Unit 1

Number and Algebra

162

1.4 Solutions of Equations

Answer **all** questions. Show your workings clearly in the space provided.

1. One of the solutions of $7x^2 - nx - 10 = 0$ is $x = 5$.
   (a) Find the value of n.
   (b) Hence, find the other solution of $7x^2 - 33x - 10 = 0$.

Answer: (a) _____ [1]

(b) _____ [2]

2. (a) Express $2x^2 + 3x - 8$ in the form $(x + a)^2 + b$, where a and b are integers.
   (b) Hence, solve $2(2y + 3)^2 + 3(2y + 3) = 8$.

Answer: (a) _____ [1]

(b) _____ [3]

3. (a) Express $2x^2 - 7x - 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants.
   (b) Hence, solve the equation $2x^2 = 13 + 7x$. Round off your answers to 4 significant figures.

Answer: (a) _____ [2]

(b) _____ [3]

4. Solve the following equation.

$$\left(5 + \frac{9}{x}\right)(11x - 6) = 3 - [6 - 2(2 + x)]$$

Answer: _____ [4]

5. Solve each of the following equations.

(a) $\sqrt[3]{0.25^{x}} \times 32^{x-1} \div \frac{1}{3} = 24$

(b) $2(2x-1)^2 - 3(2x-1) = 5$

Answer: (a) _____ [2]

(b) _____ [3]

6. (a) Simplify $\left(-\frac{1}{3}a^{-1}b^{3}\right)^{2} \div \sqrt{a^{3}b^{5}} \times \frac{5a^{0}}{(2b^{-2})^{2}}$, expressing your answer in positive index form.

(b) (i) Express $x^2 - 8x + 11$ in the form $(x+a)^2 + b$, where a and b are integers.

(ii) Hence, solve the equation $2x^4 - 16x^2 = -22$.

Answer: (a) _____ [2]

(b)(i) _____ [1]

(ii) _____ [3]

7. Solve each of the following equations.

(a) $(2x + 3)^2 - 45 = 0$

(b) $\frac{1}{3x^2 - 12} - \frac{3}{x + 2} + \frac{7}{2 - x} = 1$

Answer: (a) _____ [2]

(b) _____ [4]

8. Solve each of the following equations.

(a) $x(x-5)-(2x+1)(x-2)=3$

(b) $\frac{5}{x-2}+\frac{x}{3x^2-5x-2}=1\frac{2}{7}$

Answer: (a) _____ [3]

(b) _____ [4]

9. Solve each of the following equations.

(a) $\left(\frac{2}{x}+1\right)(x+3)=-5$

(b) $x+4-\frac{5}{2x-7}=3$

Answer: (a) _____ [3]

(b) _____ [4]

10. Study the number pattern below.

Line 1: $1+\frac{1}{2}=\frac{3}{2}$

Line 2: $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

Line 3: $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$

Line 4: $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$

Line 5: $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$

⋮

(a) Write down line 10 of the pattern in a similar format.

(b) Write down line n of the pattern in a similar format.

(c) With reference to the pattern, find the values of x, y and z such that $\frac{1}{x}+\frac{1}{y}=\frac{z}{992}$.

(d) Find the value of $\frac{3}{2}-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}-\frac{13}{42}+\frac{15}{56}-\frac{17}{72}$ by using the number pattern. Show your working clearly.

Answer: (a) _____ [1]

(b) _____ [1]

(c) _____ [3]

(d) _____ [2]

11. Consider the number pattern below.

Line 1: $2^2 - 3 = 1$

Line 2: $3^2 - 7 = 2$

Line 3: $4^2 - 11 = 5$

Line 4: $5^2 - 15 = 10$

(a) Write down line 5 of the pattern in a similar format and evaluate it.

(b) The expression for the nth term, $T_n = an^2 + bn + c$, where a, b and c are constants. Find the values of a, b and c.

(c) Find the 28^{th} term of the sequence.

(d) Does the number 1268 lie in the sequence? Explain your answer. [3]

Answer: (a) _____ [1]

(b) _____ [3]

(c) _____ [1]

12. (a) (i) Factorise $12p - 3p^3$ completely.

(ii) Hence, solve $12p = 3p^3$.

(b) (i) Express $\frac{3}{2x^2+5x-3} + \frac{2}{x+3}$ as a single fraction in its simplest form.

(ii) Hence, solve $\frac{3}{2x^2+5x-3} + \frac{2}{x+3} = -1$.

Answer: (a)(i) _____ [2]

(ii) _____ [2]

(b)(i) _____ [2]

(ii) _____ [3]

13. On a particular day, a ship has a speed of 54 km/h in still waters. The speed of the ocean current on that day is x km/h.

(a) Find an expression, in terms of x, for the time needed by the ship to travel a distance of 72 km against the ocean current.

(b) Find an expression, in terms of x, for the time needed by the ship to travel a distance of 72 km along the same direction as the ocean current.

It is known that the difference in the time taken by the ship in parts (a) and (b) is 27 minutes.

(c) Form an equation in x, and show that it reduces to $x^2 + 320x - 2916 = 0$. [3]

(d) Solve the equation $x^2 + 320x - 2916 = 0$, rounding off your answers to 2 decimal places.

(e) Hence, calculate the time taken by the ship to travel 100 km against the ocean current on that particular day. Give your answer in hours, minutes and seconds, correct to the nearest second.

Answer: (a) _____ [1]

(b) _____ [1]

(d) _____ [3]

(e) _____ [1]

14. Mrs Lee bought x pencil cases and paid \$84 in total. All the pencil cases were of the same price.

(a) Write down and simplify an expression, in terms of x, for the price of each pencil case.

(b) Mrs Lee gave 4 pencil cases away and sold each of the remaining pencil cases at \$2.80 more than she paid for it. Write down and simplify an expression, in terms of x, for the total amount that she collected from the sale of the remaining pencil cases.

(c) Given that Mrs Lee made a profit of 50%, form an equation in x and show that it reduces to $x^2 - 19x - 120 = 0$. [3]

(d) Solve the equation $x^2 - 19x - 120 = 0$.

(e) Hence, find

(i) the cost price of each pencil case,

(ii) the percentage profit that Mrs Lee would have made if she sold all the pencil cases without giving any away.

Answer: (a) _____ [1]

(b) _____ [2]

(d) _____ [2]

(e)(i) _____ [1]

(ii) _____ [1]

15. Jie Lun can complete a puzzle in t min. Nurul is able to complete an identical puzzle 90 s before Jie Lun.

(a) Find, in terms of t, the fraction of the puzzle that Jie Lun can complete in 1 min.

(b) Find, in terms of t, the fraction of the puzzle that Nurul can complete in 1 min.

If Nurul and Jie Lun work on the puzzle together, they would take 3 min to complete the puzzle.

(c) Form an equation in t, and show that it reduces to $2t^2 - 15t + 9 = 0$. [3]

(d) Solve the equation $2t^2 - 15t + 9 = 0$. Round off your answer to 3 decimal places.

(e) Explain why one of the solutions in (d) is not applicable for this question. [1]

(f) Find the time taken by Nurul to complete the puzzle if he works alone.

Answer: (a) _____ [1]

(b) _____ [1]

(d) _____ [3]

(f) _____ [1]

16. Ismail and Jun Wei were cycling from town A to town B which is 72 km apart. For the first half of the journey, each of them cycled at an average speed of x km/h. For the rest of the journey, Ismail's average speed increased by 3 km/h while Jun Wei's average speed decreased by 2 km/h.

(a) Express in terms of x, the total time taken (in hours) by Ismail to cycle from town A to town B.

(b) Express in terms of x, the total time taken (in hours) by Jun Wei to cycle from town A to town B.

The difference in time taken by Ismail and Jun Wei was 21.6 min.

(c) Form an equation in x, and show that it reduces to $x^2 + x - 506 = 0$. [3]

(d) Solve the equation $x^2 + x - 506 = 0$.

(e) Hence, find the mean time taken by Ismail and Jun Wei to cycle from town A to town B. Give your answer in hours and minutes, correct to the nearest minute.

Answer: (a) _____ [1]

(b) _____ [1]

(d) _____ [3]

(c) _____ [2]

17. In 2016, Vincent spent a total of \$210 on x books.
In 2017, Vincent spent 24% more than in 2016 and purchased 2 more books than he did in 2016.

(a) Write down an expression, in terms of x, for the average cost of 1 book in 2016.

(b) Write down an expression, in terms of x, for the average cost of 1 book in 2017.

The average cost of each book in 2016 is 21 cents more than the average cost of each book in 2017.

(c) Form an equation in x, and show that it reduces to $x^2 + 242x - 2000 = 0$. [3]

(d) Solve the equation $x^2 + 242x - 2000 = 0$.

(e) In 2018, Vincent spent 50% more on books than in 2017. The number of books that he bought in 2018 was the sum of the number of books that he bought in 2016 and 2017. Find the average cost of each book in 2018.

Answer: (a) _____ [1]

(b) _____ [1]

(d) _____ [3]

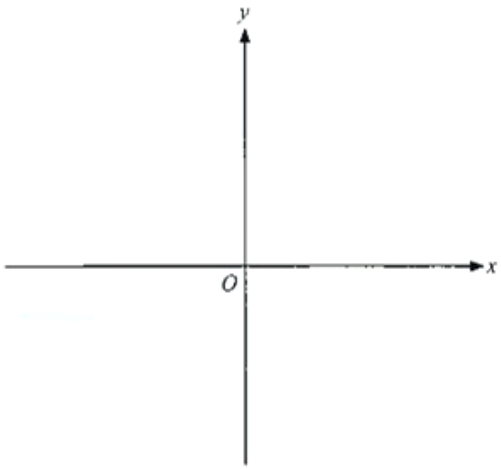
(e) _____ [2]

18. (a) Express $7 - 5x - x^2$ in the form $a - (x - b)^2$, where a and b are constants.
(b) Hence,
(i) state the coordinates of the maximum point of $y = 7 - 5x - x^2$,
(ii) sketch the graph of $y = 7 - 5x - x^2$ on the axes below. [3]

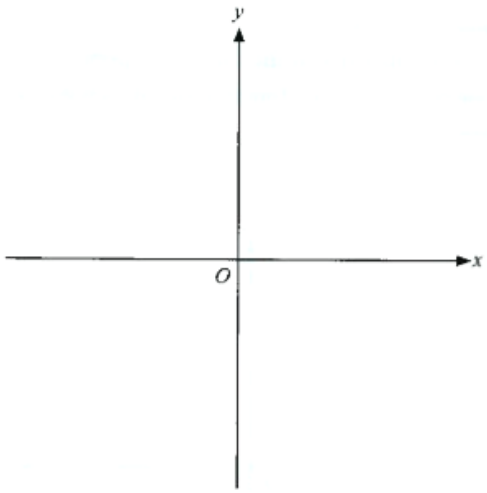
Answer: (a) _____ [1]

(b)(i) _____ [1]

Answer: (b)(ii)



19. (a) Sketch the graph of $y = 3(x - 1)(5 + 2x)$ on the axes below, indicating clearly the x-and y-intercepts as well as the turning point. [3]



(b) State the equation of the line of symmetry of $y = 3(x - 1)(5 + 2x)$.

(c) The equation $3(x - 1)(5 + 2x) = k$ has no solution. State the greatest possible integer value of k.

Answer: (b) _____ [1]

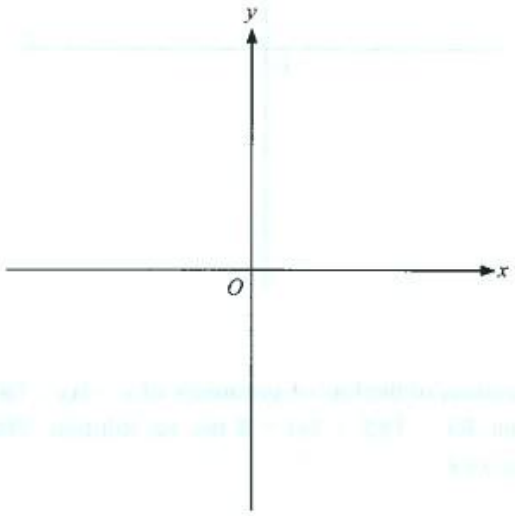
(c) _____ [1]

20. The equation of a graph is $y = ax^2 + bx + c$ where a, b and c are integers. It is given that $a^2 = 4$, the curve cuts the x-axis at $x = 1$ and the x-coordinate of the maximum point of the curve is -2.

(a) Find the values of a, b and c.

(b) Hence, find the coordinates of the maximum point of $y = ax^2 + bx + c$.

(c) Using the values of a, b and c found in (a), sketch the graph of $y = ax^2 + bx + c$ on the axes below. [2]



Answer: (a) _____ [4]

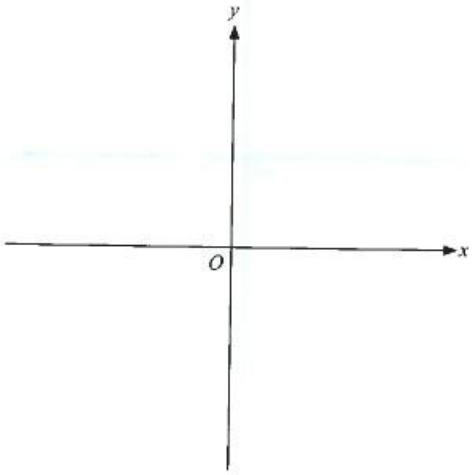
(b) _____ [1]

21. (a) Express $10 - 3x - x^2$ in the form $a - (x + b)^2$, where a and b are constants.

(b) Hence,

(i) state the equation of the line of symmetry of $y = 10 - 3x - x^2$,

(ii) sketch the graph of $y = 10 - 3x - x^2$, indicating clearly the x- and y-intercepts and the turning point of the curve. [3]

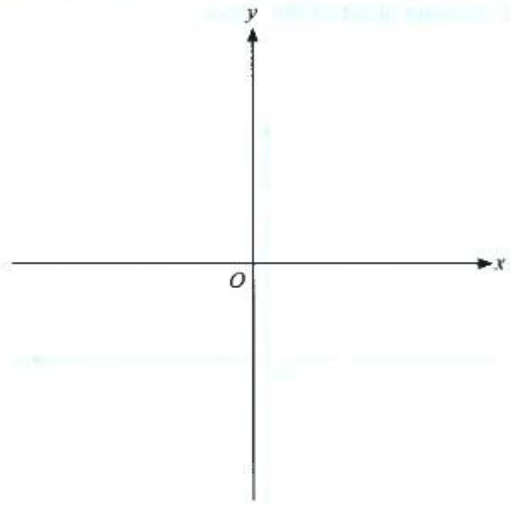


Answer: (a) _____ [1]

(b)(i) _____ [1]

22. (a) Sketch the graph of $y = \frac{1}{2}x(5x - 8)$ on the axes below, indicating clearly the intercepts and the turning point of the curve. [3]

(b) Hence, state the number of solutions to the equation $x(5x - 8) = -4$.



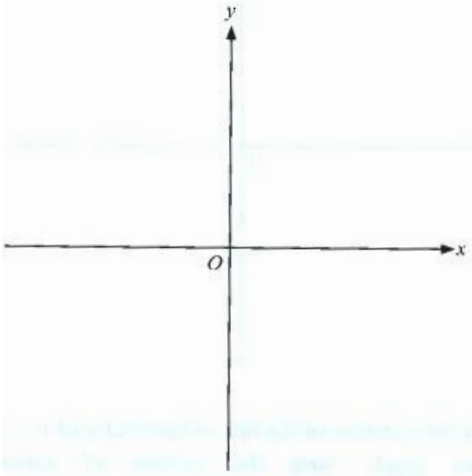
Answer: (b) _____ [1]

23. (a) Express $2x^2 - 8x - 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

(b) Hence,

(i) state the coordinates of the minimum point of $y = 2x^2 - 8x - 7$,

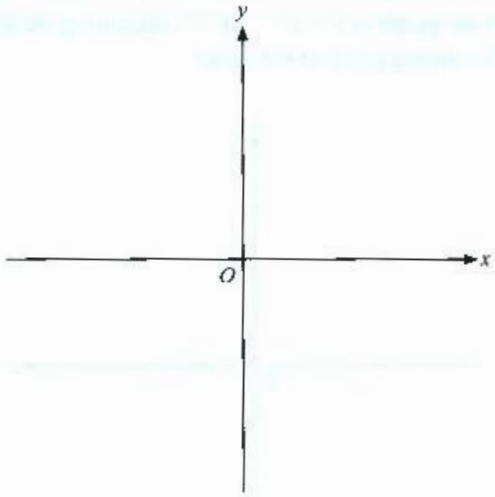
(ii) sketch the graph of $y = 2x^2 - 8x - 7$, indicating clearly the x- and y-intercepts and the turning point of the curve. [3]



Answer: (a) _____ [1]

(b)(i) _____ [1]

24. (a) Sketch the graph of $y = 11x + 2x^2$ on the axes below, indicating clearly the intercepts and the turning point of the curve. [3]



(b) Write down the equation of the line of symmetry of $y = 11x + 2x^2$.

(c) Using your graph, state the number of solutions to the equation $4x^2 + 22x + 30.25 = 0$.

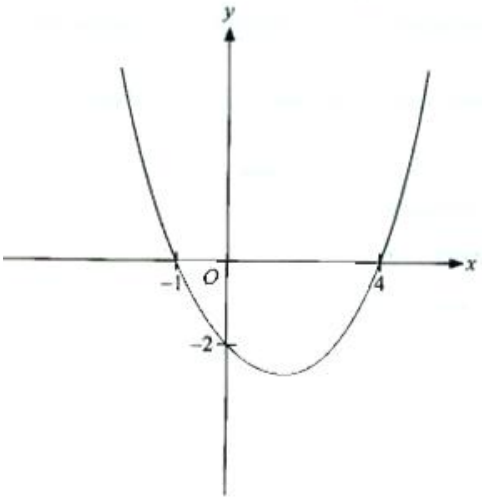
Answer: (b) _____ [1]

(c) _____ [1]

25. The equation of the curve below is $y = px^2 + qx + r$, where p, q and r are constants.

(a) Find the values of p, q and r.

(b) Hence, find the distance between the minimum point of the curve and the origin.



Answer: (a) _____ [4]

(b) _____ [2]