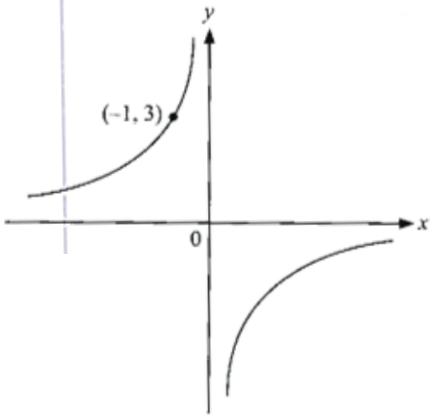


Answer: (a) _____ [1]

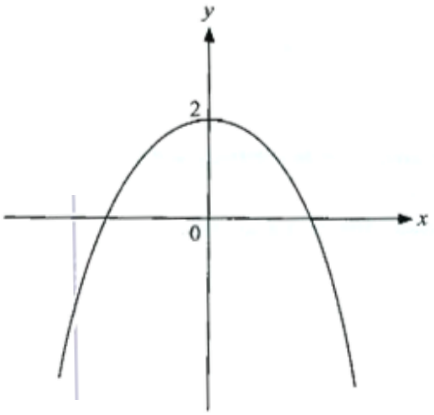
(b) _____ [2]

4. Write down a possible equation for each of the graphs shown below.

(a)



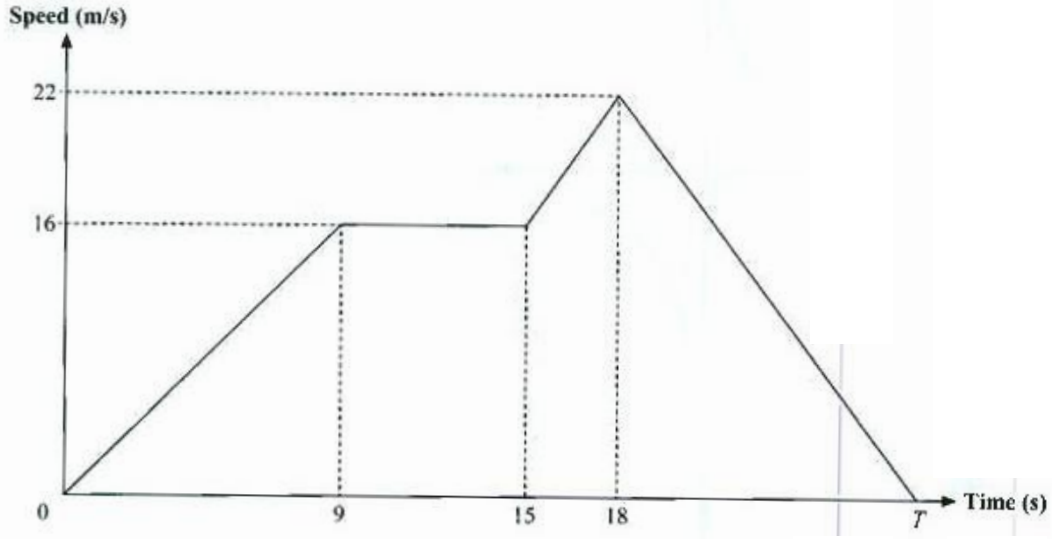
(b)



Answer: (a) _____ [2]

(b) _____ [1]

5. The diagram below shows the speed-time graph of a moving object. It is known that the total distance travelled by the object is 335 m.



(a) Find the value of T.

(b) Find the average speed of the object for the entire journey

(c) Find the speed of the object at 4 s.

(d) Find the acceleration of the object from 15 s to 18 s.

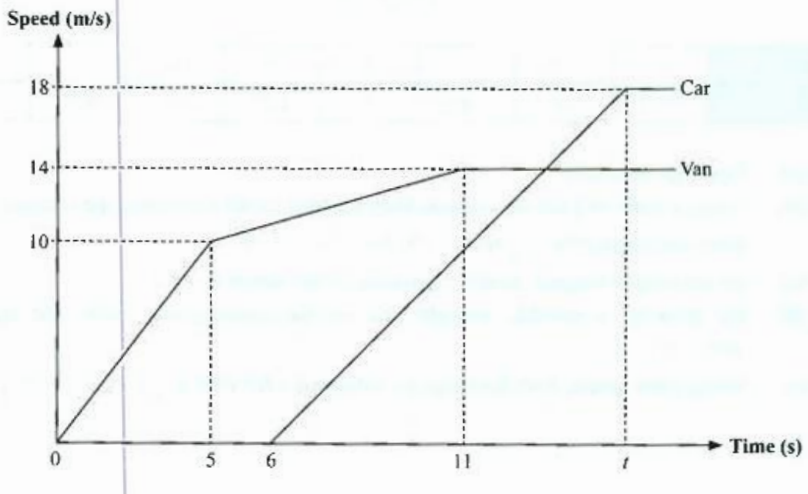
Answer: (a) _____ [2]

(b) _____ [1]

(c) _____ [2]

(d) _____ [1]

6. The diagram shows the speed-time graphs of a van and a car. The van started from rest and accelerated for 11 seconds before travelling at a constant speed of 14 m/s. 6 seconds after the van started travelling, the car accelerated from the same starting point as the van. Its rate of acceleration was thrice the acceleration of the van from 5 s to 11 s.



(a) Find the acceleration of the van from 5 s to 11 s.

(b) Find the value of t, given that t seconds represents the time at which the car first reached its constant speed of 18 m/s.

(c) Given that the car and the van were travelling on the same road, find the total time that the van would have to travel before the car and the van met.

Answer: (a) _____ [1]

(b) _____ [2]

(c) _____ [4]

7. **Answer the whole of this question on a sheet of graph paper.**
The following table shows some values of x and the corresponding values of y for the equation

$$y = \frac{1}{4}x(10 - x^2)$$

x	−3	−2	−1	0	1	2	3	4
y	−0.75	−3	p	0	2.25	3	0.75	−6

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit on both the horizontal x-axis and the vertical y-axis, draw the graph of $y = \frac{1}{4}x(10 - x^2)$ for $-3 \leqslant x \leqslant 4$. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 3$. [2]

(d) By drawing a suitable straight line on the same graph, solve the equation $x(10 - x^2) = 6$. [2]

(e) Using your graph, find the range of values of x for which $\frac{1}{4}x(10 - x^2) \geqslant \frac{1}{2}x + 1$. [3]

8. **Answer the whole of this question on a sheet of graph paper.**
The variables x and y are connected by the equation

$$y = \frac{1}{4}x^3 - \frac{1}{2}x - 5$$

Some values of x and the corresponding values of y are given in the following table.

x	−2	−1	0	1	2	3	4
y	p	−4.75	−5	−5.25	−4	q	9

(a) Calculate the value of p and of q. [2]

(b) Using a scale of 2 cm to represent 1 unit on the horizontal x-axis and 2 cm to represent 2 units on the vertical y-axis, draw the graph of $y = \frac{1}{4}x^3 - \frac{1}{2}x - 5$ for $-2 \leqslant x \leqslant 4$. [3]

(c) At $x = k$, the graph of $y = \frac{1}{4}x^3 - \frac{1}{2}x - 5$ has a gradient of 2. Find the values of k. [3]

(d) Solve each of the following equations by drawing a suitable straight line in each case where necessary.

(i) $\frac{1}{4}x^3 - \frac{1}{2}x - 5 = 0$ [1]

(ii) $x^3 - 2x - 8 = 0$ [2]

9. **Answer the whole of this question on a sheet of graph paper.**
The table below shows some values of x and the corresponding values of y of the equation

$$y = \frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x$$

x	−1	−0.5	0	0.5	1	1.5	2	2.5	3
y	p	−0.84	0	−0.16	−0.75	−1.22	−1	0.47	3.75

(a) Find the value of p. [1]

(b) Using a scale of 4 cm to represent 1 unit on the horizontal x-axis and a scale of 2 cm to represent 1 unit on the vertical y-axis, draw the graph of $y = \frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x$ for $-1 \leqslant x \leqslant 3$. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = -0.5$. [2]

(d) Use your graph to solve the equation $\frac{3}{4}x^3 + \frac{1}{2}x = 2x^2$. [2]

(e) (i) On the same axes, draw the graph of $y = 1 - x$. [1]

(ii) Write down the x-coordinate of the point where the two graphs intersect. [1]

(iii) The value of x in e(ii) is the solution of the equation $3x^3 + ax^2 + bx + c = 0$. Find the values of a, b and c. [3]