Secondary 3 Express

Worked Solutions

Last-Minute Maths Topical Drills

Unit 1 Number and Algebra

1.1 Numbers and the Four Operations

1. (a) HCF of p^2q^2r and p^4q = $\boldsymbol{p^2q}$ [1]

Must-Know Concept:
To find the HCF, we pick the factor(s) common to p^2q^2r and p^4q with the **smaller index**.

(b) $\sqrt[3]{p^3q^4r \times q^2r^8} = \sqrt[3]{p^3q^6r^9}$
$= \boldsymbol{pq^2r^3}$ [1]

Must-Know Concept:
To find the cube root, divide each of the index in p^3, q^6 and r^9 by 3.

2. (a) Greatest number that will divide 336 and 700 exactly $= 2^2 \times 7$
$= \mathbf{28}$ [1]

Must-Know Concept:
The greatest number that will divide 336 and 700 exactly is the highest common factor (HCF) of 336 and 700.
To find the HCF, we pick the factor(s) common to the numbers with the **smaller index**.

(b) $336 \times 700 = (2^4 \times 3 \times 7) \times (2^2 \times 5^2 \times 7)$
$= 2^6 \times 3 \times 5^2 \times 7^2$

$\frac{336 \times 700}{n} = \frac{2^6 \times 3 \times 5^2 \times 7^2}{3}$
$= 2^6 \times 5^2 \times 7^2$
$= (2^3 \times 5 \times 7)^2$, which is a perfect square

∴ Smallest integer $n = \mathbf{3}$ [1]

Must-Know Concept:
A number is a perfect square if each index on its prime factors is an even number.

3. (a) $5.74\% = \frac{5.74}{100}$
$= \boldsymbol{\frac{287}{5000}}$ [1]

Must-Know Concept:
To convert a percentage to a fraction/decimal, divide by 100.

(b) $8.6 \text{ m/s} = \frac{8.6 \text{ m}}{1 \text{ s}}$
$= \frac{(8.6 \div 1000) \text{ km}}{\frac{1}{3600} \text{ h}}$ [1]
$= \mathbf{30.96 \text{ km/h}}$ [1]

Must-Know Concept:
1 km = 1000 m
1 h = 60 min
1 min = 60 s
1 h = 60 × 60 s
= 3600 s

4. (a) $-2y \div 4x^{-5} = \frac{-2y}{4x^{-5}}$
$= \boldsymbol{-\frac{x^5y}{2}}$ [1]

Must-Know Concept:
$a^{-n} = \frac{1}{a^n}$

(b) $\left(\frac{p^2}{2r^3}\right)^{-2} \div \sqrt[4]{(r^6p^4)^3} \times (5p)^0 = \left(\frac{2r^3}{p^2}\right)^2 \div (r^6p^4)^{\frac{3}{4}} \times 1$
$= \frac{4r^6}{p^4} \div r^{\frac{9}{2}}p^3$ [1]
$= 4r^6p^{-4} \div r^{\frac{9}{2}}p^3$
$= 4r^{6-\frac{9}{2}}p^{-4-3}$
$= 4r^{\frac{3}{2}}p^{-7}$
$= \boldsymbol{\frac{4r^{\frac{3}{2}}}{p^7}}$ [1]

Must-Know Concept:
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$a^0 = 1$

5. (a) $5^{3y+x} = 5^{3y} \times 5^x$
$= (5^y)^3 \times 5^x$
$= 3^3 \times 7$
$= \mathbf{189}$ [1]

Must-Know Concept:
$a^m \times a^n = a^{m+n}$
$(a^m)^n = a^{mn}$

(b) $25^{3+x-\frac{1}{2}y} = 5^{2\left(3+x-\frac{1}{2}y\right)}$
$= 5^{6+2x-y}$
$= 5^6 \times 5^{2x} \div 5^y$ [1]
$= 5^6 \times (5^x)^2 \div 5^y$
$= 15\,625 \times 7^2 \div 3$
$= \mathbf{255\,208\frac{1}{3}}$ [1]

Must-Know Concept:
$a^m \div a^n = a^{m-n}$

6. (a) $1 \div \left(\frac{7p^2}{4p^{-1}q^2}\right)^{-2} \times (8p^2q^3)^0 = 1 \div \left(\frac{4p^{-1}q^2}{7p^2}\right)^2 \times 1$
$= 1 \div \frac{16p^{-2}q^4}{49p^4}$ [1]
$= 1 \div \frac{16q^4}{49p^6}$
$= 1 \times \frac{49p^6}{16q^4}$
$= \mathbf{\frac{49p^6}{16q^4}}$ [1]

Must-Know Concept:
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$a^0 = 1$

(b)

Indices with Base 3	Last digit
$3^1 = 3$	3
$3^2 = 9$	9
$3^3 = 27$	7
$3^4 = 81$	1
$3^5 = 243$	3
$3^6 = 729$	9
$3^7 = 2187$	7
$3^8 = 6561$	1

Notice that the last digit of 3^n follows the pattern $\underline{3, 9, 7, 1}$, 3, 9, 7, 1, …
$3^{2018} = 3^{4(504)+2}$

$\therefore$ Last digit of $3^{2018} = \mathbf{9}$ [1]

Must-Know Concept:
Note that 2018 leaves a remainder of 2 when divided by 4.
Therefore, the last digit of 3^{2018} is the same as the last digit of 3^2.

7. (a)

Indices with Base 2	Last digit
$2^1 = 2$	2
$2^2 = 4$	4
$2^3 = 8$	8
$2^4 = 16$	6
$2^5 = 32$	2
$2^6 = 64$	4
$2^7 = 128$	8
$2^8 = 256$	6

Notice that the last digit of 2^n follows the pattern $\underline{2, 4, 8, 6}$, 2, 4, 8, 6, …
$2^{9999} = 2^{4(2499)+3}$
$\therefore$ Last digit of $2^{9999} = \mathbf{8}$ [1]

Must-Know Concept:
Note that 9999 leaves a remainder of 3 when divided by 4.
Therefore, the last digit of 2^{9999} is the same as the last digit of 2^3.

(b)

Indices with Base 5	Last digit
$5^1 = 5$	5
$5^2 = 25$	5
$5^3 = 125$	5
$5^4 = 625$	5
$5^5 = 3125$	5

Notice that the last digit of 5^n is always 5.
$\therefore$ Last digit of $5^{323} = 5$ [1]
Last digit of $2(5^{323}) = 0$
Last digit of $2(5^{323}) + 7 = 0 + 7$
$= \mathbf{7}$ [1]

Must-Know Concept:
If a number that has a last digit of 5 is multiplied by 2, its last digit becomes 0.
E.g. $2 \times 125 = 250$

8. (a)

$$\sqrt{3^x} \times 27^{x+1} \div 9^{3x-4} = \frac{1}{81}$$
$$(3^x)^{\frac{1}{2}} \times 3^{3(x+1)} \div 3^{2(3x-4)} = \frac{1}{3^4}$$
$$3^{\frac{1}{2}x} \times 3^{3x+3} \div 3^{6x-8} = 3^{-4}$$
$$3^{\frac{1}{2}x+3x+3-(6x-8)} = 3^{-4}$$
$$3^{\frac{1}{2}x+3x+3-6x+8} = 3^{-4} \quad [1]$$
$$3^{-2.5x+11} = 3^{-4}$$
$$\therefore -2.5x + 11 = -4$$
$$-2.5x = -15$$
$$x = \frac{-15}{-2.5}$$
$$= \mathbf{6} \quad [1]$$

Must-Know Concept:
List of numbers which can be expressed in index notation with a common base of 3: 3, 9, 27, 81, 243, …

(b)

$$(3x)^{2a} \times 5^{2a} = (xy)^a$$
$$3^{2a}x^{2a} \times 5^{2a} = x^a y^a$$
$$9^a x^{2a} \times 25^a = x^a y^a \quad [1]$$
$$\frac{(9 \times 25)^a x^{2a}}{x^a} = y^a$$
$$225^a x^a = y^a$$
$$(225x)^a = y^a$$
$$\therefore \mathbf{y = 225x} \quad [1]$$

Must-Know Concept:
Note that $(pq)^n = p^n q^n$.

9. (a)

$$\left(\frac{-2x^2y^{-3}}{5x^{-4}y^5}\right)^2 \div \left(\frac{3x^{-5}y^{-1}}{x^2y^6}\right)^{-3} = \frac{4x^4y^{-6}}{25x^{-8}y^{10}} \div \left(\frac{x^2y^6}{3x^{-5}y^{-1}}\right)^3$$
$$= \frac{4x^4y^{-6}}{25x^{-8}y^{10}} \div \frac{x^6y^{18}}{27x^{-15}y^{-3}}$$
$$= \frac{4x^4y^{-6}}{25x^{-8}y^{10}} \times \frac{27x^{-15}y^{-3}}{x^6y^{18}} \quad [1]$$
$$= \frac{108x^{-11}y^{-9}}{25x^{-2}y^{28}}$$
$$= \frac{108x^{-11-(-2)}y^{-9-28}}{25}$$
$$= \frac{108x^{-9}y^{-37}}{25}$$
$$= \mathbf{\frac{108}{25x^9y^{37}}} \quad [1]$$

Must-Know Concept:
Note that $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.
$\frac{w}{x} \div \frac{y}{z} = \frac{w}{x} \times \frac{z}{y}$

(b)

$$\left(\frac{1}{2}\right)^{x+1} \div 32^{x+2} \times 72 = 4^{2x} \times 3^2$$
$$2^{-1(x+1)} \div 2^{5(x+2)} \times 72 = 2^{2(2x)} \times 9$$
$$2^{-x-1} \div 2^{5x+10} \times 72 \div 9 = 2^{4x}$$
$$2^{-x-1} \div 2^{5x+10} \times 8 = 2^{4x}$$
$$2^{-x-1} \div 2^{5x+10} \times 2^3 = 2^{4x} \quad [1]$$
$$2^{-x-1-(5x+10)+3} = 2^{4x}$$
$$\therefore -x - 1 - 5x - 10 + 3 = 4x$$
$$-6x - 8 = 4x$$
$$10x = -8$$
$$x = -\frac{8}{10}$$
$$= \mathbf{-\frac{4}{5}} \quad [1]$$

Must-Know Concept:
Note that $\frac{1}{2} = 2^{-1}$.
List of numbers which can be expressed in index notation with a common base of 2: 2, 4, 8, 16, 32, …

10. (a)

$$\frac{(-2a^2b)^2}{5c} \div \left(\frac{3ac}{b^3}\right)^{-2} \times \frac{c^5b}{(8a)^0} = \frac{4a^4b^2}{5c} \div \left(\frac{b^3}{3ac}\right)^2 \times \frac{c^5b}{1}$$
$$= \frac{4a^4b^2}{5c} \div \frac{b^6}{9a^2c^2} \times c^5b$$
$$= \frac{4a^4b^2}{5c} \times \frac{9a^2c^2}{b^6} \times c^5b \quad [1]$$
$$= \frac{36a^6b^3c^7}{5cb^6}$$
$$= \mathbf{\frac{36a^6c^6}{5b^3}} \quad [1]$$

Must-Know Concept:
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$a^0 = 1$

(b)

$$\frac{\sqrt[3]{7} \times 343}{49^{\frac{2}{5}}} = \frac{1}{7^y}$$
$$\frac{7^{\frac{1}{3}} \times 7^3}{(7^2)^{\frac{2}{5}}} = 7^{-y} \quad [1]$$
$$\frac{7^{\frac{1}{3}} \times 7^3}{7^{\frac{4}{5}}} = 7^{-y}$$
$$7^{\frac{1}{3}+3-\frac{4}{5}} = 7^{-y}$$
$$-y = \frac{1}{3} + 3 - \frac{4}{5}$$
$$-y = 2\frac{8}{15}$$
$$y = \mathbf{-2\frac{8}{15}} \quad [1]$$

Must-Know Concept:
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$a^{-n} = \frac{1}{a^n}$
List of numbers which can be expressed in index notation with a common base of 7: 7, 49, 343, 2401, …

11. (a) $\left(\frac{\sqrt[3]{x^2} \times 7y^0}{2x^{-1}y^3}\right)^{-2} = \left(\frac{2x^{-1}y^3}{\sqrt[3]{x^4} \times 7y^0}\right)^2$

$= \left(\frac{2x^{-1}y^3}{(x^4)^{\frac{1}{3}} \times 7}\right)^2$ [1]

$= \left(\frac{2x^{-1}y^3}{7x^{\frac{4}{3}}}\right)^2$

$= \frac{4x^{-2}y^6}{49x^{\frac{8}{3}}}$

$= \mathbf{\frac{4y^6}{49x^{\frac{14}{3}}}}$ [1]

> **Must-Know Concept:**
> $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
> $\sqrt[n]{a} = a^{\frac{1}{n}}$
> $a^0 = 1$

(b) $\frac{8^{x-1} \times 32^x}{4^{2-3x}} = \sqrt{2}$

$\frac{2^{3(x-1)} \times 2^{5x}}{2^{2(2-3x)}} = 2^{\frac{1}{2}}$

$\frac{2^{3x-3} \times 2^{5x}}{2^{4-6x}} = 2^{\frac{1}{2}}$

$2^{3x-3+5x-(4-6x)} = 2^{\frac{1}{2}}$ [1]

$\therefore 3x - 3 + 5x - 4 + 6x = \frac{1}{2}$

$14x = 7\frac{1}{2}$

$x = 7\frac{1}{2} \div 14$

$= \mathbf{\frac{15}{28}}$ [1]

> **Must-Know Concept:**
> $\sqrt[n]{a} = a^{\frac{1}{n}}$
> List of numbers which can be expressed in index notation with a common base of 2: 2, 4, 8, 16, 32, …

12. (a) $2 \times 9^{x-1} \div \sqrt[3]{3^x} = 54$

$3^{2(x-1)} \div (3^x)^{\frac{1}{3}} = \frac{54}{2}$

$3^{2x-2} \div 3^{\frac{1}{3}x} = 27$

$3^{2x-2-\frac{1}{3}x} = 3^3$ [1]

$\therefore 2x - 2 - \frac{1}{3}x = 3$

$\frac{5}{3}x = 5$

$x = 5 \div \frac{5}{3}$

$= \mathbf{3}$ [1]

> **Must-Know Concept:**
> List of numbers which can be expressed in index notation with a common base of 3: 3, 9, 27, 81, 243, …

(b) $5^{2y-1} = \frac{1}{25}$

$5^{2y-1} = \frac{1}{5^2}$

$5^{2y-1} = 5^{-2}$ [1]

$2y - 1 = -2$

$2y = -1$

$y = -\frac{1}{2}$ [1]

$7^{y+\frac{1}{2}} = 7^{-\frac{1}{2}+\frac{1}{2}}$

$= 7^0$

$= \mathbf{1}$ [1]

> **Must-Know Concept:**
> $a^{-n} = \frac{1}{a^n}$
> $a^0 = 1$
> List of numbers which can be expressed in index notation with a common base of 5: 5, 25, 125, 625, …

13. (a) 1.9×10^{-5} metres

$= 1.9 \times 10^{-5} \times 10^6$ micrometres

$= 19$ micrometres [1]

$\therefore k = \mathbf{19}$

> **Must-Know Concept:**
> 1 micrometre = 10^{-6} metre
> $\therefore$ 1 metre = 10^6 micrometres

(b) Diameter of each organism

$= 2 \times 1.9 \times 10^{-5}$ metres

$= 3.8 \times 10^{-5}$ metres

Total length formed by the organisms

$= 156 \times 3.8 \times 10^{-5}$ metres [1]

$= 0.005\ 928$ metres

$= 0.5928$ centimetres

$= \mathbf{(5.928 \times 10^{-1})}$ **centimetres** [1]

> **Must-Know Concept:**
> The total length formed by these organisms is dependent on two factors:
> - The number of organisms
> - The diameter of each organism
> 1 metre = 100 centimetres

14. (a) $\frac{\sqrt[3]{3.5826} \times (-2.5512)^2}{12.95} = 0.769\ 045$

$= \mathbf{0.7690}$ (4 s.f.) [1]

> **Must-Know Concept:**
> The 4th significant figure of 0.769 045 is 0. The number on its right is 4. Since 4 is less than 5, we round down.

(b) 300 million m/s = 300×10^6 m/s

$= \frac{300 \times 10^6 \text{ m}}{1 \text{ s}}$

$= \frac{(300 \times 10^6 \div 1000) \text{ km}}{\frac{1}{60 \times 60} \text{ h}}$ [1]

= 1 080 000 000 km/h

= **(1.08×10^9) km/h** [1]

Must-Know Concept:
1 million = 10^6
1 km = 1000 m
1 h = 60 min
1 min = 60 s
$\therefore$ 1 h = (60×60) s
= 3600 s

15. (a) 140 000 000 = 140×10^6
= 140 million

$\therefore k =$ **140** [1]

Must-Know Concept:
1 million = 1 000 000
= 10^6

(b) 1 TB = 1×10^{12} B
1 GB = 1×10^9 B
1 MB = 1×10^6 B
Size of 28 movies = 28×4.3 GB
= $28 \times 4.3 \times 10^9$ B [1]
= 120.4×10^9 B
Size of 572 songs = 572×3.6 MB
= $572 \times 3.6 \times 10^6$ B [1]
= 2059.2×10^6 B
Remaining space
= $(1 \times 10^{12}) - (120.4 \times 10^9) - (2059.2 \times 10^6)$
= $8.775\ 408 \times 10^{11}$ B
= $8.775\ 408 \times 10^{11} \times 10^{-9}$ GB
= **$8.775\ 408 \times 10^2$ GB** [1]

Must-Know Concept:
We can work on this question systematically by converting all the given storage and file sizes to the same unit (bytes).

16. (a) $2x + y = (2 \times 2.21 \times 10^n) + (470 \times 10^n)$
$= (4.42 \times 10^n) + (470 \times 10^n)$
$= (4.42 + 470) \times 10^n$
$= 474.42 \times 10^n$ [1]
$= 4.7442 \times 10^2 \times 10^n$
= **$4.7442 \times 10^{n+2}$** [1]

Must-Know Concept:
Note that $10^a \times 10^b = 10^{a+b}$.
Do not round off 4.7442 as the value is exact.

(b) $x^2y = (2.21 \times 10^n)^2 \times 470 \times 10^n$
$= (4.8841 \times 10^{2n} \times 470 \times 10^n)$
$= (4.8841 \times 470) \times 10^{2n+n}$
$= 2295.527 \times 10^{3n}$ [1]
$= 2.295527 \times 10^3 \times 10^{3n}$
= **$2.295527 \times 10^{3n+3}$** [1]

Must-Know Concept:
Note that $(ab)^n = a^n b^n$.
Do not round off 2.295 527 as the value is exact.

17. (a) 825 000 milligrams
= $825\ 000 \times 10^{-3}$ grams
= $825\ 000 \times 10^{-3} \times 10^{-9}$ gigagrams
= $8.25 \times 10^5 \times 10^{-12}$ gigagrams
= **8.25×10^{-7} gigagrams** [1]

Must-Know Concept:
1 milligram = 10^{-3} gram
1 gigagram = 10^9 grams
$\therefore$ 1 gram = 10^{-9} gigagram

(b) (i) 149.6 million km = $149.6 \times 10^6 \times 10^3$ m
= $1.496 \times 10^2 \times 10^9$ m
= **1.496×10^{11} m** [1]

Must-Know Concept:
1 million = 10^6
1 km = 10^3 m
Do not round off 1.496 as the answer is exact.

(ii) 300 Mm = 300×10^6 m
Time taken
$= \frac{1.496 \times 10^{11} \text{ m}}{300 \times 10^6 \text{ m/s}}$ [1]
= 498.666 s
= 8.31111 min
= **8 min 19 s** (nearest second) [1]

Must-Know Concept:
1 megametre = 10^6 metres
Time Taken = $\frac{\text{Distance}}{\text{Speed}}$
1 min = 60 s

18. (a) 337 780 = $0.337\ 78 \times 10^6$
= 0.337 78 million
$\therefore k =$ **0.337 78** [1]

Must-Know Concept:
1 million = 10^6

(b) Population of Iceland in 2017

$= \frac{100}{100.82} \times 337\ 780$

$= 335\ 032.73$

$=$ **335 033** (nearest whole number) [1]

Must-Know Concept:
The population of Iceland in 2018 is $(100 + 0.82 =)$ 100.82% of its population in 2017.

(c) Average number of people per km^2 in Iceland $= \frac{337\ 780}{1.03 \times 10^5}$

$= 3.279\ 42$ [1]

Average number of people per km^2 in Australia $= \frac{24\ 769\ 768}{7.6823 \times 10^6}$

$= 3.224\ 26$ [1]

Iceland has the higher average number of people per square kilometre. [1]

Must-Know Concept:
To find the average number of people per square kilometre in a country, we divide its population by its area.

1.2 Algebraic Expressions and Formulae

1. (a) $4a^2 + 6a + 2 = \mathbf{(2a + 1)(2a + 2)}$ [1]

Must-Know Concept:
Factorise $4a^2 + 6a + 2$ using the multiplication frame.

(b) Given any whole number a, $(2a + 1)$ and $(2a + 2)$ are two consecutive positive integers, of which one is odd and the other is even.
Since the **product of an odd number and an even number is even,**
$4a^2 + 6a + 2 = (2a + 1)(2a + 2)$ is always an even number. [1]

Must-Know Concept:
The product of an odd number and an even number is always even.
The product of two odd numbers is odd.
The product of two even numbers is even.

2. (a) $2p^2 + 6pq - 8q^2 = 2(p^2 + 3pq - 4q^2)$ [1]

$= \mathbf{2(p + 4q)(p - q)}$ [1]

Must-Know Concept:
After performing factorisation, always check if the result can be factorised further.

(b) $3a^2 - 12b^2 + 8a + 16b$

$= 3(a^2 - 4b^2) + 8(a + 2b)$

$= 3(a - 2b)(a + 2b) + 8(a + 2b)$ [1]

$= (a + 2b)[3(a - 2b) + 8]$

$= \mathbf{(a + 2b)(3a - 6b + 8)}$ [1]

Must-Know Concept:
$x^2 - y^2 = (x - y)(x + y)$

3. (a) $\frac{49 \times 5^{299} - 7 \times 5^{300}}{14 \times 5^{298}} = \frac{7^2 \times 5^{299} - 7 \times 5^{300}}{14 \times 5^{298}}$

$= \frac{7 \times 5^{299}(7 - 5)}{14 \times 5^{298}}$ [1]

$= \frac{7 \times 2 \times 5^{299}}{14 \times 5^{298}}$

$= \frac{14 \times 5^{299}}{14 \times 5^{298}}$

$= \mathbf{5}$ [1]

Must-Know Concept:
To simplify the given fraction, we factorise 7×5^{299} from the numerator.

(b) $\frac{a + 3b}{5a - b} = \frac{3}{7}$

$7(a + 3b) = 3(5a - b)$

$7a + 21b = 15a - 3b$

$8a = 24b$ [1]

$\frac{a}{b} = \frac{24}{8}$

$= 3$

$\frac{2a}{7b} = \frac{2}{7} \times 3$

$= \mathbf{\frac{6}{7}}$ [1]

Must-Know Concept:
Note that $\frac{2a}{7b} = \frac{2}{7} \times \frac{a}{b}$.

(c) $\frac{1}{y} + \frac{1}{x^3} = \frac{3}{5z}$

$\frac{1}{x^3} = \frac{3}{5z} - \frac{1}{y}$

$\frac{1}{x^3} = \frac{3y - 5z}{5yz}$ [1]

Perform cross-multiplication:

$x^3(3y - 5z) = 5yz$

$x^3 = \frac{5yz}{3y - 5z}$ [1]

$x = \sqrt[3]{\frac{5yz}{3y - 5z}}$ [1]

Must-Know Concept:
To solve the equation, we simplify the fractions so as to perform cross-multiplication.

4. (a) Method 1

$$1+\frac{2}{a}+\frac{2}{b}=\frac{ab+2b+2a}{ab} \quad [1]$$
$$=\frac{ab+2(a+b)}{ab}$$
$$=\frac{8+2(5)}{8}$$
$$=\frac{18}{8}$$
$$=\frac{9}{4} \quad [1]$$

Must-Know Concept:
Express as $1+\frac{2}{a}+\frac{2}{b}$ as a single fraction first.

Method 2

$$1+\frac{2}{a}+\frac{2}{b}=1+\frac{2b+2a}{ab} \quad [1]$$
$$=1+\frac{2(a+b)}{ab}$$
$$=1+\frac{2(5)}{8}$$
$$=\frac{9}{4} \quad [1]$$

Must-Know Concept:
Express $\frac{2}{a}+\frac{2}{b}$ as a single fraction first.

(b)
$$2p^2-32q^2=92$$
$$2(p^2-16q^2)=92$$
$$2(p-4q)(p+4q)=92 \quad [1]$$
$$2(4)(p+4q)=92$$
$$p+4q=\frac{92}{8}$$
$$=11.5 \quad [1]$$
$$p^2+8q+16q^2=(p+4q)^2$$
$$=11.5^2$$
$$=\mathbf{132.25} \quad [1]$$

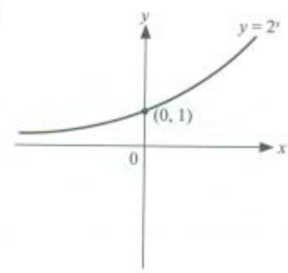
Must-Know Concept:
$x^2-y^2=(x-y)(x+y)$
Note that $p^2+8q+16q^2$ can be factorised using the multiplication frame.

(c)
$$3x=-2z\sqrt{\frac{y-3}{w+3y}}$$
$$(3x)^2=\left(-2z\sqrt{\frac{y-3}{w+3y}}\right)^2$$
$$9x^2=4z^2\left(\frac{y-3}{w+3y}\right) \quad [1]$$
$$9x^2=\frac{4z^2y-12z^2}{w+3y}$$
$$9x^2(w+3y)=4z^2y-12z^2 \quad [1]$$
$$9x^2w+27x^2y=4z^2y-12z^2$$
$$4z^2y-27x^2y=9x^2w+12z^2$$
$$y(4z^2-27x^2)=9x^2w+12z^2$$
$$y=\mathbf{\frac{9x^2w+12z^2}{4z^2-27x^2}} \quad [1]$$

Must-Know Concept:
The square of a negative number is positive.
E.g. $(-2z)^2=4z^2$

1.3 Functions and Graphs

1. (a)

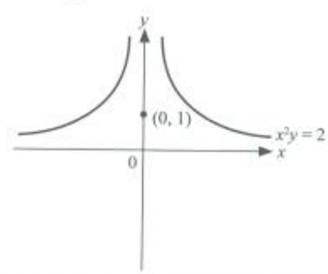


[1]

Must-Know Concept:
When $x=0$, $y=2^0=1$.
The graph of $y=2^x$ passes through (0, 1).

(b) $x^2y=2$

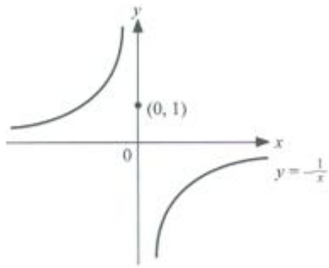
$$y=\frac{2}{x^2}$$



[1]

Must-Know Concept:
The graph of $y=\frac{k}{x^2}$, where k is a nonzero constant, is undefined when $x=0$.

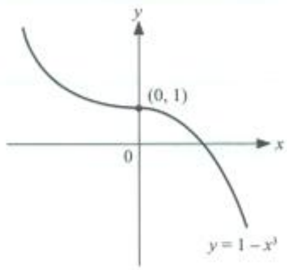
(c)



[1]

Must-Know Concept:
The graph of $y = \frac{k}{x}$, where k is a nonzero constant, is undefined when $x = 0$.

(d)



[1]

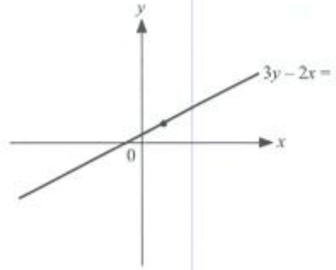
Must-Know Concept:
When $x = 0$, $y = 1 - (0)^3 = 1$.
$\therefore$ The graph of $y = 1 - x^3$ passes through (0, 1).
To draw the graph of $y = 1 - x^3$, shift the graph of $y = -x^3$ up by 1 unit.

2. (a) $3y - 2x = 1$

$$3y = 2x + 1$$
$$y = \frac{2}{3}x + \frac{1}{3}$$

Note that both the gradient and the y-intercept of the line are positive.

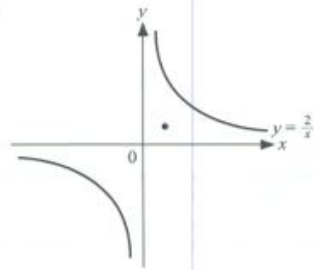
When $x = 1$,
$$y = \frac{2}{3}(1) + \frac{1}{3}$$
$$= 1$$



[1]

Must-Know Concept:
The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept.
The gradient of an upward-sloping line is positive and the gradient of a downward-sloping line is negative.

(b)

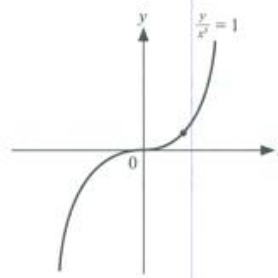


[1]

Must-Know Concept:
When $x = 1$, $y = \frac{2}{1} = 2$.
$\therefore$ The graph of $y = \frac{2}{x}$ does not pass through (1, 1).

(c) $\frac{y}{x^3} = 1$

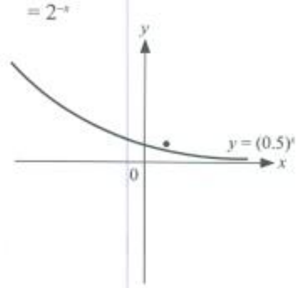
$$y = x^3$$



[1]

Must-Know Concept:
When $x = 1$, $y = 1^3 = 1$.
$\therefore$ The graph of $\frac{y}{x^3} = 1$ passes through (1, 1).

(d) $y = 0.5^x$
$= \left(\frac{1}{2}\right)^x$
$= (2^{-1})^x$
$= 2^{-x}$



[1]

Must-Know Concept:
When $x = 1$, $y = 0.5^1 = 0.5$.
$\therefore$ The graph of $y = (0.5)^x$ does not pass through (1, 1).

3. (a) $\boldsymbol{y = x^3 + 1}$ [1]

Must-Know Concept:
Based on the shape of the curve, its equation is of the form $y = kx^3$ where $k > 0$.
Since it crosses the y-axis at 1, we add 1 to its equation.
Marking scheme:
Accept any answer written in the form of $y = kx^3 + 1$, where $k > 0$.

(b) The equation of the curve is of the form $y = \frac{k}{x^2}$ where $k > 0$.
When $x = 2$, $y = \frac{1}{2}$.
$\therefore \frac{1}{2} = \frac{k}{x^2}$
$k = \frac{1}{2} \times 4$
$= 2$ [1]
$\therefore \boldsymbol{y = \frac{2}{x^2}}$ [1]

Must-Know Concept:
Based on the shape of the curve, its equation is of the form $y = \frac{k}{x^2}$ where $k > 0$.
To find the value of k, we substitute the given value of x and of y into the equation.

4. (a) The equation of the curve is of the form $y = \frac{k}{x}$, where $k < 0$.
When $x = -1$, $y = 3$.
$\therefore 3 = \frac{k}{-1}$
$k = 3(-1)$
$= -3$ [1]
$\therefore \boldsymbol{y = -\frac{3}{x}}$ [1]

Must-Know Concept:
Based on the shape of the curve, its equation is of the form $y = \frac{k}{x}$ where $k < 0$.
To find the value of k, we substitute the given value of x and of y into the equation.

(b) $\boldsymbol{y = 2 - x^2}$ [1]

Must-Know Concept:
Based on the shape of the curve, its equation is of the form $y = kx^2$ where $k < 0$.
Since it crosses the y-axis at 2, we add 2 to its equation.
Marking scheme:
Accept any answer written in the form of $y = kx^2 + 2$, where $k < 0$.

5. (a) Since the total distance travelled by the object is 335 m,
$\frac{1}{2}(9)(16) + (6)(16) + \frac{1}{2}(16 + 22)(3)$
$+ \frac{1}{2}(T - 18)(22) = 335$ [1]
$72 + 96 + 57 + 11(T - 18) = 335$
$11(T - 18) = 110$
$T - 18 = \frac{110}{11}$
$T - 18 = 10$
$T = \boldsymbol{28}$ [1]

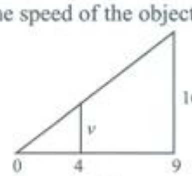
Must-Know Concept:
Given a speed-time graph of an object,
Distance travelled by the object = Area under the graph

(b) Average speed of the object $= \frac{335}{28}$
$= \boldsymbol{11\frac{27}{28}}$ **m/s** [1]

Must-Know Concept:
Average Speed = $\frac{\text{Total distance}}{\text{Total time taken}}$

(c) Method 1

Let the speed of the object at 4 s be v.



By similar triangles,

$\frac{v}{16} = \frac{4}{9}$ [1]

$9v = 64$

$v = \frac{64}{9}$

$= \mathbf{7\frac{1}{9}}$ **m/s** [1]

> **Must-Know Concept:**
> The ratios of corresponding lengths of similar figures are equal.

Method 2

Let the speed of the object at 4 s be v.

$\frac{v-0}{4-0} = \frac{16-0}{9-0}$

$\frac{v}{4} = \frac{16}{9}$ [1]

$9v = 64$

$v = \frac{64}{9}$

$= \mathbf{7\frac{1}{9}}$ **m/s** [1]

> **Must-Know Concept:**
> The points (0, 0), (4, v) and (9, 16) lie on a straight line with a fixed gradient.

(d) Acceleration of the object from 15 s to 18 s

$= \frac{22-16}{18-15}$

$= \mathbf{2}$ **m/s²** [1]

> **Must-Know Concept:**
> Given a speed-time graph of an object,
> Acceleration = Gradient of the graph

6. (a) Acceleration of the van from 5 s to 11 s

$= \frac{14-10}{11-5}$

$= \mathbf{\frac{2}{3}}$ **m/s²** [1]

> **Must-Know Concept:**
> Given a speed-time graph of an object,
> Acceleration = Gradient of the graph

(b) Acceleration of the car $= 3 \times \frac{2}{3}$ m/s²

$= 2$ m/s²

$\frac{18-0}{t-6} = 2$ [1]

$2(t-6) = 18$

$2t - 12 = 18$

$2t = 30$

$t = \mathbf{15}$ [1]

(c) Total distance travelled by the van in the first 15 s

$= \frac{1}{2}(5)(10) + \frac{1}{2}(6)(10+14) + (14)(15-11)$

$= 25 + 72 + 56$

$= 153$ m [1]

Total distance travelled by the car from 6 s to 15 s $= \frac{1}{2}(15-6)(18)$

$= 81$ m [1]

Let the additional travelling time needed for the car and the van to meet be x s.

$153 + 14x = 81 + 18x$ [1]

$4x = 72$

$x = \frac{72}{4}$

$= 18$ s

Total time that the van has to travel

$= 15 + 18$

$= \mathbf{33}$ **s** [1]

> **Must-Know Concept:**
> When the van and the car meets, the total distance travelled by the van and by the car are equal.
> Given a speed-time graph of an object,
> Distance travelled by the object = Area under the graph

7. (a) $p = \frac{1}{4}(-1)[10 - (-1)^2]$

$= \mathbf{-2.25}$ [1]

> **Must-Know Concept:**
> Substitute $x = -1$ into the given equation to find the value of p.

(b) Refer to Appendix 1.

> **Must-Know Concept:**
> Do not forget to label the axes and the equation of the curve.
> *Marking scheme:*
> *Correct scale used for both axes, and axes are labelled [1]*
> *Curve smooth and drawn correctly [1]*
> *Origin and equation of curve labelled [1]*

(c) Using the points (3, 0.75) and (4, –3.5),

Gradient $= \frac{0.75-(-3.5)}{3-4}$

$= \mathbf{-4.25}$ [1]

Must-Know Concept:
Using two points (x_1, y_1) and (x_2, y_2) on the tangent,
Gradient $= \frac{y_2-y_1}{x_2-x_1}$
Marking scheme:
Accept $-4.35 \leqslant m \leqslant -4.15$
Award the remaining mark if tangent line is drawn correctly.

(d) $x(10-x^2)=6$

$\frac{1}{4}x(10-x^2)=1.5$

From the graph, when $y = 1.5$,

$x = \mathbf{0.6}$ or $x = \mathbf{2.8}$

[1] [1]

Must-Know Concept:
Let the left-hand-side of the given equation be y.
Marking scheme:
Accept $0.55 \leqslant x \leqslant 0.65$ and $2.75 \leqslant x \leqslant 2.85$

(e) $\frac{1}{4}x(10-x^2) \geqslant \frac{1}{2}x+1$

$y \geqslant \frac{1}{2}x+1$

To determine the range of values, we first draw the line $y=\frac{1}{2}x+1$.

x	–2	0	2
y	0	1	2

From the graph, when $y=\frac{1}{2}x+1$,

$x = 0.5$ or $x = 2.55$

Range of values of is $\mathbf{0.5 \leqslant x \leqslant 2.55}$ [2]

Must-Know Concept:
To draw the line $y=\frac{1}{2}x+1$, we substitute some values of x into the equation and plot the points on the graph paper.
Marking scheme:
Award the remaining mark if the line $y=\frac{1}{2}x+1$ is drawn correctly and its equation is labelled.
Deduct 1 mark if the inequality signs were written as '<'.

8\. (a) When $x = -2$,

$p=\frac{1}{4}(-2)^3-\frac{1}{2}(-2)-5$

$= \mathbf{-6}$ [1]

When $x = 3$,

$q=\frac{1}{4}(3)^3-\frac{1}{2}(3)-5$

$= \mathbf{0.25}$ [1]

Must-Know Concept:
Substitute $x = -2$ and $x = 3$ into the given equation to find the values of p and q respectively.

(b) Refer to Appendix 2.

Must-Know Concept:
Do not forget to label the axes and the equation of the curve.
Marking scheme:
Correct scale used for both axes, and axes are labelled [1]
Curve smooth and drawn correctly [1]
Origin and equation of curve labelled [1]

(c) The problem can be solved by first constructing the line $y = 2x$, which has a gradient of 2.

x	–2	0	2
y	–4	0	4

Next, we construct the lines parallel to $y = 2x$ which are tangent to $y=\frac{1}{4}x^3-\frac{1}{2}x-5$.

From the graph, $k = \mathbf{1.8}$ or $\mathbf{-1.8}$

Must-Know Concept:
Parallel lines have the same gradient.
Marking scheme:
Correct tangent lines drawn [1]
Correct values of k: 1 mark each [2]
(Accept $-1.85 \leqslant k \leqslant -1.75$ and $1.75 \leqslant k \leqslant 1.85$)

(d) (i) $\frac{1}{4}x^3-\frac{1}{2}x-5=0$

$y = 0$

From the graph, when $y = 0$,

$x = \mathbf{2.95}$ [1]

Marking scheme:

Accept $2.9 \leqslant x \leqslant 3$

(ii) $x^3 - 2x - 8 = 0$

$\frac{1}{4}x^3 - \frac{1}{2}x - 2 = 0$

$\frac{1}{4}x^3 - \frac{1}{2}x = 2$

$\frac{1}{4}x^3 - \frac{1}{2}x - 5 = -3$ [1]

$y = -3$

From the graph, when $y = -3$,

$x = \mathbf{2.28}$ [1]

> **Must-Know Concept:**
> Let the left-hand side of the given equation be y.
> Marking scheme:
> Accept $2.23 \leqslant x \leqslant 2.33$

9\. (a) $p = \frac{3}{4}(-1)^3 - 2(-1)^2 + \frac{1}{2}(-1)$

$= \mathbf{-3.25}$ [1]

> **Must-Know Concept:**
> Substitute $x = -1$ into the given equation.

(b) Refer to Appendix 3.

> **Must-Know Concept:**
> Do not forget to label the axes and the equation of the curve.
> *Marking scheme:*
> *Correct scale used for both axes, and axes are labelled [1]*
> *Curve smooth and drawn correctly [1]*
> *Origin and equation of curve labelled [1]*

(c) Using the points $(-0.5, -0.84)$ and $(0.1, 1)$,

Gradient $= \frac{1-(-0.84)}{0.1-(-0.5)}$

≈ 3.06666

$= \mathbf{3.07}$ (3 s.f.) [1]

> **Must-Know Concept:**
> Using two points (x_1, y_1) and (x_2, y_2) on the tangent,
> Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$
> *Marking scheme:*
> *Accept $3.04 \leqslant m \leqslant 3.1$*
> *Award the remaining mark if tangent line is drawn correctly.*

(d) $\frac{3}{4}x^3 + \frac{1}{2}x = 2x^2$

$\frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x = 0$

From the graph, when $y = 0$,

$x = \mathbf{0}$ or $x = \mathbf{0.28}$ or $x = \mathbf{2.39}$

> **Must-Know Concept:**
> Make the left-hand-side of the given equation to be y.
> *Marking scheme:*
> *Accept $0.25 \leqslant x \leqslant 0.32$ and $2.36 \leqslant x \leqslant 2.42$*
> *(Award 1 mark if 2 answers are correct.*
> *Award 2 marks if all 3 answers are correct.)*

(e) (i) $y = 1 - x$

x	-1	1	3
y	2	0	-2

(On graph paper)

> **Must-Know Concept:**
> To draw the line $y = 1 - x$, we substitute some values of x into the equation and plot the points on the graph paper.
> *Marking scheme:*
> *Award the mark if the line $y - x = 1$ is drawn correctly and its equation is labelled.*

(ii) $x = \mathbf{2}$ [1]

> **Must-Know Concept:**
> The line and the curve intersect at the point $(2, -1)$

(iii) Equation of curve: $y = \frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x$

Equation of line: $y = 1 - x$

When the two graphs intersect,

$\frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x = 1 - x$ [1]

$\frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x + x - 1 = 0$

$\frac{3}{4}x^3 - 2x^2 + \frac{3}{2}x - 1 = 0$

Multiply the equation by 4:

$3x^3 - 8x^2 + 6x - 4 = 0$ [1]

$\therefore a = \mathbf{-8}, b = \mathbf{6}, c = \mathbf{-4}$ [1]

> **Must-Know Concept:**
> At the point where the two graphs intersect, the value of y is the same for both graphs.

1.4 Solutions of Equations

1. (a) Since $x = 5$ is a solution of the equation,

$$7(5)^2 - n(5) - 10 = 0$$
$$5n = 165$$
$$n = \mathbf{33} \quad [1]$$

> **Must-Know Concept:**
> Substitute $x = 5$ into the given equation.

(b)
$$7x^2 - 33x - 10 = 0$$
$$(7x + 2)(x - 5) = 0 \quad [1]$$
$$7x + 2 = 0 \quad \text{or} \quad x - 5 = 0$$
$$7x = -2 \quad \text{or} \quad x = 5$$
$$x = -\frac{2}{7}$$

The other solution is $\boldsymbol{x = -\frac{2}{7}}$. [1]

> **Must-Know Concept:**
> Make use of the value of n found in (a).
> The equation can be solved by factorising $7x^2 - 33x - 10$ using the multiplication frame.

2. (a)
$$2x^2 + 3x - 8 = 2[x^2 + 1.5x - 4]$$
$$= 2\left[x^2 + 1.5x + \left(\frac{1.5}{2}\right)^2 - 4 - \left(\frac{1.5}{2}\right)^2\right]$$
$$= 2[(x + 0.75)^2 - 4.5625]$$
$$= \mathbf{2(x + 0.75)^2 - 9.125} \quad [1]$$

> **Must-Know Concept:**
> When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(b)
$$2(2y + 3)^2 + 3(2y + 3) = 8$$
$$2(2y + 3)^2 + 3(2y + 3) - 8 = 0$$
$$2(2y + 3 + 0.75)^2 - 9.125 = 0 \quad [1]$$
$$2(2y + 3.75)^2 = 9.125$$
$$(2y + 3.75)^2 = 4.5625$$
$$2y + 3.75 = \pm\sqrt{4.5625} \quad [1]$$
$$2y + 3.75 \approx 2.136\,00 \quad \text{or} \quad 2y + 3.75 \approx -2.136\,00$$
$$2y = -1.614 \qquad 2y = -5.886$$
$$y = \mathbf{-0.807} \text{ (3 s.f.)} \qquad y = -2.943 = \mathbf{-2.94} \text{ (3 s.f.)} \quad [1]$$

> **Must-Know Concept:**
> Notice that $2(2y + 3)^2 + 3(2y + 3) - 8$ has the same form as $2x^2 + 3x - 8$, where $x = 2y + 3$.
> Therefore, we can make use of the answer for a.
> *Marking scheme:*
> *Award the final mark only if both of the answers are correct.*

3. (a)
$$2x^2 - 7x - 13$$
$$= 2(x^2 - 3.5x - 6.5)$$
$$= 2\left[x^2 - 3.5x + \left(-\frac{3.5}{2}\right)^2 - 6.5 - \left(-\frac{3.5}{2}\right)^2\right] \quad [1]$$
$$= 2[(x - 1.75)^2 - 9.5625]$$
$$= \mathbf{2(x - 1.75)^2 - 19.125} \quad [1]$$

> **Must-Know Concept:**
> When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(b)
$$2x^2 = 13 + 7x$$
$$2x^2 - 7x - 13 = 0$$
$$2(x - 1.75)^2 - 19.125 = 0$$
$$2(x - 1.75)^2 = 19.125$$
$$(x - 1.75)^2 = 9.5625 \quad [1]$$
$$x - 1.75 = \pm\sqrt{9.5625}$$
$$x - 1.75 \approx 3.092\,33 \quad \text{or} \quad x - 1.75 \approx -3.092\,33$$
$$x = 4.842\,33 \qquad x = -1.342\,33$$
$$x = \mathbf{4.842} \text{ (4 s.f.) [1]} \qquad x = \mathbf{-1.342} \text{ (4 s.f.) [1]}$$

> **Must-Know Concept:**
> When taking square roots on both sides of the equation, do not forget the ± sign.

4.
$$\left(5 + \frac{9}{x}\right)(11x - 6) = 3 - [6 - 2(2 + x)]$$
$$55x - 30 + 99 - \frac{54}{x} = 3 - [6 - 4 - 2x]$$
$$55x + 69 - \frac{54}{x} = 3 - [2 - 2x]$$
$$55x + 69 - \frac{54}{x} = 3 - 2 + 2x$$
$$53x + 68 - \frac{54}{x} = 0$$

Multiply the equation by x:

$$53x^2 + 68x - 54 = 0 \quad [1]$$
$$x = \frac{-68 \pm \sqrt{68^2 - 4(53)(-54)}}{2(53)} \quad [1]$$
$$x = \frac{-68 \pm \sqrt{16\,072}}{106}$$
$$x \approx 0.554\,485 \quad \text{or} \quad x \approx -1.837\,50$$
$$x = \mathbf{0.554} \text{ (3 s.f.) [1]} \qquad x = \mathbf{-1.84} \text{ (3 s.f.) [1]}$$

> **Must-Know Concept:**
> $(a + b)(c + d) = ac + ad + bc + bd$
> Note that $\frac{9}{x} \times 11x = \frac{99x}{x} = 99$.

5. (a) $\sqrt[3]{0.25^x} \times 32^{x-1} \div \frac{1}{3} = 24$

$\sqrt[3]{\left(\frac{1}{4}\right)^x} \times 2^{5(x-1)} = 24 \times \frac{1}{3}$

$\left(\frac{1}{2^2}\right)^{x\left(\frac{1}{3}\right)} \times 2^{5x-5} = 8$

$\frac{1}{2^{\frac{2}{3}x}} \times 2^{5x-5} = 2^3$ [1]

$2^{-\frac{2}{3}x+5x-5} = 2^3$

$\therefore -\frac{2}{3}x + 5x - 5 = 3$

$\frac{13}{3}x = 8$

$x = 8 \div \frac{13}{3}$

$= \mathbf{1\frac{11}{13}}$ [1]

Must-Know Concept:
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$a^{-n} = \frac{1}{a^n}$
List of numbers which can be expressed in index notation with a common base 2: 2, 4, 8, 16, 32, …

(b) Method 1

$2(2x-1)^2 - 3(2x-1) = 5$

$2[(2x)^2 - 2(2x)(1) + 1^2] - 6x + 3 = 5$

$2(4x^2 - 4x + 1) - 6x + 3 = 5$

$8x^2 - 8x + 2 - 6x + 3 - 5 = 0$

$8x^2 - 14x = 0$ [1]

$x(8x - 14) = 0$

$x = \mathbf{0}$ or $8x - 14 = 0$

[1] $x = \mathbf{1.75}$ [1]

Method 2

$2(2x-1)^2 - 3(2x-1) = 5$

Let $y = 2x - 1$

$2y^2 - 3y = 5$

$2y^2 - 3y - 5 = 0$

$(2y - 5)(y + 1) = 0$ [1]

$2y - 5 = 0$ or $y + 1 = 0$

$y = 2.5$ $\quad y = -1$

$\therefore 2x - 1 = 2.5$ or $2x - 1 = -1$

$2x = 3.5$ $\quad 2x = 0$

$x = \mathbf{1.75}$ $\quad x = \mathbf{0}$

[1] [1]

Must-Know Concept:
Note that $2(2x-1)^2 - 3(2x-1) = 5$ has the same form as $2y^2 - 3y = 5$, where $y = 2x - 1$.

6. (a) $\left(-\frac{1}{3}a^{-1}b^3\right)^2 \div \sqrt{a^3b^5} \times \frac{5a^0}{(2b^{-2})^2}$

$= \frac{1}{9}a^{-2}b^6 \div (a^3b^5)^{\frac{1}{2}} \times \frac{5}{4b^{-4}}$ [1]

$= \frac{1}{9}a^{-2}b^6 \div a^{\frac{3}{2}}b^{\frac{5}{2}} \times \frac{5b^4}{4}$

$= \frac{5}{36}a^{-2-\frac{3}{2}}b^{6-\frac{5}{2}+4}$

$= \frac{5}{36}a^{-\frac{7}{2}}b^{\frac{15}{2}}$

$= \mathbf{\frac{5b^{\frac{15}{2}}}{36a^{\frac{7}{2}}}}$ [1]

Must-Know Concept:
$\sqrt[n]{a} = a^{\frac{1}{n}}$
$a^0 = 1$
$a^{-n} = \frac{1}{a^n}$

(b) (i) $x^2 - 8x + 11$

$= x^2 - 8x + \left(-\frac{8}{2}\right)^2 + 11 - \left(-\frac{8}{2}\right)^2$

$= \mathbf{(x-4)^2 - 5}$ [1]

Must-Know Concept:
When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(ii) $2x^4 - 16x^2 = -22$

$x^4 - 8x^2 = -11$

$x^4 - 8x^2 + 11 = 0$

$(x^2 - 4)^2 - 5 = 0$ [1]

$(x^2 - 4)^2 = 5$

$x^2 - 4 = \pm\sqrt{5}$

$x^2 - 4 \approx 2.236\ 07$

$x^2 = 6.236\ 07$

$x = \pm\sqrt{6.236\ 07}$

$x = \mathbf{2.50}$ or $x = \mathbf{-2.50}$ [1]

or

$x^2 - 4 = -2.236\ 07$

$x^2 = 1.763\ 93$

$x = \pm\sqrt{1.763\ 93}$

$x = \mathbf{1.33}$ or $x = \mathbf{-1.33}$ [1]

(3 s.f.) (3 s.f.)

Must-Know Concept:
Note that $x^4 - 8x^2 + 11 = (x^2)^2 - 8(x^2) + 11$. It has the same form as $x^2 - 8x + 11$.
Hence, we can make use of our answer in (b)(i).

7. (a) $(2x+3)^2 - 45 = 0$

$(2x+3)^2 = 45$

$2x+3 = \pm\sqrt{45}$ [1]

$2x+3 \approx 6.708\,20$ or $2x+3 \approx -6.708\,20$

$2x = 3.708\,20$ $\quad$ $2x = -9.708\,20$

$x = 1.8541$ $\quad$ $x = -4.8541$

$= \mathbf{1.85}$ (3 s.f.) $\quad$ $= \mathbf{-4.85}$ [1] (3 s.f.)

> **Must-Know Concept:**
> When taking square roots on both sides of the equation, do not forget the ± sign.

(b) $\frac{1}{3x^2-12} - \frac{3}{x+2} + \frac{7}{2-x} = 1$

$\frac{1}{3(x^2-4)} - \frac{3}{x+2} + \frac{7}{2-x} = 1$

$\frac{1}{3(x-2)(x+2)} - \frac{3}{x+2} + \frac{7}{2-x} = 1$

$\frac{1}{3(x-2)(x+2)} - \frac{3}{x+2} - \frac{7}{x-2} = 1$ [1]

$\frac{1-9(x-2)-21(x+2)}{3(x-2)(x+2)} = 1$

$\frac{1-9x+18-21x-42}{3(x-2)(x+2)} = 1$

$\frac{-30x-23}{3(x-2)(x+2)} = 1$

Perform cross-multiplication:

$-30x-23 = 3(x-2)(x+2)$ [1]

$-30x-23 = 3x^2-12$

$3x^2+30x+11 = 0$

$x = \frac{-30 \pm \sqrt{30^2-4(3)(11)}}{2(3)}$ [1]

$x = \frac{-30 \pm \sqrt{768}}{6}$

$x \approx -0.381\,198$ or $x \approx -9.618\,80$

$x = \mathbf{-0.381}$ (3 s.f.) $\quad$ $x = \mathbf{-9.62}$ [1] (3 s.f.)

> **Must-Know Concept:**
> To add or subtract fractions, a common denominator is required.
> Note that $2-x = -(x-2)$.
> *Marking scheme:*
> *The final mark should be awarded only if both answers are correct.*

8. (a) $x(x-5)-(2x+1)(x-2) = 3$

$x^2-5x-(2x^2-4x+x-2) = 3$

$x^2-5x-2x^2+4x-x+2-3 = 0$

$-x^2-2x-1 = 0$

$x^2+2x+1 = 0$ [1]

×	x	$+1$
x	x^2	x
$+1$	x	1

$x + x = 2x$

$(x+1)^2 = 0$ [1]

$x+1 = 0$

$x = \mathbf{-1}$ [1]

> **Must-Know Concept:**
> $(a+b)(c+d) = ac+ad+bc+bd$

(b) $\frac{5}{x-2} + \frac{x}{3x^2-5x-2} = 1\frac{2}{7}$

$\frac{5}{x-2} + \frac{x}{(3x+1)(x-2)} = 1\frac{2}{7}$

$\frac{5(3x+1)+x}{(3x+1)(x-2)} = \frac{9}{7}$

$\frac{15x+5+x}{(3x+1)(x-2)} = \frac{9}{7}$

$\frac{16x+5}{(3x+1)(x-2)} = \frac{9}{7}$ [1]

Perform cross-multiplication:

$9(3x+1)(x-2) = 7(16x+5)$

$9(3x^2-5x-2) = 112x+35$

$27x^2-45x-18-112x-35 = 0$

$27x^2-157x-53 = 0$ [1]

$x = \frac{-(-157) \pm \sqrt{(-157)^2-4(27)(-53)}}{2(27)}$ [1]

$x = \frac{157 \pm \sqrt{30\,373}}{54}$

$x = \frac{157 + \sqrt{30\,373}}{54}$ or $x = \frac{157 - \sqrt{30\,373}}{54}$

$x \approx 6.13479$ $\quad$ $x \approx -0.319972$

$x = \mathbf{6.13}$ (3 s.f.) $\quad$ $x = \mathbf{-0.320}$ (3 s.f.) [1]

> **Must-Know Concept:**
> To simplify the given fractions, factorise $3x^2-5x-2$ using the multiplication frame.
> A quadratic equation of the form $ax^2+bx+c=0$ can be solved using the quadratic formula:
> $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

9. (a) $\left(\frac{2}{x}+1\right)(x+3)=-5$

$2+\frac{6}{x}+x+3=-5$

$x+10+\frac{6}{x}=0$

Multiply the equation by x:

$x^2+10x+6=0$ [1]

$x=\frac{-10\pm\sqrt{10^2-4(1)(6)}}{2(1)}$ [1]

$x=\frac{-10\pm\sqrt{76}}{2}$

$x=\frac{-10+\sqrt{76}}{2}$ or $x=\frac{-10-\sqrt{76}}{2}$

$\approx -0.641\ 101$ $\approx -9.358\ 90$

$= \mathbf{-0.641}$ (3 s.f.) $= \mathbf{-9.36}$ (3 s.f.) [1]

> **Must-Know Concept:**
>
> $(a+b)(c+d)=ac+ad+bc+bd$
>
> A quadratic equation of the form $ax^2+bx+c=0$ can be solved using the quadratic formula:
>
> $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
>
> *Marking scheme:*
> *The final mark should be awarded only if both answers are correct.*

(b) $x+4-\frac{5}{2x-7}=3$

$x+4-3=\frac{5}{2x-7}$

$x+1=\frac{5}{2x-7}$

Perform cross-multiplication:

$(x+1)(2x-7)=5$ [1]

$2x^2-7x+2x-7=5$

$2x^2-5x-12=0$ [1]

$(2x+3)(x-4)=0$ [1]

$2x+3=0$ or $x-4=0$

$2x=-3$ $\mathbf{x=4}$

$\mathbf{x=-1.5}$ [1]

> **Must-Know Concept:**
>
> The equation $2x^2-5x-12=0$ can be solved by either:
>
> (1) Factorising $2x^2-5x-12$ using the multiplication frame or
>
> (2) Using the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
>
> *Marking scheme:*
> *The final mark should be awarded only if both answers are correct.*

10. (a) Line 10: $\frac{1}{10}+\frac{1}{11}=\frac{21}{110}$ [1]

(b) Line n: $\frac{1}{n}+\frac{1}{n+1}=\frac{2n+1}{n(n+1)}$ [1]

> **Must-Know Concept:**
>
> Line 1: $1+\frac{1}{2}=\frac{1+2}{1\times 2}=\frac{3}{2}$
>
> Line 2: $\frac{1}{2}+\frac{1}{3}=\frac{2+3}{2\times 3}=\frac{5}{6}$
>
> Line 3: $\frac{1}{3}+\frac{1}{4}=\frac{3+4}{3\times 4}=\frac{7}{12}$
>
> $\therefore$ Line n: $\frac{1}{n}+\frac{1}{n+1}=\frac{n+n+1}{n(n+1)}=\frac{2n+1}{n(n+1)}$

(c) Since $xy=992$ and $y=x+1$,

$x(x+1)=992$

$x^2+x-992=0$

$(x-31)(x+32)=0$

$x-31=0$ or $x+32=0$

$\mathbf{x=31}$ [1] $x=-32$ [1] (rejected)

$y=31+1$

$=\mathbf{32}$ [1]

$z=31+32$

$=\mathbf{63}$ [1]

(d) $\frac{3}{2}-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}-\frac{13}{42}+\frac{15}{56}-\frac{17}{72}$

$=\left(1+\frac{1}{2}\right)-\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)-\ldots+\left(\frac{1}{7}+\frac{1}{8}\right)-\left(\frac{1}{8}+\frac{1}{9}\right)$ [1]

$=1+\cancel{\frac{1}{2}}-\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{4}}-\ldots+\cancel{\frac{1}{7}}+\cancel{\frac{1}{8}}-\cancel{\frac{1}{8}}-\frac{1}{9}$ [1]

$=1-\frac{1}{9}$

$=\mathbf{\frac{8}{9}}$ [1]

> **Must-Know Concept:**
>
> Write each of the fractions in $\frac{3}{2}-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}-\frac{13}{42}+\frac{15}{56}-\frac{17}{72}$ as a sum of two fractions first.

11. (a) Line 5: $6^2 - 19 = 17$ [1]

(b) $T_n = (n+1)^2 - [3 + 4(n-1)]$ [1]

$= n^2 + 2n + 1 - [3 + 4n - 4]$

$= n^2 + 2n + 1 - [4n - 1]$

$= n^2 + 2n + 1 - 4n + 1$

$= n^2 - 2n + 2$ [1]

$a = \mathbf{1}$, $b = \mathbf{-2}$ **and** $c = \mathbf{2}$ [1]

> **Must-Know Concept:**
> First, write down a separate expression for each of the following patterns
>
> | Pattern | Expression |
> |---|---|
> | $2^2, 3^2, 4^2, 5^2, \ldots$ | $(n+1)^2$ |
> | $3, 7, 11, 15, \ldots$ | $4n - 1$ |
>
> Next, we subtract the second expression from the first to obtain the answer.

(c) $T_{28} = (28)^2 - 2(28) + 2$

$= \mathbf{730}$ [1]

> **Must-Know Concept:**
> Substitute $n = 28$ into the expression found in (b).

(d) Suppose that 1268 lies in the sequence.

Then $T_n = 1268$

$n^2 - 2n + 2 = 1268$

$n^2 - 2n - 1266 = 0$

$n = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1266)}}{2(1)}$ [1]

$n = \dfrac{2 \pm \sqrt{5068}}{2}$

$n = \dfrac{2 + \sqrt{5068}}{2}$ or $n = \dfrac{2 - \sqrt{5068}}{2}$

$n \approx 36.5949$ $\quad n \approx -34.5949$ [1]

Since n is not a positive integer, 1268 does not lie in the sequence. [1]

> **Must-Know Concept:**
> Since n represents the position of a term in a number sequence, n must be a positive integer.

12. (a) (i) $12p - 3p^3 = 3p(4 - p^2)$ [1]

$= \mathbf{3p(2 - p)(2 + p)}$ [1]

> **Must-Know Concept:**
> After performing factorisation, always check if the result can be factorised further.
> $a^2 - b^2 = (a - b)(a + b)$

(ii) $12p = 3p^3$

$12p - 3p^3 = 0$

$3p(2 - p)(2 + p) = 0$

$3p = 0$ or $2 - p = 0$ or $2 + p = 0$ [1]

$p = 0$ $\quad p = 2$ $\quad p = -2$ [1]

$\therefore p = \mathbf{-2, 0}$ **and** $\mathbf{2}$

> **Must-Know Concept:**
> Make use of the answer in (a)(i).

(b) (i) $\dfrac{3}{2x^2 + 5x - 3} + \dfrac{2}{x+3}$

$= \dfrac{3}{(2x-1)(x+3)} + \dfrac{2}{x+3}$ [1]

$= \dfrac{3 + 2(2x-1)}{(2x-1)(x+3)}$

$= \dfrac{3 + 4x - 2}{(2x-1)(x+3)}$

$= \mathbf{\dfrac{4x+1}{(2x-1)(x+3)}}$ [1]

> **Must-Know Concept:**
> Factorise $2x^2 + 5x - 3$ using the multiplication frame first.
> Note that $-(1 - 2x) = 2x - 1$.

(ii) $\dfrac{3}{2x^2 + 5x - 3} + \dfrac{2}{x+3} = -1$

$\dfrac{4x+1}{(2x-1)(x+3)} = -1$

Perform cross-multiplication:

$4x + 1 = -(2x - 1)(x + 3)$

$4x + 1 = -(2x^2 + 5x - 3)$

$4x + 1 = -2x^2 - 5x + 3$

$2x^2 + 9x - 2 = 0$ [1]

$x = \dfrac{-9 \pm \sqrt{9^2 - 4(2)(-2)}}{2(2)}$ [1]

$x = \dfrac{-9 \pm \sqrt{97}}{4}$

$x = \dfrac{-9 + \sqrt{97}}{4}$ or $x = \dfrac{-9 - \sqrt{97}}{4}$

$x \approx 0.212\,214$ or $x \approx -4.712\,21$

$x = \mathbf{0.212}$ or $x = \mathbf{-4.71}$ [1]

(3 s.f.) $\quad$ (3 s.f.)

> **Must-Know Concept:**
> A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
> *Marking scheme:*
> *The final mark should be awarded only if both answers are correct.*

13. (a) Actual speed of the ship = $(54 - x)$ km/h

Time needed = $\frac{72}{54 - x}$ **h** [1]

Must-Know Concept:
Since the ship travels against the ocean current, its actual speed is less than 54 km/h.
Time taken = $\frac{\text{Distance}}{\text{Speed}}$

(b) Actual speed of the ship = $(54 + x)$ km/h

Time needed = $\frac{72}{54 + x}$ **h** [1]

Must-Know Concept:
Since the ship travels along the same direction as the ocean current, its actual speed is more than 54 km/h.
Time taken = $\frac{\text{Distance}}{\text{Speed}}$

(c) Since the difference in the time taken is 27 minutes,

$$\frac{72}{54 - x} - \frac{72}{54 + x} = \frac{27}{60}$$ [1]

$$\frac{72(54 + x) - 72(54 - x)}{(54 - x)(54 + x)} = \frac{27}{60}$$

$$\frac{3888 + 72x - 3888 + 72x}{(54 - x)(54 + x)} = \frac{27}{60}$$

$$\frac{144x}{(54 - x)(54 + x)} = \frac{9}{20}$$

Perform cross-multiplication:

$9(54 - x)(54 + x) = 2880x$ [1]

$9(2916 - x^2) = 2880x$

$2916 - x^2 = 320x$

$\mathbf{x^2 + 320x - 2916 = 0}$ (shown) [1]

Must-Know Concept:
The time taken for the ship to travel against the current is more than the time taken for it to travel along the same direction as the current. Therefore, we subtract $\frac{72}{54 + x}$ from $\frac{72}{54 - x}$.
Do not forget to convert 27 min to hour.
Marking scheme:
Award 1 mark for formulating a correct equation at the start.
Award 1 mark for simplifying the equation into one that does not involve fractions.
Award the final mark for simplifying the equation to the one stated in the question.

(d) $x^2 + 320x - 2916 = 0$

$$x = \frac{-320 \pm \sqrt{320^2 - 4(1)(-2916)}}{2(1)}$$ [1]

$$x = \frac{-320 \pm \sqrt{114\,064}}{2(1)}$$

$x = \frac{-320 + \sqrt{114\,064}}{2(1)}$ or $x = \frac{-320 - \sqrt{114\,064}}{2(1)}$

$x \approx \mathbf{8.866\,81}$ $\quad$ $x \approx -328.867$

$= \mathbf{8.87}$ (2 d.p.) [1] $\quad$ $= -328.87$ (2 d.p.) (rejected) [1]

Must-Know Concept:
A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
In this question, x represents the speed of the ocean current. Therefore, it cannot be negative.

(e) Time taken by the ship

$= \frac{100 \text{ km}}{(54 - 8.866\,81) \text{ km/h}}$

$\approx 2.215\,66$ h

$=$ **2 h 12 min 56 s** (nearest second) [1]

Must-Know Concept:
Time taken = $\frac{\text{Distance}}{\text{Speed}}$

14. (a) $\mathbf{\$\left(\frac{84}{x}\right)}$ [1]

(b) Number of pencil cases left

$= (x - 4)$

Selling price of each pencil case

$= \$\left(\frac{84}{x} + 2.80\right)$

Total amount that she collected

$= \$\left(\frac{84}{x} + 2.80\right)(x - 4)$ [1]

$= \$\left(84 - \frac{336}{x} + 2.8x - 11.20\right)$

$= \mathbf{\$\left(72.80 - \frac{336}{x} + 2.8x\right)}$ [1]

Must-Know Concept:
$(a - b)(a - b) = a^2 - ab - ab + b^2$

(c) Profit that Mrs Lee made $= \frac{50}{100} \times \$84$
$= \$42$ [1]

Since she made a profit of \$42,
$72.80 - \frac{336}{x} + 2.8x - 84 = 42$ [1]
$2.8x - 53.2 - \frac{336}{x} = 0$
Multiply the equation by x:
$2.8x^2 - 53.2x - 336 = 0$
$\mathbf{x^2 - 19x - 120 = 0}$ (shown) [1]

Must-Know Concept:
Profit = Total selling price – Total cost price
Marking scheme:
Award 1 mark for formulating a correct equation at the start.
Award 1 mark for simplifying the equation into one that does not involve fractions.
Award the final mark for simplifying the equation to the one stated in the question.

(d) $x^2 - 19x - 120 = 0$
$(x - 24)(x + 5) = 0$ [1]
$x - 24 = 0$ or $x + 5 = 0$
$\mathbf{x = 24}$ or $x = -5$ [1]
(rejected)

Must-Know Concept:
x represents the number of pencil cases that Mrs Lee has bought. Hence, it cannot be a negative number.

(e) (i) Cost price of each pencil case
$= \$\frac{84}{24}$
$= \mathbf{\$3.50}$ [1]

Must-Know Concept:
Substitute the value of x found in (d) to the expression found in (a).

(ii) Profit she would have made if she sold all the pencil cases $= 24 \times \$2.80$
$= \$67.20$
Percentage profit $= \frac{\$67.20}{\$84} \times 100\%$
$= \mathbf{80\%}$ [1]

Must-Know Concept:
Percentage Profit $= \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$

15. (a) $\frac{1}{t}$ [1]

(b) $90 \text{ s} = \frac{90}{60}$ min
$= 1.5$ min
Time taken for Nurul to complete the puzzle
$= (t - 1.5)$ min
Fraction of the puzzle that Nurul can complete in 1 min $= \mathbf{\frac{1}{t - 1.5}}$ [1]

Must-Know Concept:
1 min = 60 s

(c) Fraction of the puzzle that they can complete in 1 min if they work together $= \frac{1}{3}$
$\therefore \frac{1}{t} + \frac{1}{t - 1.5} = \frac{1}{3}$ [1]
$\frac{t - 1.5 + t}{t(t - 1.5)} = \frac{1}{3}$
$\frac{2t - 1.5}{t(t - 1.5)} = \frac{1}{3}$
Perform cross-multiplication:
$t(t - 1.5) = 3(2t - 1.5)$ [1]
$t^2 - 1.5t = 6t - 4.5$
$t^2 - 7.5t + 4.5 = 0$
$\mathbf{2t^2 - 15t + 9 = 0}$ (shown) [1]

Must-Know Concept:
Make use of the fractions written in parts (a) and (b).
Marking scheme:
Award 1 mark for formulating a correct equation at the start.
Award 1 mark for simplifying the equation into one that does not involve fractions.
Award the final mark for simplifying the equation to the one stated in the question.

(d) $2t^2 - 15t + 9 = 0$
$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(9)}}{2(2)}$ [1]
$t = \frac{15 \pm \sqrt{153}}{4}$
$t = \frac{15 + \sqrt{153}}{4}$ or $t = \frac{15 - \sqrt{153}}{4}$
$t \approx 6.842\ 33$ or $t \approx 0.657\ 671$
$\mathbf{t = 6.842}$ (3 d.p.) [1] or $t = 0.658$ (3 d.p.) (rejected) [1]

Must-Know Concept:
A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(e) If $t = 0.658$, the time taken by Ismail to complete the puzzle $(t - 1.5)$ s will be a negative number, which is not possible. Therefore, $t = 0.658$ is not applicable for this question. [1]
(Alternative answer: The time taken by Jie Lun to complete the puzzle, t min, cannot be less than the time taken by Ismail and Jie Lun to work on the puzzle together. Therefore, $t = 0.658$ is not applicable for this question.)

(f) Time taken by Ismail to complete the puzzle if he works alone
$= (6.842\ 33 - 1.5)$ min
$= 5.342\ 33$ min
$= 5$ min 20.5398 s
$=$ **5 min 21 s** (nearest second) [1]

> **Must-Know Concept:**
> Make use of the greater value of x found in (d).

16. (a) Half of the distance between town A to town B $= 72$ km $\div 2$
$= 36$ km
Time taken by Ismail to travel the first half of the journey $= \frac{36}{x}$ h
Time taken by Ismail to travel the remaining part of the journey $= \frac{36}{x+3}$ h
Total time taken by Ismail to travel from town A to town B $= \left(\frac{36}{x} + \frac{36}{x+3}\right)$ **h** [1]

> **Must-Know Concept:**
> Ismail's speed for the remaining part of the journey $= (x + 3)$ km/h
> Time taken $= \frac{\text{Distance}}{\text{Speed}}$

(b) Time taken by Jun Wei to travel the first half of the journey $= \frac{36}{x}$ h
Time taken by Jun Wei to travel the remaining part of the journey $= \frac{36}{x-2}$ h
Total time taken by Jun Wei to travel from town A to town B $= \left(\frac{36}{x} + \frac{36}{x-2}\right)$ **h** [1]

> **Must-Know Concept:**
> Jun Wei's speed for the remaining part of the journey $= (x - 2)$ km/h
> Time taken $= \frac{\text{Distance}}{\text{Speed}}$

(c) Since the difference in time taken is 21.6 min,

$$\frac{36}{x} + \frac{36}{x-2} - \left(\frac{36}{x} + \frac{36}{x+3}\right) = \frac{21.6}{60} \quad [1]$$
$$\frac{36}{x} + \frac{36}{x-2} - \frac{36}{x} - \frac{36}{x+3} = \frac{9}{25}$$
$$\frac{36}{x-2} - \frac{36}{x+3} = \frac{9}{25}$$
$$\frac{36x + 108 - 36x + 72}{(x-2)(x+3)} = \frac{9}{25}$$
$$\frac{180}{(x-2)(x+3)} = \frac{9}{25}$$

Perform cross-multiplication:

$$9(x-2)(x+3) = 4500 \quad [1]$$
$$9(x^2 + 3x - 2x - 6) = 4500$$
$$9(x^2 + x - 6) = 4500$$
$$x^2 + x - 6 = 500$$
$$\mathbf{x^2 + x - 506 = 0} \text{ (shown)} \quad [1]$$

> **Must-Know Concept:**
> Since Jun Wei's speed for the remaining journey was slower, the total time taken by Jun Wei was more than Ismail. Therefore, we subtract $\left(\frac{36}{x} + \frac{36}{x+3}\right)$ from $\left(\frac{36}{x} + \frac{36}{x-2}\right)$.
> Do not forget to convert 21.6 min to hour.
> *Marking scheme:*
> *Award 1 mark for formulating a correct equation at the start.*
> *Award 1 mark for simplifying the equation into one that does not involve fractions.*
> *Award the final mark for simplifying the equation to the one stated in the question.*

(d) <u>Method 1</u>
$x^2 + x - 506 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-506)}}{2(1)} \quad [1]$$
$$x = \frac{-1 \pm \sqrt{2025}}{2}$$
$$x = \frac{-1 + \sqrt{2025}}{2} \quad \text{or} \quad x = \frac{-1 - \sqrt{2025}}{2}$$

$x = \mathbf{22}$ [1] or $x = -23$ (rejected) [1]

Method 2

$x^2 + x - 506 = 0$

$(x - 22)(x + 23) = 0$ [1]

$x - 22 = 0$ or $x + 23 = 0$

$x = \mathbf{22}$ or $x = -23$

[1] (rejected)

[1]

Must-Know Concept:
A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
In this question, x represents the cycling speed. Therefore, it cannot be negative.

(e) Total time taken by Ismail and Jun Wei

$= \frac{36}{22} + \frac{36}{22+3} + \frac{36}{22} + \frac{36}{22-2}$ [1]

$= 6\frac{141}{275}$ h

Mean time taken by Ismail and Jun Wei

$= 6\frac{141}{275}$ h $\div 2$

$= 3\frac{141}{550}$ h

$= 3$ h 15.3818 min

$=$ **3 h 15 min** (nearest minute) [1]

Must-Know Concept:
Mean of n numbers $= \frac{\text{Sum of } n \text{ numbers}}{n}$

17. (a) $\mathbf{\$\left(\frac{210}{x}\right)}$ [1]

Must-Know Concept:
Average cost of each book $= \frac{\text{Total cost of all books}}{\text{Total number of books}}$

(b) Amount of money that Vincent spent in 2017 $= \frac{124}{100} \times \$210$

$= \$260.40$

Number of books that Vincent purchased in 2017 $= x + 2$

Average cost of 1 book in 2017 $= \mathbf{\$\left(\frac{260.40}{x+2}\right)}$

[1]

(c) Since the average cost of each book in 2016 is 21 cents more,

$\frac{210}{x} - \frac{260.40}{x+2} = \frac{21}{100}$ [1]

$\frac{210(x+2) - 260.40x}{x(x+2)} = \frac{21}{100}$

$\frac{210x + 420 - 260.40x}{x(x+2)} = \frac{21}{100}$

Perform cross-multiplication:

$21x(x+2) = 100(420 - 50.4x)$

[1]

$21x^2 + 42x = 42\,000 - 5040x$

$21x^2 + 5082x - 42\,000 = 0$

$\mathbf{x^2 + 242x - 2000 = 0}$ (shown) [1]

Must-Know Concept:
Do not forget to convert 21 cents to dollar.
$1 = 100 cents
Award 1 mark for formulating a correct equation at the start.
Award 1 mark for simplifying the equation into one that does not involve fractions.
Award the final mark for simplifying the equation to the one stated in the question.

(d) Method 1

$x^2 + 242x - 2000 = 0$

$x = \frac{-242 \pm \sqrt{(242)^2 - 4(1)(-2000)}}{2(1)}$ [1]

$x = \frac{-242 \pm \sqrt{66\,564}}{2}$

$x = \frac{-242 + \sqrt{66\,564}}{2}$ or $x = \frac{-242 - \sqrt{66\,564}}{2}$

$x = \mathbf{8}$ or $x = -250$

[1] (rejected)

[1]

Method 2

$x^2 + 242x - 2000 = 0$

$(x - 8)(x + 250) = 0$ [1]

$x - 8 = 0$ or $x + 250 = 0$

$x = \mathbf{8}$ or $x = -250$

[1] (rejected)

[1]

Must-Know Concept:
A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
In this question, x represents the number of books that Vincent bought. Therefore, it cannot be negative.

(e) Amount of money that Vincent spent in 2018 $= \frac{150}{100} \times \$260.40$

$= \$390.60$ [1]

Number of books that Vincent bought in 2018 $= 8 + 8 + 2$

$= 18$

Average cost of each book in 2018

$= \frac{\$390.60}{18}$

$=$ **\$21.70** [1]

> **Must-Know Concept:**
> Make use of the positive value of x found in (d).

18. (a) $7 - 5x - x^2 = -(x^2 + 5x - 7)$

$= -\left[x^2 + 5x + \left(\frac{5}{2}\right)^2 - 7 - \left(\frac{5}{2}\right)^2\right]$

$= -[(x + 2.5)^2 - 13.25]$

$= -(x + 2.5)^2 + 13.25$

$= \mathbf{13.25 - (x + 2.5)^2}$ [1]

> **Must-Know Concept:**
> When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(b) (i) $y = 13.25 - (x + 2.5)^2$

For coordinates of maximum point,

$x + 2.5 = 0$

$x = -2.5$

$y = 13.25 - 0^2$

$= 13.25$

Coordinates of maximum point

$=$ **(–2.5, 13.25)** [1]

> **Must-Know Concept:**
> The turning point of a quadratic curve lies on its line of symmetry.
> The equation of the line of symmetry of $y = 7 - 5x - x^2$ is $x = -2.5$.

(ii) When $x = 0$,

$y = 13.25 - (0 + 2.5)^2$

$= 7$

When $y = 0$,

$13.25 - (x + 2.5)^2 = 0$

$(x + 2.5)^2 = 13.25$

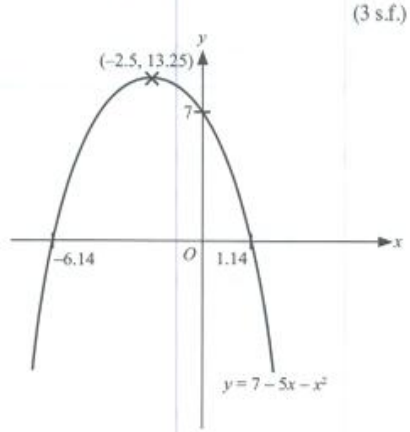
$x + 2.5 = \pm\sqrt{13.25}$

$x + 2.5 \approx 3.640\,05$ or $x + 2.5 \approx -3.640\,05$

$x = 1.140\,05$ $\quad$ $x = -6.140\,05$

≈ 1.14 $\quad$ ≈ -6.14

(3 s.f.)



> **Must-Know Concept:**
> To sketch the graph, first find the **x-intercepts** and the **y-intercept** of $y = 13.25 - (x + 2.5)^2$.
> *Marking scheme:*
> *Both x-intercepts are correctly labelled [1]*
> *y-intercept correctly labelled [1]*
> *Correct shape of curve [1]*

19. (a) When $x = 0$,

$y = 3(0 - 1)[5 + 2(0)]$

$= -15$

When $y = 0$,

$3(x - 1)(5 + 2x) = 0$

$(x - 1)(5 + 2x) = 0$

$x - 1 = 0$ or $5 + 2x = 0$

$x = 1$ or $2x = -5$

$x = -2.5$

For turning point,

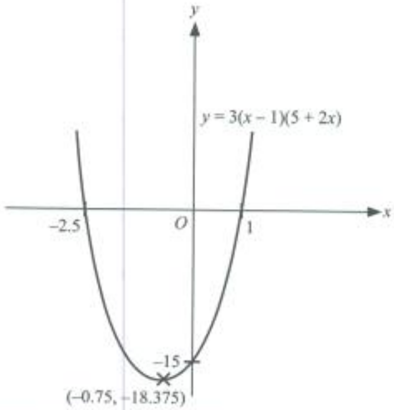
$x = \frac{1 + (-2.5)}{2}$

$= -0.75$

When $x = -0.75$,

$y = 3(-0.75 - 1)[5 + 2(-0.75)]$

$= -18.375$



Must-Know Concept:
To find the x-intercepts, substitute $y = 0$ into the given equation.
To find the y-intercept, substitute $x = 0$ into the given equation.
The turning point of a quadratic curve occurs on its line of symmetry.
Marking scheme:
Both x-intercepts are correctly labelled [1]
y-intercept and turning point are correctly labelled [1]
Correct shape of curve [1]

(b) $\boldsymbol{x = -0.75}$ [1]

Must-Know Concept:
The line of symmetry of a quadratic curve is a vertical line. It cuts the curve into two halves which are mirror images of each other.

(c) $k = \mathbf{-19}$ [1]

Must-Know Concept:
Any line with the equation $y = k$, where k is a constant, is a horizontal line.
Note that if $k < -18.375$, the horizontal line will not intersect the curve. Therefore $3(x - 1)(5 + 2x) = k$ will have no solution.

20. (a) Given that $a^2 = 4$,

$a = \pm\sqrt{4}$
$= \pm 2$

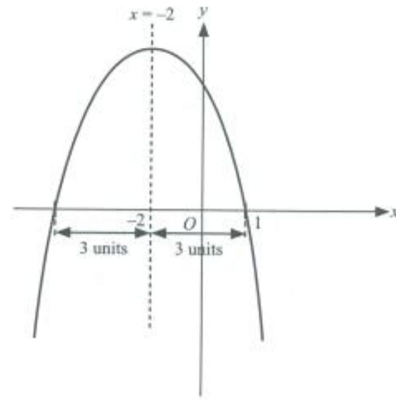
Since the curve has a maximum point, $a < 0$.

$\therefore a = \mathbf{-2}$ [1]

$y = -2x^2 + bx + c$

Since the curve cuts the x-axis at $x = 1$, $(1, 0)$ lies on the curve.

$\therefore -2(1)^2 + b(1) + c = 0$
$-2 + b + c = 0$
$b + c = 2$ ....(1)



Since the x-coordinate of the maximum point of the curve is -2,

The other x-intercept $= -2 - 3$
$= -5$

$\therefore -2(-5)^2 + b(-5) + c = 0$ [1]
$-50 - 5b + c = 0$
$c = 5b + 50$ ....(2)

Sub (2) into (1):

$b + 5b + 50 = 2$
$6b = -48$
$b = \mathbf{-8}$ [1]

Sub $b = -8$ into (2):

$c = 5(-8) + 50$
$c = \mathbf{10}$ [1]

Must-Know Concept:
For the quadratic curve $y = ax^2 + bx + c$
If $a < 0$, the curve has a maximum point.
If $a > 0$, the curve has a minimum point.

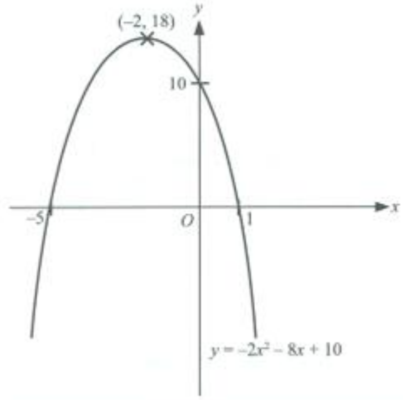
(b) $y = -2x^2 - 8x + 10$

When $x = -2$,
$y = -2(-2)^2 - 8(-2) + 10$
$= 18$

Coordinates of maximum point = **(−2, 18)** [1]

Must-Know Concept:
Substitute $x = -2$ and the values of a, b and c into y.

(c) When $x = 0$,
$y = -2(0)^2 - 8(0) + 10$
$= 10$



Must-Know Concept:
When the curve crosses the x-axis, $y = 0$.
When the curve crosses the y-axis, $x = 0$.
Marking scheme:
x-intercepts and y-intercept are labelled correctly [1]
Correct shape of curve and coordinates of turning point are shown [1]

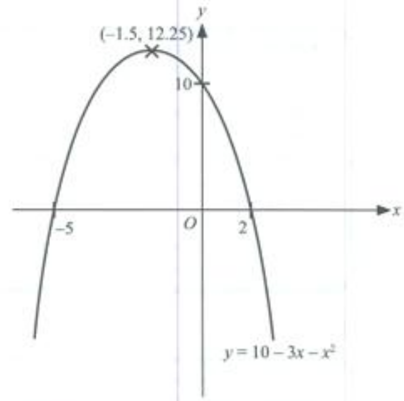
21. (a) $10 - 3x - x^2 = -(x^2 + 3x - 10)$
$= -\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - 10 - \left(\frac{3}{2}\right)^2\right]$
$= -[(x + 1.5)^2 - 12.25]$
$= \mathbf{12.25 - (x + 1.5)^2}$ [1]

Must-Know Concept:
When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(b) (i) $x + 1.5 = 0$
$\mathbf{x = -1.5}$ [1]

Must-Know Concept:
The line of symmetry of a quadratic curve is a vertical line. It cuts the curve into two halves which are mirror images of each other.

(b) (ii) When $x = 0$,
$y = 12.25 - (0 + 1.5)^2$
$= 10$
When $y = 0$,
$12.25 - (x + 1.5)^2 = 0$
$(x + 1.5)^2 = 12.25$
$x + 1.5 = \pm\sqrt{12.25}$
$x + 1.5 = 3.5$ or $x + 1.5 = -3.5$
$x = 2$ or $x = -5$
For turning point,
$x + 1.5 = 0$
$x = -1.5$
$y = 12.25 - 0^2$
$= 12.25$



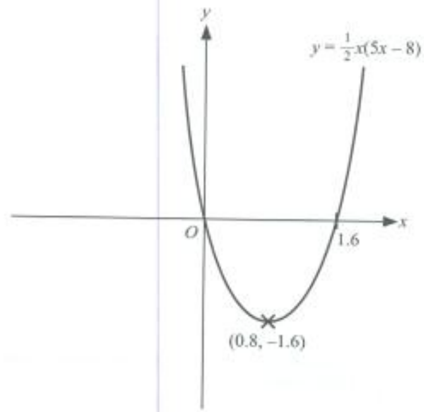
Must-Know Concept:
To find the x-intercepts of the curve, substitute $y = 0$.
To find the y-intercepts of the curve, substitute $x = 0$.
The turning point of a quadratic curve lies on its line of symmetry.
Marking scheme:
Both x-intercepts are correctly labelled [1]
y-intercept and turning point are correctly labelled [1]
Correct shape of curve [1]

22. (a) When $x = 0$,
$y = \frac{1}{2}(0)[5(0) - 8] = 0$
When $y = 0$,
$\frac{1}{2}x(5x - 8) = 0$
$\frac{1}{2}x = 0$ or $5x - 8 = 0$
$x = 0$ or $5x = 8$
$x = 1.6$
For turning point,
$x = \frac{0 + 1.6}{2}$
$= 0.8$

When $x = 0.8$,

$y = \frac{1}{2}(0.8)[5(0.8) - 8]$

$= -1.6$

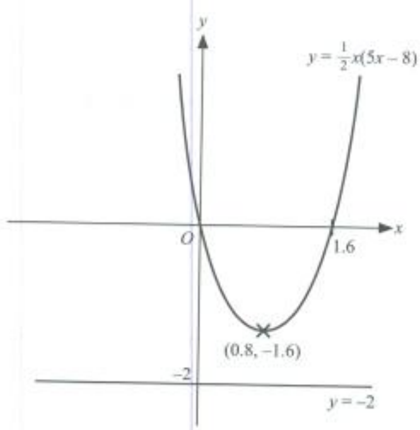


Must-Know Concept:
To find the x-intercepts of the curve, substitute $y = 0$.
To find the y-intercepts of the curve, substitute $x = 0$.
The turning point of a quadratic curve lies on its line of symmetry.
Marking scheme:
Both x-intercepts are correctly labelled [1]
Turning point is correctly labelled [1]
Correct shape of curve [1]

(b) $x(5x - 8) = -4$

$\frac{1}{2}x(5x - 8) = -2$

Draw the line $y = -2$ on the graph.



Note that the line $y = -2$ does not intersect the curve $y = \frac{1}{2}x(5x - 8)$.

$\therefore$ Number of solutions = **0** [1]

Must-Know Concept:
Any line of the form $y = c$, where c is a constant, is a horizontal line.

23. (a) $2x^2 - 8x - 7 = 2[x^2 - 4x - 3.5]$

$= 2\left[x^2 - 4x + \left(-\frac{4}{2}\right)^2 - 3.5 - \left(-\frac{4}{2}\right)^2\right]$

$= 2[(x-2)^2 - 7.5]$

$= \mathbf{2(x-2)^2 - 15}$ [1]

Must-Know Concept:
When performing 'completing the square' on an expression, note that: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

(b) (i) For turning point,

$x - 2 = 0$

$x = 2$

$y = 2(0)^2 - 15$

$= -15$

Coordinates of minimum point = **(2, −15)** [1]

Must-Know Concept:
The coordinates of the turning point of a quadratic curve lies on its line of symmetry.
Note that the equation of the line of symmetry of $y = 2x^2 - 8x - 7$ is $x = 2$.

(b) (ii) $y = 2x^2 - 8x - 7$

$= 2(x-2)^2 - 15$

When $x = 0$,

$y = 2(0-2)^2 - 15$

$= -7$

When $y = 0$,

$2(x-2)^2 - 15 = 0$

$2(x-2)^2 = 15$

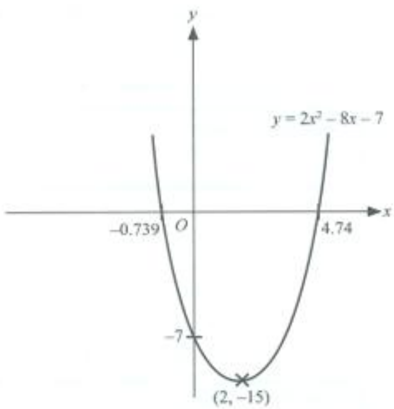
$(x-2)^2 = 7.5$

$x - 2 = \pm\sqrt{7.5}$

$x - 2 \approx 2.738\ 61$ or $x - 2 \approx -2.738\ 61$

$x = 4.738\ 61$ or $x = -0.738\ 61$

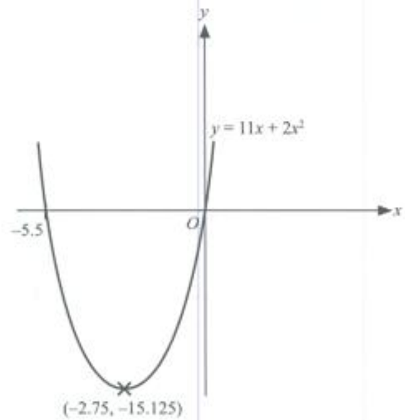
$= 4.74$ (3 s.f.) or $= -0.739$ (3 s.f.)



Must-Know Concept:
To find the x-intercepts of the curve, substitute $y = 0$.
To find the y-intercepts of the curve, substitute $x = 0$.
Marking scheme:
Both x-intercepts are correctly labelled [1]
y-intercept and turning point are correctly labelled [1]
Correct shape of curve [1]

24. (a) When $x = 0$,
$y = 11(0) + 2(0)^2$
$= 0$
When $y = 0$,
$11x + 2x^2 = 0$
$x(11 + 2x) = 0$
$x = 0$ or $11 + 2x = 0$
$2x = -11$
$x = -5.5$

For turning point,
$x = \frac{0 + (-5.5)}{2}$
$x = -2.75$
$y = 11(-2.75) + 2(-2.75)^2$
$= -15.125$

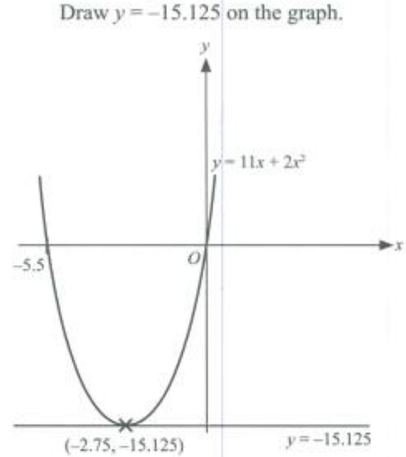


Must-Know Concept:
To find the x-intercepts of the curve, substitute $y = 0$.
To find the y-intercepts of the curve, substitute $x = 0$.
The turning point of a quadratic curve lies on its line of symmetry.
Marking scheme:
Both x-intercepts are correctly labelled [1]
Turning point is correctly labelled [1]
Correct shape of curve [1]

(b) $x = -2.75$ [1]

Must-Know Concept:
The line of symmetry of a quadratic curve is a vertical line. It cuts the curve into two halves which are mirror images of each other.

(c) $4x^2 + 22x + 30.25 = 0$
$2x^2 + 11x + 15.125 = 0$
$11x + 2x^2 = -15.125$
Draw $y = -15.125$ on the graph.



Note that the line $y = -15.125$ intersects the curve $y = 11x + 2x^2$ at its turning point.
$\therefore$ Number of solutions = **1** [1]

Must-Know Concept:
Any line of the form $y = c$, where c is a constant, is a horizontal line.

25. (a) When $y = 0$,
$x = -1$ or $x = 4$
$x + 1 = 0$ $x - 4 = 0$
$(x + 1)(x - 4) = 0$
$\therefore y = k(x + 1)(x - 4)$, where k is a constant.
When $x = 0$, $y = -2$.
$\therefore -2 = k(0 + 1)(0 - 4)$
$-2 = -4k$
$k = 0.5$
$y = 0.5(x + 1)(x - 4)$ [1]
$= 0.5(x^2 - 3x - 4)$
$= 0.5x^2 - 1.5x - 2$
$p = \mathbf{0.5}$, $q = \mathbf{-1.5}$ and $r = \mathbf{-2}$ [3]

Must-Know Concept:
Given the x- and y-intercepts of a quadratic curve, we can derive the equation of the curve.
Note that $p > 0$ since the curve has a minimum point.

(b) For minimum point:
$x = \frac{-1 + 4}{2}$
$= 1.5$
$y = 0.5(1.5)^2 - 1.5(1.5) - 2$
$= -3.125$
Coordinates of minimum point
$= (1.5, -3.125)$ [1]
Distance between minimum point and the origin $= \sqrt{(1.5 - 0)^2 + (-3.125 - 0)^2}$
$\approx 3.466\ 36$
$= \mathbf{3.47}$ **units** (3 s.f.) [1]

Must-Know Concept:
The turning point of a quadratic curve lies on its line of symmetry.
Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

1.5 Inequalities

1. (a) Greatest value of $y^2 - x^2 = 7^2 - 0^2$
$= \mathbf{49}$ [1]

Must-Know Concept:
Note that $a^2 \geq 0$ for any real value of a.

(b) Least possible value of $\frac{y}{x} = \frac{7}{-1}$
$= \mathbf{-7}$ [1]

Must-Know Concept:
Note that $\frac{7}{-1} < \frac{7}{-3}$.

(c) Greatest possible value of $\frac{1}{x} + \frac{1}{y} = \frac{1}{1} + \frac{1}{3}$
$= \mathbf{1\frac{1}{3}}$ [1]

Must-Know Concept:
Note that $\frac{1}{1} > \frac{1}{4}$.

2. (a) Least possible value of $x + 3y = -1 + 3(1)$
$= \mathbf{2}$ [1]

Must-Know Concept:
The least possible value of $x + 3y$ occurs when both x and y are the least.

(b) Least possible value of $\frac{y}{x} = \frac{7}{-1}$
$= \mathbf{-7}$ [1]

Must-Know Concept:
Note that $\frac{7}{-1} < \frac{1}{-1}$.

(c) Greatest possible value of $(x - y)^2 = (-1 - 7)^2$
$= \mathbf{64}$ [1]

Must-Know Concept:
Note that $a^2 \geq 0$ for any real value of a.

3. (a) Greatest possible value of
$2y - 3x = 2(7) - 3(-2)$
$= \mathbf{20}$ [1]

Must-Know Concept:
The greatest possible value of $(2y - 3x)$ occurs when y is the greatest and x is the least.

(b) Least possible value of $\frac{x}{y} = \frac{-2}{1}$
$= -2$ [1]

Must-Know Concept:
Note that $\frac{-2}{1} < \frac{-2}{7}$.

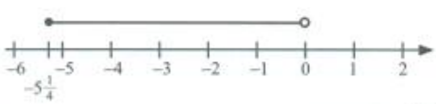
(c) Least possible value of $(x+y)^2 = (-1+1)^2$
$= 0$ [1]

Must-Know Concept:
Note that $a^2 \geqslant 0$ for any real value of a.

4\. (a)
$$\frac{3x+7}{5} \leqslant \frac{1+2x}{2} + 3$$
$$\frac{3x+7}{5} \leqslant \frac{1+2x+6}{2}$$
$$\frac{3x+7}{5} \leqslant \frac{2x+7}{2}$$
$$2(3x+7) \leqslant 5(2x+7) \quad [1]$$
$$6x + 14 \leqslant 10x + 35$$
$$-4x \leqslant 21$$
$$4x \geqslant -21$$
$$x \geqslant -5\frac{1}{4} \quad [1]$$

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(b) Since $x \geqslant -5\frac{1}{4}$ and $x < 0$,



[1]

$$-5\frac{1}{4} \leqslant x < 0$$

Must-Know Concept:
For the inequality signs $\leqslant$ and $\geqslant$, shade the circle on the number line.
For the inequality signs $<$ and $>$, do not shade the circle on the number line.
Marking scheme:
No mark should be awarded if the circles are not drawn correctly, or if $-5\frac{1}{4}$ is not labelled on the number line.

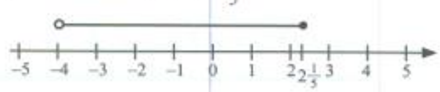
5\. (a) $\frac{3k}{4} - 4 < k - 3 \leqslant \frac{5-3k}{2}$

$\frac{3k}{4} - 4 < k - 3$ and	$k - 3 \leqslant \frac{5-3k}{2}$
$\frac{3k}{4} - k < -3 + 4$	$2(k-3) \leqslant 5 - 3k$
$-\frac{1}{4}k < 1$	$2k - 6 \leqslant 5 - 3k$
$\frac{1}{4}k > -1$	$5k \leqslant 11$
$k > -4$	$k \leqslant 2\frac{1}{5}$
[1]	[1]

$\therefore -4 < k \leqslant 2\frac{1}{5}$ [1]

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(b) $-4 < k \leqslant 2\frac{1}{5}$



[1]

Must-Know Concept:
For the inequality signs $\leqslant$ and $\geqslant$, shade the circle on the number line.
For the inequality signs $<$ and $>$, do not shade the circle on the number line.
Marking scheme:
No mark should be awarded if the circles are not drawn correctly, or if $2\frac{1}{5}$ is not labelled on the number line.

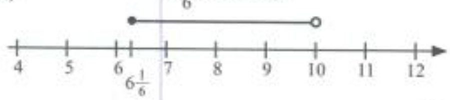
6\. (a)

$1 + x < 3 + \frac{4x}{5}$ and	$\frac{3x+13}{3} \leqslant \frac{6x+5}{4}$
$x - \frac{4x}{5} < 3 - 1$	$4(3x+13) \leqslant 3(6x+5)$
$\frac{1}{5}x < 2$	$12x + 52 \leqslant 18x + 15$
$x < 10$	$12x - 18x \leqslant 15 - 52$
[1]	$-6x \leqslant -37$
	$6x \geqslant 37$
	$x \geqslant 6\frac{1}{6}$ [1]

$\therefore 6\frac{1}{6} \leqslant x < 10$ [1]

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(b) $6\frac{1}{6} \leqslant x < 10$



[1]

Must-Know Concept:
For the inequality signs ⩽ and ⩾, shade the circle on the number line
For the inequality signs < and >, do not shade the circle on the number line.
Marking scheme:
No mark should be awarded if the circles are not drawn correctly, or if $6\frac{1}{6}$ is not labelled on the number line.

(c) 7, 8, 9 [1]

Must-Know Concept:
List of integers:
…, −3, −2, −1, 0, 1, 2, 3, …

7. (a) $\frac{4a+1}{3} < a + 2\frac{1}{2} \leqslant 5 + \frac{3a}{2}$

$\frac{4a+1}{3} < a + 2\frac{1}{2}$ or $a + 2\frac{1}{2} \leqslant 5 + \frac{3a}{2}$

$4a + 1 < 3\left(a + 2\frac{1}{2}\right)$ $\qquad a - \frac{3a}{2} \leqslant 5 - 2\frac{1}{2}$

$4a + 1 < 3a + 7\frac{1}{2}$ $\qquad -\frac{1}{2}a \leqslant 2\frac{1}{2}$

$4a - 3a < 7\frac{1}{2} - 1$ $\qquad \frac{1}{2}a \geqslant -2\frac{1}{2}$

$a < 6\frac{1}{2}$ $\qquad a \geqslant -5$

[1] [1]

$\therefore \mathbf{-5 \leqslant a < 6\frac{1}{2}}$ [1]

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(b) Largest possible prime value of a = **5** [1]

Must-Know Concept:
A prime number is only divisible by 1 and itself.
Note that 1 is not a prime number. 6 is not a prime number as it is divisible by 1, 2, 3 and 6.

8. (a) $\frac{3x+1}{2} - \frac{5(x-2)}{3} = \frac{3(3x+1) - 10(x-2)}{6}$ [1]

$= \frac{9x + 3 - 10x + 20}{6}$

$= \mathbf{\frac{23-x}{6}}$ [1]

Must-Know Concept:
To add or subtract fractions, a common denominator is required.
In this case, a common multiple of 2 and 3 is 6.

(b) (i) $1 < \frac{3x+1}{2} - \frac{5(x-2)}{3} \leqslant \frac{x}{4} + 2$

$1 < \frac{23-x}{6} \leqslant \frac{x}{4} + 2$

$1 < \frac{23-x}{6}$ and $\frac{23-x}{6} \leqslant \frac{x}{4} + 2$

$6 < 23 - x$ $\qquad \frac{23-x}{6} \leqslant \frac{x+8}{4}$

$x < 23 - 6$ $\qquad 4(23-x) \leqslant 6x + 48$

$x < 17$ $\qquad 92 - 4x \leqslant 6x + 48$

[1] $\qquad -10x \leqslant -44$

$10x \geqslant 44$

$x \geqslant 4\frac{2}{5}$ [1]

$\therefore \mathbf{4\frac{2}{5} \leqslant x < 17}$ [1]

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(ii) Largest possible prime value of x = **13** [1]

Must-Know Concept:
A prime number is only divisible by 1 and itself.
In this question, the largest possible integer value of x is 16. However, 16 is not a prime number.

9. (a) (i) Greatest possible value of
$3y - 2x = 3(8) - 2(-2)$
$= 24 + 4$
$= \mathbf{28}$ [1]

Must-Know Concept:
To obtain the greatest possible value, we subtract a negative x value.

(ii) Least possible value of $\frac{y}{x} = \frac{8}{-1}$
$= \mathbf{-8}$ [1]

Must-Know Concept:
Note that $\frac{8}{-1} < \frac{8}{-2}$.

(iii) Least possible value of
$x^3 + y^2 = (-2)^3 + 0^2$
$= \mathbf{-8}$ [1]

Must-Know Concept:
Note that $a^2 \geqslant 0$ for any real value of a.

(b) (i) Since $\left(\frac{3}{2}p+5\right)^\circ$ is a reflex angle,

$180^\circ < \left(\frac{3}{2}p+5\right)^\circ < 360^\circ$ [1]

$180 < \frac{3}{2}p + 5$ and $\frac{3}{2}p + 5 < 360$

$175 < \frac{3}{2}p$ $\qquad$ $\frac{3}{2}p < 355$

$116\frac{2}{3} < p$ $\qquad$ $p < 236\frac{2}{3}$

$\therefore \mathbf{116\frac{2}{3} < p < 236\frac{2}{3}}$ [1]

Must-Know Concept:
A reflex angle is greater than 180° but less than 360°.
Marking scheme:
Do not award the marks if the inequality signs are written as '≤'.

(ii) Largest possible integer value of p = **236** [1]

Must-Know Concept:
List of integers:
…,–3, –2, –1, 0, 1, 2, 3, …

10. (a) $\left(\frac{1}{p}-p\right)^2 = \left(\frac{1}{p}\right)^2 - 2\left(\frac{1}{p}\right)(p) + p^2$

$= \frac{1}{p^2} - 2 + p^2$

$= p^2 + \frac{1}{p^2} - 2$

$= 32 - 2$

$= 30$ [1]

$\frac{1-p^2}{p} = \frac{1}{p} - p$

$= \pm\sqrt{30}$

$\approx \pm 5.477\ 23$

$= \mathbf{\pm 5.48}$ (3 s.f.) [1]

Must-Know Concept:
Note that $\frac{1-p^2}{p} = \frac{1}{p} - \frac{p^2}{p}$
$= \frac{1}{p} - p$
$(a-b)^2 = a^2 - 2ab + b^2$

(b) (i) $\frac{2y-1}{3} < \frac{3y+5}{2} \leqslant y + 3\frac{1}{3}$

$\frac{2y-1}{3} < \frac{3y+5}{2}$ and $\frac{3y+5}{2} \leqslant y + 3\frac{1}{3}$

$2(2y-1) < 3(3y+5)$ $\qquad$ $\frac{3}{2}y + \frac{5}{2} \leqslant y + 3\frac{1}{3}$

$4y - 2 < 9y + 15$ $\qquad$ $\frac{3}{2}y - y \leqslant 3\frac{1}{3} - \frac{5}{2}$

$-5y < 17$ $\qquad$ $\frac{1}{2}y \leqslant \frac{5}{6}$

$5y > -17$ $\qquad$ $y \leqslant 1\frac{2}{3}$

$y > -3\frac{2}{5}$ $\qquad$ [1]

[1]

$\therefore \mathbf{-3\frac{2}{5} < y \leqslant 1\frac{2}{3}}$ [1]

Must-Know Concept:
When both sides of an inequality are multiplied/divided by a negative number, the inequality sign changes direction.

(ii) –3, –2, –1, 0, 1 [1]

Must-Know Concept:
List of integers:
…,–3, –2, –1, 0, 1, 2, 3, …

11. Let the number of files that he could buy be x.
$\therefore$ Number of pens that he could buy $= 2x$
Since he would like to spend at most \$50,
$3.70(x) + 2.20(2x) + 3(1.90) \leqslant 50$ [1]
$3.7x + 4.4x + 5.7 \leqslant 50$
$8.1x \leqslant 50 - 5.7$
$8.1x \leqslant 44.3$
$x \leqslant \frac{44.3}{8.1}$
$x \leqslant 5.47$ (3 s.f.) [2]

Largest number of files that could be bought = 5
Largest number of pens that could be bought
= 2(5)
= **10** [1]

Must-Know Concept:
Since Mr Fadly would like to spend **at most** \$50, the inequality sign should be '≤' instead of '<'.
Marking scheme:
Deduct 1 mark if the inequality sign was written as '<'.

1.6 Problems in Real-World Context

1. Since the total interest earned was $1648.64,

$\$20\,000 + \$1648.64 = \$20\,000\left(1 + \frac{\frac{r}{2}}{100}\right)^{2\times 2}$

$21\,648.64 = 20\,000\left(1 + \frac{r}{200}\right)^4$ [1]

$\left(1 + \frac{r}{200}\right)^4 = \frac{21\,648.64}{20\,000}$

$1 + \frac{r}{200} = \sqrt[4]{\frac{21\,648.64}{20\,000}}$

$1 + \frac{r}{200} \approx 1.019\,999$

$\frac{r}{200} = 0.019\,999$

$r = 3.9998$

$\approx$ **4** (nearest whole number) [1]

> **Must-Know Concept:**
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> Since the interest was compounded half-yearly, the values of r and n have to be adjusted accordingly.

2. (a) Total amount $= \$60\,000\left(1 + \frac{5.8 \div 12}{100}\right)^{3\frac{1}{2}\times 12}$ [1]

$\approx \$73\,468.413$

Total interest that he will earn

$= \$73\,468.413 - \$60\,000$

$=$ **$13 468.41** (nearest cent) [1]

> **Must-Know Concept:**
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> Since the interest was compounded half-yearly, the values of r and n have to be adjusted accordingly.

(b) Percentage profit $= \frac{\$13\,468.41}{\$60\,000} \times 100\%$

$\approx 22.4474\%$

$=$ **22.4%** (3 s.f.) [1]

> **Must-Know Concept:**
> Percentage Profit $= \frac{\text{Profit}}{\text{Original Value}} \times 100\%$

3. Amount of money that Azizah invests in bank A $= \frac{75}{100} \times \$4500$

$= \$3375$ [1]

Amount of money that Azizah invests in bank B $= \$4500 - \$3375$

$= \$1125$

Interest earned from Bank A $= \frac{3375 \times 6.5 \times 2}{100}$

$= \$438.75$ [1]

Total amount that she will have in bank B after 2 years $= \$1125\left(1 + \frac{\frac{6}{12}}{100}\right)^{2\times 12}$

$\approx \$1268.055$ [1]

Total amount of money that she will have after 2 years $= \$3375 + \$438.75 + \$1268.055$

$=$ **$5082** (nearest dollar) [1]

> **Must-Know Concept:**
> Simple Interest
> Interest, $I = \frac{PRT}{100}$
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> The simplest interest formula returns only the interest, while the compound interest formula returns the total amount.

4. Let the amount of money that Mr Wee invested in bank A be $\$x$.

Amount of money that he invested in bank B

$= \$(5000 - x)$

Interest earned from bank A $= \frac{x(5.5)(3)}{100}$ [1]

$= \$0.165x$

Total amount in bank A at the end of 3 years

$= \$x + \$0.165x$

$= \$1.165x$

Total amount in bank B at the end of 3 years

$= (5000 - x)\left(1 + \frac{5}{100}\right)^3$ [1]

$= (5000 - x)(1.157\,625)$

$= \$(5788.125 - 1.157\,625x)$ [1]

Since the total amount that he had in both banks is $5810.25,

$1.165x + 5788.125 - 1.157\,625x = 5810.25$ [1]

$0.007\,375x = 22.125$

$x = \frac{22.125}{0.007\,375}$

$= 3000$

Amount of money that he invested in bank A

$=$ **$3000** [1]

> **Must-Know Concept:**
> Simple Interest
> Interest, $I = \frac{PRT}{100}$
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> The simplest interest formula returns only the interest, while the compound interest formula returns the total amount.

5. (a) Total amount $= \$50\ 000\left(1 + \frac{\frac{6.2}{2}}{100}\right)^{4 \times 2}$ [1]
$= \mathbf{\$63\ 832.13}$ (nearest cent) [1]

> **Must-Know Concept:**
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> Since the interest was compounded half-yearly, the values of r and n have to be adjusted accordingly.

(b) Let the amount of money that Bryan borrowed be $\$P$.
$P + 3467.75 = P\left(1 + \frac{5}{100}\right)^3$ [1]
$P + 3467.75 = 1.157\ 625P$
$1.157\ 625P - P = 3467.75$
$0.157\ 625P = 3467.75$
$P = \frac{3467.75}{0.157\ 625}$
$= 22\ 000$
$\therefore$ Amount of money that he borrowed
$= \mathbf{\$22\ 000}$ [1]

> **Must-Know Concept:**
> Since Bryan borrowed $\$P$ and the interest that he paid was \$3467.75,
> Total amount $= \$(P + 3467.75)$

6. (a) Deposit $= \frac{15}{100} \times \$2299$
$= \$344.85$ [1]
Total monthly instalments $= 24 \times \$129$
$= \$3096$
Total amount that Aaron will pay altogether
$= \$344.85 + \$3096$
$= \mathbf{\$3440.85}$ [1]

> **Must-Know Concept:**
> Hire purchase price = Deposit + Total instalments

(b) Let the rate of simple interest be R.
Principal amount $= \$2299 - \$344.85$
$= \$1954.15$ [1]
Total interest charged over 2 years
$= \$3440.85 - \$2299$
$= \$1141.85$
$\therefore 1141.85 = \frac{1954.15 \times R \times 2}{100}$ [1]
$1141.85 = 39.083R$
$R = \frac{1141.85}{39.083}$
$\approx 29.2160\%$
$= \mathbf{29.2\%}$ (3 s.f.) [1]

> **Must-Know Concept:**
> Simple Interest
> Interest, $I = \frac{PRT}{100}$

(c) Price that the seller purchased the laptop at
$= \frac{70}{100} \times \$2299$
$= \$1609.30$ [1]
Percentage profit that the seller made
$= \frac{\$3440.85 - \$1609.30}{\$1609.30} \times 100\%$
$\approx 113.810\%$
$= \mathbf{114\%}$ (3 s.f.) [1]

> **Must-Know Concept:**
> Percentage Profit $= \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$

7. (a) Since he earned a total interest of \$1500,
$30\ 000 + 1500 = 30\ 000\left(1 + \frac{\frac{r}{4}}{100}\right)^{3 \times 4}$ [1]
$31\ 500 = 30\ 000\left(1 + \frac{r}{400}\right)^{12}$
$\frac{31\ 500}{30\ 000} = \left(1 + \frac{r}{400}\right)^{12}$
$1 + \frac{r}{400} = \sqrt[12]{\frac{31\ 500}{30\ 000}}$
$1 + \frac{r}{400} \approx 1.004\ 07$
$\frac{r}{400} = 0.004\ 07$
$r = 400 \times 0.004\ 07$
$= 1.628$
$= \mathbf{1.63}$ (3 s.f.) [1]

> **Must-Know Concept:**
> Compound Interest
> Total Amount, $A = p\left(1 + \frac{r}{100}\right)^n$
> Since the interest was compounded quarterly, the values of r and n have to be adjusted accordingly.

(b) Let the amount that Jacelyn should invest be $P.

Since she wishes to see at least $10 000 in her bank at the end of 3 years,

$$P + \frac{P \times 2.5 \times 3}{100} \geqslant 10\ 000 \quad [1]$$

$$P + \frac{7.5P}{100} \geqslant 10\ 000$$

$$1.075P \geqslant 10\ 000$$

$$P \geqslant \frac{10\ 000}{1.075}$$

$$P \geqslant 9302.33 \quad [1]$$

Minimum amount that she should invest = **$9303** (nearest whole number) [1]

> **Must-Know Concept:**
> Simple Interest
> Interest, $I = \frac{PRT}{100}$
> Since she wishes to see **at least** $10 000 in her bank at the end of 3 years, the inequality sign should be written as '$\geqslant$' instead of '$>$'.
> *Marking scheme:*
> *Deduct 1 mark if the inequality sign was written as '$>$'.*

(c) Selling price of the sports shoes during the sale $= \frac{60}{100} \times \$320$

= $192 [1]

(100 + 20 =) 120% of the cost price of the shoes ⟶ $192

Cost price of the shoes $= \frac{100}{120} \times \$192$

= $160 [1]

Percentage profit that the seller would have made if he sold the sports shoes at

$\$320 = \frac{\$320 - \$160}{\$160} \times 100\%$

= **100%** [1]

> **Must-Know Concept:**
> Percentage Profit $= \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$

8. (a) Note that Australia is a destination in Zone 3.

Postage rate for first 500 g = $17.00

Remaining weight of the parcel

= 960 g – 500 g

= 460 g

Postage rate for remaining 460 g

= 5 × $3.50 [1]

= $17.50

Total amount that Jenny has to be pay altogether = $17.00 + $17.50 + $2.50

= **$37** [1]

> **Must-Know Concept:**
> Do not forget to add $2.50 as this is a registered article.

(b) (i) Note that Hong Kong is a destination in Zone 2.

Cost of first 500 g of the parcel

= $12.00

Cost of remaining weight of the parcel

= $19.50 – $12.00

= $7.50

Maximum possible remaining weight

$= \frac{\$7.50}{\$2.50} \times 100$ g

= 300 g [1]

Maximum possible total weight of the parcel = 500 g + 300 g

= **800 g** [1]

> **Must-Know Concept:**
> Since Ramesh sent a non-registered parcel, no additional fee is required.

(ii) $700 < x \leqslant 800$ [1]

> **Must-Know Concept:**
> Since Ramesh paid $19.50, his parcel weighed more than 700 g and at most 800 g.

(iii) HKD $1 = S$0.17

HKD $105 = S$0.17 × 105

= S$17.85 [1]

Since $17.85 is less than $19.50, the amount Ramesh charged was **not sufficient** to cover for the postage fees. [1]

9. (a) Mr Wong's total annual income for 2018

= 12($4200) + 1.5($4200)

= **$56 700** [1]

> **Must-Know Concept:**
> A person's annual income includes not only his monthly salary but also bonuses and other earnings (if any).

(b) Mr Wong's chargeable income
= $56 700 – $14 000
= $42 700 [1]
Amount of tax that Mr Wong had to pay for the first $40 000 of his chargeable income
= $550
Remaining chargeable income
= $42 700 – $40 000
= $2700
Amount of tax that Mr Wong had to pay for the remaining $2700 = $\frac{7}{100} \times \$2700$
= $189 [1]
Total amount of tax that Mr Wong had to pay = $550 + $189
= **$739** [1]

> **Must-Know Concept:**
> Chargeable Income
> = Total Annual Income – Total Tax Reliefs

(c) (i) For the first $80 000 of Mr Sheriff's chargeable income, he paid $3350.
Remaining amount of tax paid
= $4730 – $3350
= $1380
Remaining chargeable income
= $\frac{\$1380}{11.5} \times 100$
= $12 000 [1]
His chargeable income for 2018
= $80 000 + $12 000
= **$92 000** [1]

> **Must-Know Concept:**
> Using the table (Row 4), we can see that the income tax rate for his remaining chargeable income is 11.5%.
> ∴ 11.5% of his remaining chargeable income = $1380

(ii) His annual income for 2018
= $92 000 + $16 000
= **$108 000** [1]

> **Must-Know Concept:**
> Chargeable Income
> = Total Annual Income – Total Tax Reliefs
> Total Annual Income
> = Chargeable Income + Total Tax Reliefs

10. (a) (i) Amount of money that was credited to her Medisave Account each month
= $\frac{9}{37} \times \frac{37}{100} \times \$3800$
= **$342** [1]

> **Must-Know Concept:**
> The amount of money credited to her Medisave account is $\frac{9}{37}$ of her total monthly CPF contribution.

(ii) Jocelyn's new salary = $\frac{120}{100} \times \$3800$
= $4560
Total monthly contributions made to Jocelyn's CPF Medisave Account
= $\frac{10}{37} \times \frac{37}{100} \times \$4560$
= $456 [1]
Percentage increase in contributions
= $\frac{\$456 - \$342}{\$342} \times 100\%$
= $\mathbf{33\frac{1}{3}\%}$ [1]

> **Must-Know Concept:**
> At 46 years old, the monthly CPF Medisave contribution is $\frac{10}{37}$ of the total monthly CPF contribution.
> Percentage Increase = $\frac{\text{Increase}}{\text{Original Value}} \times 100\%$

(b) Amount of money credited to his CPF account in end January
= $\frac{37}{100} \times \$3300$
= $1221
Amount of money credited to his Ordinary account = $\frac{23}{37} \times \$1221$
= $759 [1]
Amount of money credited to his Medisave and Special Accounts = $1221 – $759
= $462
Interest earned from his Ordinary account
= $\frac{759 \times \frac{2.5}{12} \times 11}{100}$
= $17.394 [1]
Interest earned from his Medisave and Special accounts = $\frac{462 \times \frac{4}{12} \times 11}{100}$
= $16.94 [1]

Total interest that he will earn
= \$17.394 + \$16.94
= **\$34.33** (nearest cent) [1]

> **Must-Know Concept:**
> The duration from end January to end December is 11 months.
> Since the interest will only be credited to his account by the end of each year, the interest is compounded yearly and not monthly.

(c) It was assumed that the monthly computation of interest was done at the end of each month and after his CPF contribution was credited in end January. [1]

Unit 2 Geometry and Measurement

2.1 Congruence and Similarity

1. $\triangle \mathbf{DOC}$ is congruent to $\triangle AOB$. [1]
In $\triangle AOB$ and $\triangle DOC$,
$OA = OD$ (radii of circle)
$OB = OC$ (radii of circle) } [1]
$\angle AOB = \angle DOC$ (vertically opposite $\angle$s) [1]
$\therefore$ $\triangle AOB$ is congruent to $\triangle DOC$ (SAS). [1]

> **Must-Know Concept:**
> To prove that two triangles are congruent using the SAS rule, we show that:
> – 2 corresponding lengths of the two triangles are equal
> – The angles subtended between the 2 corresponding lengths are equal

2. (a) Since the two solids are geometrically similar,

$$\left(\frac{V_A}{V_B}\right) = \left(\frac{h_A}{h_B}\right)^3$$

$$\frac{h_A}{h_B} = \sqrt[3]{\frac{V_A}{V_B}}$$

$$\frac{h_A}{h_B} = \sqrt[3]{\frac{40.5}{768}}$$

$$= \frac{3}{8}$$

Ratio of the height of solid A to height of solid B = **3 : 8** [1]

> **Must-Know Concept:**
> For two similar figures, $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$.

(b) The cost of painting is dependent on the surface area of the solids.

$$\therefore \frac{\$4.50}{\text{Cost of painting solid } B} = \left(\frac{3}{8}\right)^2 \quad [1]$$

$$\frac{\$4.50}{\text{Cost of painting solid } B} = \frac{9}{64}$$

$$\text{Cost of painting solid } B = \frac{\$4.50 \times 64}{9}$$

$$= \mathbf{\$32} \quad [1]$$

> **Must-Know Concept:**
> For two similar figures, $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$.

(c)

$$\frac{\text{Mass of solid } A}{256 \text{ g}} = \left(\frac{3}{8}\right)^3$$

$$\text{Mass of solid } A = \left(\frac{3}{8}\right)^3 \times 256 \text{ g}$$

$$= \mathbf{13.5\ g} \quad [1]$$

> **Must-Know Concept:**
> For two similar figures made of the same material, $\frac{m_1}{m_2} = \left(\frac{h_1}{h_2}\right)^3$.

3. (a)

$$\left(\frac{\text{Height of pyramid } X}{12}\right)^2 = \frac{78}{487.5}$$

$$\frac{\text{Height of pyramid } X}{12} = \sqrt{\frac{78}{487.5}} \quad [1]$$

$$\frac{\text{Height of pyramid } X}{12} = \frac{2}{5}$$

$$\text{Height of pyramid } X = \frac{2}{5} \times 12 \text{ cm}$$

$$= \mathbf{4.8\ cm} \quad [1]$$

> **Must-Know Concept:**
> For two similar figures, $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$.

(b)

$$\frac{\text{Mass of pyramix } X}{1.625 \text{ kg}} = \left(\frac{2}{5}\right)^3$$

$$\text{Mass of pyramid } X = \left(\frac{2}{5}\right)^3 \times 1.625 \text{ kg}$$

$$= 0.104 \text{ kg}$$

$$= \mathbf{104\ g} \quad [1]$$

> **Must-Know Concept:**
> For two similar figures made of the same material, $\frac{m_1}{m_2} = \left(\frac{h_1}{h_2}\right)^3$.

(c) $\dfrac{\$7.80}{\text{Cost of coating pyramid } Y} = \dfrac{78}{487.5}$

78(Cost of coating pyramid Y) = \$3802.50

Cost of coating pyramid $Y = \dfrac{\$3802.50}{78}$

$= \mathbf{\$48.75}$ [1]

> **Must-Know Concept:**
> Since the coating is done on the **surface areas** of the pyramids, the cost of coating is proportional to the areas of the pyramids.

4. (a) Since the two tins are similar,

$\left(\dfrac{6}{\text{Radius of tin } B}\right)^2 = \dfrac{264\pi}{412.5\pi}$ [1]

$\dfrac{6}{\text{Radius of tin } B} = \sqrt{\dfrac{264\pi}{412.5\pi}}$

$\dfrac{6}{\text{Radius of tin } B} = \dfrac{4}{5}$

4(Radius of tin B) = 30

Radius of tin $B = 30 \div 4$

$= \mathbf{7.5\ cm}$ [1]

> **Must-Know Concept:**
> For two similar figures, $\dfrac{A_1}{A_2} = \left(\dfrac{r_1}{r_2}\right)^2$

(b) $\dfrac{\text{Volume of tin } A}{1125\pi} = \left(\dfrac{4}{5}\right)^3$

Volume of tin $A = \left(\dfrac{4}{5}\right)^3 \times 1125\pi$

$= \mathbf{576\pi\ cm^3}$ [1]

> **Must-Know Concept:**
> For two similar figures, $\dfrac{V_1}{V_2} = \left(\dfrac{r_1}{r_2}\right)^3$

(c) Cost of 1 cm^3 of paint in tin A
= \$18.50 ÷ 576π
= \$0.010223
Cost of 1 cm^3 of paint in tin B
= \$35.10 ÷ 1125π
= \$0.009931 [1]

Tin B has the better value for money as the cost of each cm^3 of paint in tin B is lower than that of tin A. [1]

> **Must-Know Concept:**
> Compare the cost of 1 cm^3 of paint between tin A and tin B.

5. (a) $\angle PQR = 180° - 90°$ (adjacent $\angle$s on a straight line)
$= 90°$

$\angle PQR = \angle RQS = 90°$ [1]

Let $\angle RPQ = x°$

$\angle PRQ = 180° - 90° - x°$ ($\angle$ sum of $\triangle$)
$= 90° - x°$

$\angle SRQ = 90° - (90° - x°)$
$= 90° - 90° + x°$
$= x°$

$\therefore \angle RPQ = \angle SRQ$

$\angle PRQ = \angle RSQ$ ($\angle$ sum of $\triangle$) [1]

$\therefore$ **$\triangle PQR$ is similar to $\triangle RQS$ (3 pairs of corresponding $\angle$s are equal)** [1]

> **Must-Know Concept:**
> If all the corresponding angles in two triangles are equal, the two triangles are similar.

(b) $\triangle \boldsymbol{PRS}$ is similar to $\triangle PQR$. [1]

> **Must-Know Concept:**
> Notice that in $\triangle PQR$ and $\triangle PRS$,
> $\angle QPR = \angle RPS$ (common $\angle$)
> $\angle PQR = \angle PRS = 90°$ (given)

(c) Since $\triangle PQR$ and $\triangle RQS$ are similar,

$\dfrac{PQ}{RQ} = \dfrac{RQ}{QS}$

$\dfrac{4}{RQ} = \dfrac{RQ}{9}$ [1]

$(RQ)^2 = 36\ cm^2$

$RQ = \sqrt{36\ cm^2}$

$= \mathbf{6\ cm}$ [1]

> **Must-Know Concept:**
> The ratios of corresponding lengths of two similar figures are equal.

6. (a) (i) $\dfrac{\text{Curved surface area of } X}{\text{Curved surface area of entire cone}} = \left(\dfrac{3}{3+1+2}\right)^2$

$= \mathbf{\dfrac{1}{4}}$ [1]

> **Must-Know Concept:**
> If a cone is cut horizontally, the resulting cone is similar to the original cone.
> For two similar figures, $\dfrac{A_1}{A_2} = \left(\dfrac{h_1}{h_2}\right)^2$

(ii) $\frac{\text{Curved surface area of } X}{\text{Curved surface area of } X+Y} = \left(\frac{3}{3+1}\right)^2$

$= \frac{9}{16}$ [1]

$\frac{\text{Curved surface area of } X}{\text{Curved surface area of } Y} = \frac{9}{16-9}$

$= \mathbf{1\frac{2}{7}}$ [1]

Must-Know Concept:
Note that X is not similar to Y since X is a cone while Y is a frustum.

(iii) $\frac{\text{Volume of } X+Y}{\text{Volume of entire cone}} = \left(\frac{3+1}{3+1+2}\right)^3$

$= \frac{8}{27}$ [1]

$\frac{\text{Volume of } Z}{\text{Volume of entire cone}} = \frac{27-8}{27}$

$= \mathbf{\frac{19}{27}}$ [1]

Must-Know Concept:
Note that Z is not similar to the entire cone since Z is a frustum.
For two similar figures, $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

(b) $\frac{\text{Mass of } X}{3.4} = \left(\frac{3}{3+1+2}\right)^3$

$\frac{\text{Mass of } X}{3.4} = \frac{1}{8}$

Mass of $X = \frac{1}{8} \times 3.4$

$= 0.425$ kg

$= \mathbf{425\ g}$ [1]

Must-Know Concept:
For two similar figures, $\frac{m_1}{m_2} = \left(\frac{h_1}{h_2}\right)^3$

7. (a) In $\triangle ADB$ and $\triangle EDC$,

$\angle ADB = \angle EDC$ (common $\angle$)
$\angle DAB = \angle DEC$ (corresponding $\angle$s, $AB \parallel EC$)
$\angle DBA = \angle DCE$ (corresponding $\angle$s, $AB \parallel EC$)

Award 2 marks for listing any 2 of these 3 statements

$\therefore$ **$\triangle ADB$ is similar to $\triangle EDC$ (3 pairs of corresponding $\angle$s are equal)** [1]

Must-Know Concept:
If all the corresponding angles in two triangles are equal, the two triangles are similar.

(b) **$\triangle AGD$** is similar to $\triangle AFE$. [1]

Must-Know Concept:
Note that in $\triangle AFE$ and $\triangle AGD$,
$\angle FAE = \angle GAD$ (common)
$\angle AFE = \angle AGD$ (corresponding $\angle$s, $FE \parallel GD$)
$\angle AEF = \angle ADG$ (corresponding $\angle$s, $FE \parallel GD$)

(c) $AE = 17.5$ cm $- 10.5$ cm

$= 7$ cm

Since $\triangle AFE$ is similar to $\triangle AGD$,

$\frac{FE}{GD} = \frac{AE}{AD}$

$\frac{FE}{11} = \frac{7}{17.5}$ [1]

$FE = \frac{7}{17.5} \times 11$ cm

$= \mathbf{4.4\ cm}$ [1]

Must-Know Concept:
The ratios of corresponding lengths of two similar figures are equal.

(d) $\frac{\text{Area of } \triangle DEC}{\text{Area of } \triangle DAB} = \left(\frac{10.5}{17.5}\right)^2$

$= \frac{9}{25}$ [1]

$\therefore \frac{\text{Area of } \triangle DEC}{\text{Area of } ABCE} = \frac{9}{25-9}$

$= \mathbf{\frac{9}{16}}$ [1]

Must-Know Concept:
For two similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

8. (a) $UT = 2 \times 3$

$= 6$ cm

$QT = 3 + 6$

$= 9$ cm

In $\triangle PRV$ and $\triangle QST$,

$PV = QT = 9$ cm [1]

$\angle RPV = \angle SQT$ (corresponding $\angle$s, $PV \parallel QT$) [1]

$\angle PRV = \angle QST$ (corresponding $\angle$s, $VR \parallel TS$) [1]

$\therefore$ **$\triangle PRV$ is congruent to $\triangle QST$ (AAS).** [1]

Must-Know Concept:
To prove that two triangles are congruent using the AAS rule, we show that:
- 1 pair of corresponding lengths of the two triangles is equal
- 2 pairs of corresponding angles of the two triangles are equal

(b) $\triangle$**QRU** is similar to $\triangle PRV$. [1]

$\angle PRV = \angle QRU$ (common $\angle$)
$\angle RPV = \angle RQU$ (corresponding $\angle$s, QU // PV)
$\angle RVP = \angle RUQ$ (corresponding $\angle$s, QU // PV)

Award 2 marks for listing any 2 of these 3 statements

$\therefore$ **$\triangle PRV$ is similar to $\triangle QRU$ (3 pairs of corresponding $\angle$s are equal)** [1]

Must-Know Concept:
The ratios of corresponding lengths of two similar triangles are equal.

(c) Since $\triangle PRV$ is congruent to $\triangle QST$,
Area of $\triangle QST = 99$ cm²
Since $\triangle PRV$ is similar to $\triangle QRU$,

$$\frac{\text{Area of } \triangle QRU}{99} = \left(\frac{3}{9}\right)^2$$

$$\text{Area of } \triangle QRU = \left(\frac{3}{9}\right)^2 \times 99 \text{ cm}^2$$
$$= 11 \text{ cm}^2$$ [1]

Area of the entire figure $PSTUV$
$= 99 \text{ cm}^2 + 99 \text{ cm}^2 - 11 \text{ cm}^2$
$=$ **187 cm²** [1]

Must-Know Concept:
Two congruent figures have the same area.
For two similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

9. (a) In $\triangle PTS$ and $\triangle QTR$,
$PS = QR$ (opposite sides of a rectangle) [1]
$ST = RT$ (given) [1]
Since $\angle PSR = \angle QRS = 90°$ ($\angle$s in a rectangle)
and $\angle RST = \angle SRT = \frac{180° - 90°}{2}$ (base $\angle$s of isosceles $\triangle$)
$= 45°$
$\angle PST = \angle QRT = 90° + 45°$
$= 135°$ [1]
$\therefore$ **$\triangle PTS$ is congruent to $\triangle QTR$ (SAS).** [1]

Must-Know Concept:
To prove that two triangles are congruent using the SAS rule, we show that:
- 2 pairs of corresponding lengths of the two triangles are equal
- The angles subtended between the 2 corresponding lengths are equal

(b) $\triangle$**TPQ** is similar to $\triangle TXY$. [1]
$\angle PTQ = \angle XTY$ (common $\angle$) [1]
Since PQ and SR are opposite sides of a rectangle, PQ // SR.
$\angle QPT = \angle YXT$ (corresponding $\angle$s, PQ // SR) [1]
$\therefore \angle PQT = \angle XYT$ ($\angle$ sum of $\triangle$)
$\therefore$ **$\triangle TXY$ is similar to $\triangle TPQ$ (3 pairs of corresponding $\angle$s are equal).** [1]

Must-Know Concept:
If all the corresponding angles in two triangles are equal, the two triangles are similar.

(c) Smallest angle of $PQRST = 180° - 135°$
$=$ **45°** [1]

Must-Know Concept:
The smallest exterior angle corresponds to the largest interior angle.

(d) $PT = 10 + 8$
$= 18$ cm
Since $\triangle TXY$ is similar to $\triangle TPQ$,

$$\frac{21}{\text{Area of } \triangle TPQ} = \left(\frac{8}{18}\right)^2$$ [1]

$$\frac{21}{\text{Area of } \triangle TPQ} = \frac{16}{81}$$

16(Area of $\triangle TPQ$) $= 1701$
Area of $\triangle TPQ \approx 106.31$ cm² [1]
Area of the entire figure $PQRTS$
$= 34 + 106.31 + 34$
$=$ **174.31 cm²** [1]

Must-Know Concept:
For two similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

2.2 Coordinate Geometry

1. (a) $2y - x = 5$

$2y = x + 5$

$y = \frac{1}{2}x + \frac{5}{2}$

Gradient of line l $= \frac{1}{2}$ [1]

Equation of line l: $y = mx + c$

$4 = \frac{1}{2}(-4) + c$

$c = 6$

$\therefore y = \frac{1}{2}x + 6$ [1]

Must-Know Concept:
Parallel lines have the same gradient.
The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.

(b) At the x-axis, $y = 0$.

$\frac{1}{2}x + 6 = 0$

$\frac{1}{2}x = -6$

$x = -12$

$\therefore A(-12, 0)$ [1]

Must-Know Concept:
If a point lies on the x-axis, its y-coordinate is 0.

(c) Let the x-coordinate of B be a.
y-coordinate of $B = 3a$
Since B lies on line l,

$3a = \frac{1}{2}a + 6$ [1]

$\frac{5}{2}a = 6$

$a = 6 \div \frac{5}{2}$

$= 2.4$

y-coordinate of $B = 3(2.4)$

$= 7.2$

$\therefore B(2.4, 7.2)$ [1]

Must-Know Concept:
Any point that lies on a line satisfies the equation of that line.

2. (a) Since A, B and C lie on a straight line,
Gradient of AB = Gradient of AC

$\frac{4-3}{p-1} = \frac{p-3}{0-1}$ [1]

$\frac{1}{p-1} = \frac{p-3}{-1}$

$(p-1)(p-3) = -1$

$p^2 - 3p - p + 3 = -1$ [1]

$p^2 - 4p + 4 = 0$

$(p-2)^2 = 0$

$p - 2 = 0$

$p = \mathbf{2}$ [1]

Must-Know Concept:
Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$.

(b) $B(2, 4)$ and $C(0, 2)$

Gradient of $BC = \frac{4-2}{2-0}$

$= 1$

Equation of BC: $y = mx + c$

$2 = 1(0) + c$ [1]

$c = 2$

$\therefore$ Equation of BC: $\boldsymbol{y = x + 2}$ [1]

Must-Know Concept:
Make use of the value of p found in (a).
The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.

3. (a) Since $(-1, 6)$ lies on the line,

$3(6) + k(-1) = 25$

$18 - k = 25$

$k = -7$ [1]

Must-Know Concept:
Any point that lies on a line satisfies the equation of that line.

(b) $3y - 7x = 25$
At the x-axis, $y = 0$,

$3(0) - 7x = 25$

$-7x = 25$

$x = \frac{25}{-7}$

$= -3\frac{4}{7}$

$\therefore A(-3\frac{4}{7}, 0)$ [1]

Must-Know Concept:
When a point lies on the x-axis, $y = 0$.

(c) $3y - 7k = 25$

$3y = 7k + 25$

$y = \frac{7}{3}k + \frac{25}{3}$

Gradient of line l $= \frac{7}{3}$

Equation of line l: $y = mx + c$

$-1 = \frac{7}{3}(6) + c$ [1]

$-1 = 14 + c$

$c = -15$

$\therefore$ Equation of line l: $\mathbf{y = \frac{7}{3}x - 15}$ [1]

> **Must-Know Concept:**
> The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.
> Note that parallel lines have the same gradient.

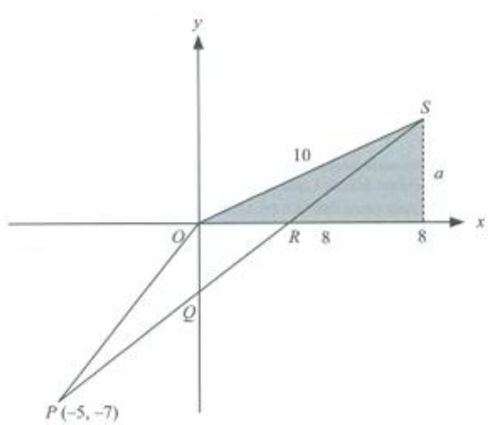
(d) When $x = 3$, $y = \frac{7}{3}(3) - 15$

$= -8$

Yes, (3, −8) lies on line l since it satisfies the equation of line l. [1]

> **Must-Know Concept:**
> If a point satisfies the equation of a line, it lies on that line.
> *Marking scheme:*
> *No marks should be awarded if no working is shown.*

4. (a) Let the y-coordinate of S be a.



By Pythagoras' Theorem,

$a^2 = 10^2 - 8^2$

$a = \sqrt{10^2 - 8^2}$

$= \mathbf{6}$ (shown) [1]

> **Must-Know Concept:**
> The Pythagoras' Theorem is applicable for a right-angled triangle.
> $c^2 = a^2 + b^2$

(b) Length of $PS = \sqrt{[8 - (-5)]^2 + [6 - (-7)]^2}$

$= \sqrt{13^2 + 13^2}$

$= 18.3848$ units

$=$ **18.4 units** (3 s.f.) [1]

> **Must-Know Concept:**
> Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
> Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(c) Gradient of $PS = \frac{6 - (-7)}{8 - (-5)}$

$= 1$ [1]

Equation of PS: $y = mx + c$

$6 = 1(8) + c$

$c = -2$

$\therefore$ Equation of PS: $\mathbf{y = x - 2}$ [1]

> **Must-Know Concept:**
> Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
> Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$
> The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.

(d) At the y-axis, $x = 0$,

$y = 0 - 2$

$= -2$

$\therefore$ **Q(0, −2)** [1]

At the x-axis, $y = 0$,

$x - 2 = 0$

$x = 2$

$\therefore$ **R(2, 0)** [1]

> **Must-Know Concept:**
> If a point lies on the x-axis, its y-coordinate is 0.
> If a point lies on the y-axis, its x-coordinate is 0.

(e) Area of $\triangle POS$

$= \text{Area of } \triangle POQ + \text{Area of } \triangle OQR + \text{Area of } \triangle ORS$

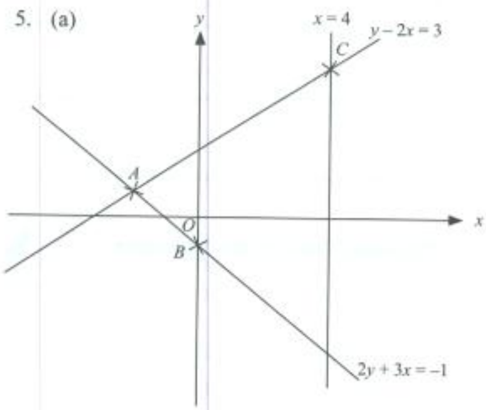
$= \frac{1}{2}(2)(5) + \frac{1}{2}(2)(2) + \frac{1}{2}(2)(6)$ [1]

$= 5 + 2 + 6$

$=$ **13 units²** [1]

> **Must-Know Concept:**
> Area of a Triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

5. (a)



$y - 2x = 3$

$y = 2x + 3$ .... (1)

$2y + 3x = -1$

$2y = -3x - 1$

$y = -1.5x - 0.5$.... (2)

(1) = (2):

$2x + 3 = -1.5x - 0.5$ [1]

$3.5x = -3.5$

$x = \frac{-3.5}{3.5}$

$= -1$ [1]

Substitute $x = -1$ into (1):

$y = 2(-1) + 3$

$= 1$

$\therefore$ **$A(-1, 1)$** [1]

Note that B lies on the line $y = -1.5x - 0.5$ and on the y-axis.

When $x = 0$,

$y = -1.5(0) - 0.5$

$= -0.5$

$\therefore$ **$B(0, -0.5)$** [1]

Note that C lies on the lines $y = 2x + 3$ and $x = 4$.

When $x = 4$,

$y = 2(4) + 3$

$= 11$

$\therefore$ **$C(4, 11)$** [1]

> **Must-Know Concept:**
> The equation of the upward-sloping line is $y - 2x = 3$ since its gradient is positive.
> The equation of the downward-sloping line is $2y + 3x = 1$ since its gradient is negative.
> The equation of a vertical line is of the form $x = c$, where c is a constant.
> When a point lies on the y-axis, its x-coordinate is 0.

(b) Length of $AC = \sqrt{[4 - (-1)]^2 + (11 - 1)^2}$

$= \sqrt{125}$

$\therefore$ **$k = 125$** [1]

> **Must-Know Concept:**
> Given two points $A(x_1, y_1)$ and $C(x_2, y_2)$,
> Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(c) Gradient of $BC = \frac{11 - (-0.5)}{4 - 0}$

$= 2.875$ [1]

Equation of BC: $y = mx + c$

$-0.5 = 2.875(0) + c$

$c = -0.5$

$\therefore y = 2.875x - 0.5$ [1]

When $x = 1$, $y = 2.875(1) - 0.5$

$= 2.375 \neq 1$

$\therefore$ **(1, 1) does not pass through the line BC.** (shown) [1]

> **Must-Know Concept:**
> Given two points $B(x_1, y_1)$ and $C(x_2, y_2)$,
> Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$.
> The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.

6. (a) Since C lies on the x-axis, its y-coordinate is 0.
Let the coordinate of C be $(x, 0)$.
Since $AB = AC$,
$\sqrt{(-3-3)^2+(-6-2)^2} = \sqrt{(x+3)^2+(0+6)^2}$ [1]
$\sqrt{100} = \sqrt{(x+3)^2+36}$
Square both sides of the equation:
$100 = (x+3)^2 + 36$
$(x+3)^2 = 64$ [1]
$x+3 = \pm\sqrt{64}$
$x+3 = \sqrt{64}$ or $x+3 = -\sqrt{64}$
$x+3 = 8$ $\quad$ $x+3 = -8$
$x = 5$ $\quad$ $x = -11$ [1]
Since the x-coordinate of C is positive,
coordinates of C is **(5, 0)**. [1]

> **Must-Know Concept:**
> If a point lies on the x-axis, its y-coordinate is 0.
> Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
> Length of $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

(b) Gradient of $AB = \frac{2-(-6)}{3-(-3)}$
$= \frac{4}{3}$ [1]
Equation of AB: $y = mx + c$
$2 = \frac{4}{3}(3) + c$
$c = -2$
$\therefore$ Equation of AB: $\mathbf{y = \frac{4}{3}x - 2}$ [1]

> **Must-Know Concept:**
> Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
> Gradient of $AB = \frac{y_2-y_1}{x_2-x_1}$
> The equation of a straight line is of the form $y = mx + c$, where m is the gradient and c is the y-intercept of the line.

(c) Let the point where the line AB crosses the x-axis be P.
When $y = 0$,
$0 = \frac{4}{3}x - 2$
$\frac{4}{3}x = 2$
$x = 2 \div \frac{4}{3}$
$= 1.5$ [1]
$\therefore$ x-coordinate of $P = 1.5$
Area of $\triangle APC = \frac{1}{2} \times (5-1.5) \times (0-(-6))$
$= \frac{1}{2} \times 3.5 \times 6$
$= 10.5$ units^2

Area of $\triangle PBC = \frac{1}{2} \times (5-1.5) \times (2-0)$
$= \frac{1}{2} \times 3.5 \times 2$
$= 3.5$ units^2
Area of $\triangle ABC = 10.5 + 3.5$ [1]
$=$ **14 units^2** [1]

> **Must-Know Concept:**
> Note that Area of $\triangle ABC$
> = Area of $\triangle PBC$ + Area of $\triangle ABC$
> Area of a Triangle = $\frac{1}{2}$ × Base × Height

(d) Coordinates of D are **(–1, –8)**. [1]

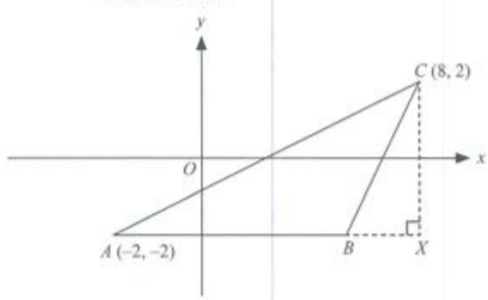
> **Must-Know Concept:**
> For $ABCD$ to be a parallelogram, $AB \parallel DC$ and $AD \parallel BC$.

2.3 Pythagoras' Theorem and Trigonometry

1. (a) Since AB is a horizontal line, y-coordinate of $B = -2$
Let the coordinates of B be $(x, -2)$
Since the gradient of BC is $\frac{4}{3}$.
$\frac{2-(-2)}{8-x} = \frac{4}{3}$ [1]
$\frac{4}{8-x} = \frac{4}{3}$
$4(8-x) = 12$
$32 - 4x = 12$
$4x = 20$
$x = 5$
Coordinates of B are **(5, –2)** [1]

> **Must-Know Concept:**
> The gradient of a line formed by two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$.

(b) To solve the problem, a right-angle triangle is constructed.



$CX = 2 - (-2)$
$= 4$ units
$AX = 8 - (-2)$
$= 10$ units

(i) $\tan \angle CAB = \frac{4}{10}$
$= \mathbf{\frac{2}{5}}$ [1]

Must-Know Concept:
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
In $\triangle CAX$, if we take $\angle CAB$ as the reference angle, CX is the opposite side while AX is the adjacent side.

(ii) Length of $BC = \sqrt{(8-5)^2 + [(2-(-2)^2]}$
$= 5$ units [1]
$\sin \angle ABC = \sin \angle CBX$
$= \mathbf{\frac{4}{5}}$ [1]

Must-Know Concept:
$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
In $\triangle CBX$, if we take $\angle CBX$ as the reference angle, CX is the opposite side while BC is the hypotenuse.
Note that $\sin (180° - \theta) = \sin \theta$

(iii) $BX = 8 - 5$
$= 3$ units
$\cos \angle ABC = -\cos \angle CBX$
$= \mathbf{-\frac{3}{5}}$ [1]

Must-Know Concept:
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
In $\triangle CBX$, if we take $\angle CBX$ as the reference angle, BX is the adjacent side while BC is the hypotenuse.
Note that $\cos (180° - \theta) = -\cos \theta$

2. (a) $RQ^2 = 29^2$
$= 841$
$PQ^2 + PR^2 = 21^2 + 20^2$
$= 841$ [1]

Since $RQ^2 = PQ^2 + PR^2$, PQR is a right-angled triangle by the concept of Pythagoras' Theorem (shown). [1]

Must-Know Concept:
Any triangle that satisfies Pythagoras' Theorem is a right-angled triangle.
Marking scheme:
Award 1 mark for correct calculation of both RQ^2 and $PQ^2 + PR^2$.
Award 1 mark for stating that $RQ^2 = PQ^2 + PR^2$ and mentioning Pythagoras' Theorem.

(b) (i) $QS = \frac{2}{3}(21)$
$= 14$ cm
$PS = 21 + 14$
$= 35$ cm
Since PRS is a right-angled triangle,
$\tan \angle PSR = \frac{20}{35}$
$= \mathbf{\frac{4}{7}}$ [1]

Must-Know Concept:
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
In $\triangle PSR$, if we take $\angle PSR$ as the reference angle, PR is the opposite side while PS is the adjacent side.

(ii) $\cos \angle SQR = -\cos \angle PQR$
$= \mathbf{-\frac{21}{29}}$ [1]

Must-Know Concept:
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
In $\triangle PQR$, if we take $\angle PQR$ as the reference angle, PQ is the adjacent side while RQ is the hypotenuse.
Note that $\cos (180° - \theta) = -\cos \theta$

(iii) $\sin (90° - \angle PQR) = \sin \angle PRQ$
$= \mathbf{\frac{21}{29}}$ [1]

Must-Know Concept:
Note that $\angle PQR + \angle PRQ = 180° - 90°$
$= 90°$
$\therefore \angle PRQ = 90° - \angle PQR$
$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
In $\triangle PQR$, if we take $\angle PRQ$ as the reference angle, PQ is the opposite side while RQ is the hypotenuse.

(c) By Pythagoras' Theorem,
$RS^2 = 20^2 + 35^2$
$RS = \sqrt{20^2 + 35^2}$
≈ 40.3113
$=$ **40.3 cm** (3 s.f.) [1]

Must-Know Concept:
The Pythagoras' Theorem is applicable for a right-angled triangle.
$c^2 = a^2 + b^2$

3. (a) (i) By Pythagoras' Theorem,

$AC^2 = 8^2 + 6^2$

$AC = \sqrt{8^2 + 6^2}$

$= 10$ units [1]

$\cos \angle ACB = \frac{8}{10}$

$= \frac{4}{5}$ [1]

> **Must-Know Concept:**
> $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
> In $\triangle ACB$, if we take $\angle ACB$ as the reference angle, BC is the adjacent side while AC is the hypotenuse.

(ii) $\sin \angle EAD = \sin \angle BAC$

$= \frac{8}{10}$

$= \frac{4}{5}$ [1]

> **Must-Know Concept:**
> $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
> In $\triangle BAC$, if we take $\angle BAC$ as the reference angle, BC is the opposite side while AC is the hypotenuse.
> Note that $\sin (180° - \theta) = \sin \theta$.

(iii) $\tan \angle BCD = -\tan \angle BCA$

$= -\frac{6}{8}$

$= -\frac{3}{4}$ [1]

> **Must-Know Concept:**
> $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
> In $\triangle BCA$, if we take $\angle BCA$ as the reference angle, AB is the opposite side while BC is the adjacent side.
> Note that $\tan (180° - \theta) = -\tan \theta$.

(b) Area of the entire figure

= Area of $\triangle ABC$ + Area of $\triangle ADE$

$= \frac{1}{2}(6)(8) + \frac{1}{2}(6)(10+3)\left(\frac{4}{5}\right)$ [1]

$= 24 + 31.2$

$=$ **55.2 cm²** [1]

> **Must-Know Concept:**
> There are 2 formulas for computing the area of a triangle:
> Area of a Triangle $= \frac{1}{2} \times$ Base $\times$ Height
> Area of a Triangle $= \frac{1}{2}ab \sin c$

4. (a) Method 1

Each interior angle of a regular pentagon

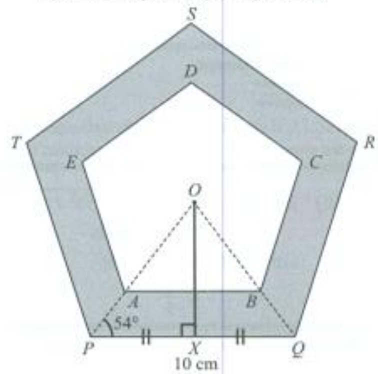
$= \frac{(5-2) \times 180°}{5}$

$= 108°$ [1]

$\angle OPQ = 108° \div 2$

$= 54°$

Let the height of $\triangle OPQ$ be OX.



$PX = 10 \div 2$

$= 5$ cm

Since OPX is a right-angled triangle,

$\tan 54° = \frac{OX}{5}$

$OX = 5 \tan 54°$

$\approx 6.881\ 91$ cm [1]

Area of $\triangle OPQ = \frac{1}{2} \times 10 \times 6.881\ 91$

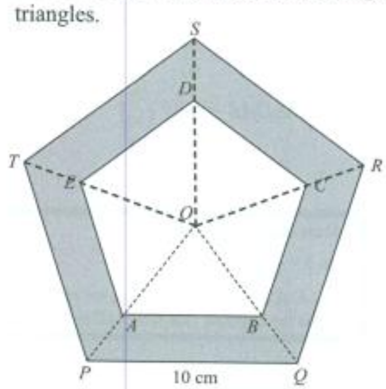
$\approx 34.409\ 55$

$=$ **34.4 cm²** (3 s.f.) [1]

> **Must-Know Concept:**
> Each interior angle of a regular n-sided polygon
> $= \frac{(n-2) \times 180°}{n}$
> $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
> In $\triangle OPX$, if we take $\angle OPX$ as the reference angle, OX is the opposite side while PX is the adjacent side.

Method 2

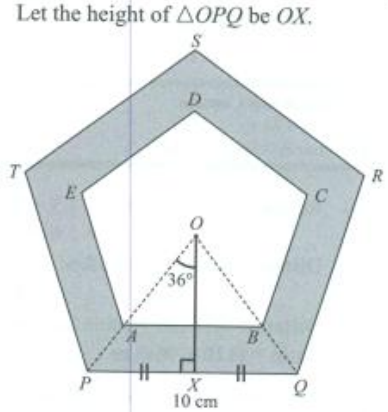
Note that $PQRST$ is made up of 5 congruent triangles.



$\angle POQ = 360° \div 5$

$= 72°$ [1]

Let the height of $\triangle OPQ$ be OX.



$\angle POX = 72° \div 2$

$= 36°$

$PX = 10 \div 2$

$= 5$ cm

Since POX is a right-angled triangle,

$\tan 36° = \frac{5}{OX}$

$OX \tan 36° = 5$

$OX = \frac{5}{\tan 36°}$

$\approx 6.881\ 91$ cm [1]

Area of $\triangle OPQ = \frac{1}{2} \times 10 \times 6.881\ 91$

$= 34.409\ 55$

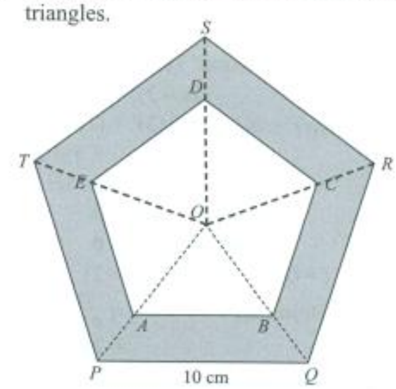
$=$ **34.4 cm²** (3 s.f.) [1]

Must-Know Concept:

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

In $\triangle OPX$, if we take $\angle POX$ as the reference angle, PX is the opposite side while OX is the adjacent side.

(b) Note that $PQRST$ is made up of 5 congruent triangles.



Area of $PQRST = 5 \times 34.409\ 55$

$= 172.048$ cm² [1]

$AB = 10 \div 1.6$

$= 6.25$ cm [1]

Since $PQRST$ is an enlargement of $ABCDE$, they are similar.

$\frac{\text{Area of } ABCDE}{172.048} = \left(\frac{6.25}{10}\right)^2$

$\frac{\text{Area of } ABCDE}{172.048} = \frac{25}{64}$

Area of $ABCDE = \frac{25}{64} \times 172.048$

$= 67.2063$ cm² [1]

Area of the shaded region

$= 172.048 - 67.2063$

$= 104.842$

$=$ **105 cm²** (3 s.f.) [1]

Must-Know Concept:

For two similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

5. (a) $BC = \frac{1}{2}(3x + 6)$ cm
$= (1.5x + 3)$ cm
By Pythagoras' Theorem,
$(2x - 3)^2 = (1.5x + 3)^2 + x^2$ [1]
$(2x)^2 - 2(2x)(3) + (3)^2 = (1.5x)^2 + 2(1.5x)(3) + 3^2 + x^2$
$4x^2 - 12x + 9 = 2.25x^2 + 9x + 9 + x^2$
$0.75x^2 - 21x = 0$ [1]
$x(0.75x - 21) = 0$
$x = 0$ or $0.75x = 21$
(Rejected) $x = \frac{21}{0.75}$
$= \mathbf{28}$ (shown) [1]

Must-Know Concept:
The Pythagoras' Theorem is applicable for a right-angled triangle.
$c^2 = a^2 + b^2$
In this question, x cannot be 0 since x represents a length.

(b) (i) $CD = 28$ cm
$BC = 1.5(28) + 3$
$= 45$ cm
$\tan \angle DBC = \mathbf{\frac{28}{45}}$ [1]

Must-Know Concept:
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
In $\triangle DBC$, if we take $\angle DBC$ as the reference angle, CD is the opposite side while BC is the adjacent side.

(ii) $BD = 2(28) - 3$
$= 53$ cm
$\cos \angle ABD = -\cos \angle DBC$
$= \mathbf{-\frac{45}{53}}$ [1]

Must-Know Concept:
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
In $\triangle DBC$, if we take $\angle DBC$ as the reference angle, BC is the adjacent side while BD is the hypotenuse.
Note that $\cos (180° - \theta) = -\cos \theta$.

(c) $AC = 2(45)$
$= 90$ cm
Since DAC is a right-angled triangle,
$\tan \angle DAB = \frac{28}{90}$ [1]
$\angle DAB = \tan^{-1}\left(\frac{28}{90}\right)$
$= 17.2815$
$= \mathbf{17.3°}$ (1 d.p.) [1]

Must-Know Concept:
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
In $\triangle DAC$, if we take $\angle DAB$ as the reference angle, CD is the opposite side while AC is the adjacent side.

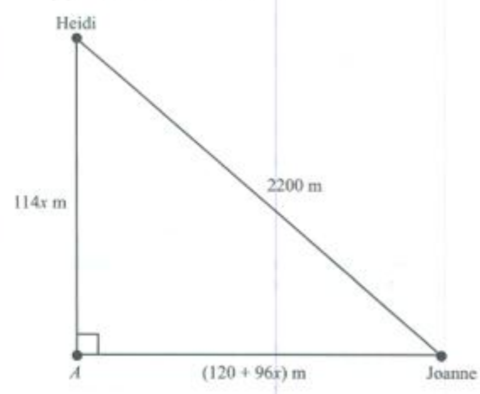
6. (a) x min $= 60 \times x$
$= 60x$ s
Distance between Heidi and point A after x min $= 1.9 \times 60x$
$= \mathbf{114x}$ **m** [1]

Must-Know Concept:
1 min = 60 s
Distance = Speed × Time

(b) Joanne's walking speed $= 1.9 - 0.3$
$= 1.6$ m/s
Distance that Joanne walked $= 1.6 \times 60x$ [1]
$= 96x$ m
Distance between Joanne and point A after x min $= \mathbf{(120 + 96x)}$ **m** [1]

Must-Know Concept:
Since Joanne started walking from point B, the distance between Joanne and point A is 120 m more than the distance that she walked.

(c) 2.2 km = 2200 m



By Pythagoras' Theorem,
$(114x)^2 + (120 + 96x)^2 = (2200)^2$ [1]
$12\,996x^2 + 120^2 + 2(120)(96x) + (96x)^2 = 4\,840\,000$
$12\,996x^2 + 14\,400 + 23\,040x + 9216x^2 - 4\,840\,000 = 0$ [1]
$22\,212x^2 + 23\,040x - 4\,825\,600 = 0$
Divide each term in the equation by 4:
$\mathbf{5553x^2 + 5760x - 1\,206\,400 = 0}$ (shown) [1]

Must-Know Concept:
The Pythagoras' Theorem is applicable for a right-angled triangle.
$c^2 = a^2 + b^2$
Do not forget to convert 2.2 km to m.
1 km = 1000 m

(d) $5553x^2 + 5760x - 1\,206\,400 = 0$

$x = \frac{-5760 \pm \sqrt{5760^2 - 4(5553)(-1\,206\,400)}}{2(5553)}$ [1]

$x = \frac{-5760 \pm \sqrt{26\,829\,734\,400}}{2(5553)}$

$x = \frac{-5760 + \sqrt{26\,829\,734\,400}}{2(5553)}$

$x \approx 14.229\,95$
$= \mathbf{14.230}$ (3 d.p.) [1]

or

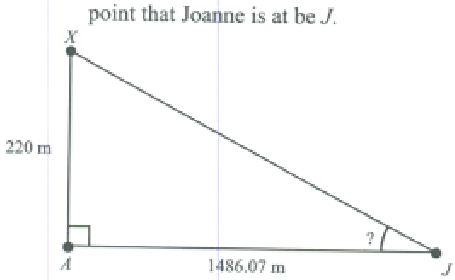
$x = \frac{-5760 - \sqrt{26\,829\,734\,400}}{2(5553)}$

$x \approx -15.267\,23$
$x = \mathbf{-15.267}$ (3 d.p.) [1]
(Rejected)

Must-Know Concept:
A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
In this question, x represents the time taken for Heidi and Anna to walk. Therefore, x cannot be negative.

(e) Distance between Joanne and point A after x min $= 120 + 96(14.229\,95)$
≈ 1486.08 m

Let the point that the bird is at be X and the point that Joanne is at be J.



$\tan \angle XJA = \frac{220}{1486.07}$ [1]

$\angle XJA = \tan^{-1}\left(\frac{220}{1486.08}\right)$
$= 8.420\,99°$
$= \mathbf{8.4°}$ (1 d.p.) [1]

∴ Angle of elevation from Joanne's position $= \mathbf{8.4°}$

Must-Know Concept:
Make use of the positive value of x found in (d).
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

7. (a) Since the area of $\triangle ABC$ is 80.75 cm²,

$\frac{1}{2}(2x - 3)(AC)\sin 30° = 80.75$ [1]

$\frac{1}{2}(2x - 3)(AC)\left(\frac{1}{2}\right) = 80.75$

$\frac{1}{4}(2x - 3)(AC) = 80.75$

$(2x - 3)(AC) = 323$

$AC = \left(\frac{323}{2x - 3}\right)$ **cm** [1]

Must-Know Concept:
Area of a Triangle $= \frac{1}{2}ab\sin c$

(b) Since the perimeter of $PQRS$ is 60 cm,
$2(x + 2) + 2PQ = 60$
$x + 2 + PQ = 30$
$PQ = 30 - x - 2$
$= \mathbf{(28 - x)}$ **cm** [1]

Must-Know Concept:
Perimeter of a Rectangle = 2(Length) + 2(Breadth)

(c) Since AC is 1 cm longer than PQ,

$\frac{323}{2x - 3} = 28 - x + 1$ [1]

$\frac{323}{2x - 3} = 29 - x$

Perform cross-multiplication:
$323 = (2x - 3)(29 - x)$ [1]
$323 = 58x - 2x^2 - 87 + 3x$
$323 = 61x - 2x^2 - 87$
$\mathbf{2x^2 - 61x + 410 = 0}$ (shown) [1]

Must-Know Concept:
$(a + b)(c + d) = ac + ad + bc + bd$

(d) $2x^2 - 61x + 410 = 0$

$x = \dfrac{-(-61) \pm \sqrt{(-61)^2 - 4(2)(410)}}{2(2)}$ [1]

$x = \dfrac{61 \pm \sqrt{441}}{4}$

$x = \dfrac{61 + \sqrt{441}}{4}$ or $x = \dfrac{61 - \sqrt{441}}{4}$

$x = \mathbf{20.5}$ [1] $\qquad x = \mathbf{10}$ [1]

> **Must-Know Concept:**
> A quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:
> $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(e) Integer value of $x = 10$

Let the shortest distance be d.

$AC = \dfrac{323}{2(10) - 3}$

$= 19$ cm

$\frac{1}{2}(19)(d) = 80.75$ [1]

$9.5d = 80.75$

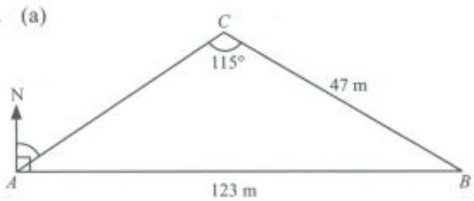
$d = \dfrac{80.75}{9.5}$

$= \mathbf{8.5\ cm}$ [1]

> **Must-Know Concept:**
> The shortest distance from C to AB is the **perpendicular** distance from C to AB.

2.4 Applications of Trigonometry

1. (a)



Using Sine Rule,

$\dfrac{47}{\sin \angle CAB} = \dfrac{123}{\sin 115°}$ [1]

$123 \sin \angle CAB = 47 \sin 115°$

$\sin \angle CAB = \dfrac{47 \sin 115°}{123}$

$\angle CAB = \sin^{-1}\left(\dfrac{47 \sin 115°}{123}\right)$

$\approx 20.262°$ [1]

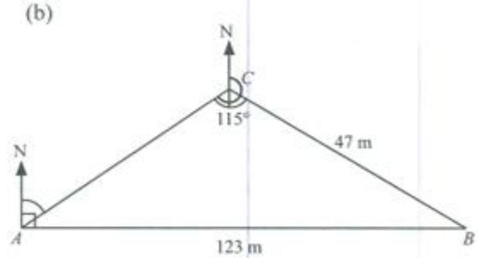
Bearing of C from $A = 90° - 20.262°$

$= 069.738°$

$= \mathbf{069.7°}$ (1 d.p.) [1]

> **Must-Know Concept:**
> For bearings, always start by facing north move in a **clockwise** direction to determine the required angle
> Bearings should always be written in 3 figures
> (E.g. 065° instead of 65°).

(b)



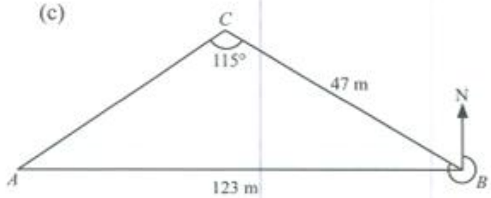
Bearing of A from $C = 180° + 69.738°$

$= 249.738°$

$= \mathbf{249.7°}$ (1 d.p.) [1]

> **Must-Know Concept:**
> The two North lines are parallel to each other. Therefore, the concept of alternate angles is applicable.

(c)



$\angle CBA = 180° - 115° - 20.262°$ ($\angle$ sum of $\triangle$)

$= 44.738°$

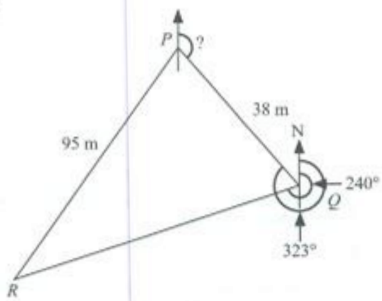
Bearing of C from $B = 270° + 44.738°$

$= 314.738°$

$= \mathbf{314.7°}$ (1 d.p.) [1]

> **Must-Know Concept:**
> Since A is due west of B, reflex $\angle ABN = 270°$.

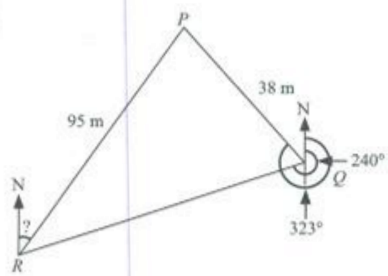
2. (a)



Bearing of Q from $P = 180° - (360° - 323°)$
$= 180° - 37°$
$= \mathbf{143°}$ [1]

Must-Know Concept:
For bearings, always start by facing north and move in a **clockwise** direction to determine the required angle
The two North lines are parallel to each other.
Therefore, the concept of alternate angles is applicable.

(b)



$\angle PQR = 323° - 240°$
$= 83°$ [1]

Using Sine Rule,

$\frac{38}{\sin \angle PRQ} = \frac{95}{\sin 83°}$ [1]

$95 \sin \angle PRQ = 38 \sin 83°$

$\sin \angle PRQ = \frac{38 \sin 83°}{95}$

$\angle PRQ = \sin^{-1}\left(\frac{38 \sin 83°}{95}\right)$
$\approx 23.3919°$ [1]

Bearing of P from R
$= (240° - 180°) - 23.3919°$
$= 60° - 23.3919°$
$= \mathbf{036.6°}$ (1 d.p.) [1]

Must-Know Concept:
Bearings should always be written in 3 figures
(E.g. 065 instead of 65°)

3. (a) By Pythagoras' Theorem,

$VA^2 = 29^2 - 20^2$
$VA = \sqrt{29^2 - 20^2}$
$= \mathbf{21\ cm}$ [1]

Must-Know Concept:
Note that VAB is a right-angled triangle.
The Pythagoras' Theorem is applicable for a right-angled triangle.
$c^2 = a^2 + b^2$

(b) $DX = \frac{3}{4}(20)$
$= 15$ cm

By Pythagoras' Theorem,
$AX^2 = 20^2 + 15^2$
$AX = \sqrt{20^2 + 15^2}$
$= 25$ cm [1]

By Pythagoras' Theorem,
$VX^2 = 21^2 + 25^2$
$VX = \sqrt{21^2 + 25^2}$
≈ 32.6497 cm
$= \mathbf{32.6\ cm}$ (3 s.f.) [1]

Must-Know Concept:
Note that $\triangle DAX$ and $\triangle VAX$ are both right-angled triangles.

(c) <u>Method 1</u>

Since DAX is a right-angled triangle,

$\tan \angle DAX = \frac{15}{20}$

$\angle DAX = \tan^{-1}\left(\frac{15}{20}\right)$
$\approx 36.8699°$ [1]

$\angle DAC = \frac{180° - 90°}{2}$ (base $\angle$s of isosceles $\triangle$)
$= 45°$ [1]

$\angle XAC = 45° - 36.8699°$
$= 8.1301$
$= \mathbf{8.1°}$ (1 d.p.) [1]

<u>Method 2</u>

$XC = 20 - 15$
$= 5$ cm

By Pythagoras' Theorem,
$AC^2 = 20^2 + 20^2$
$AC = \sqrt{20^2 + 20^2}$
≈ 28.2843 cm [1]

Using Cosine Rule,

$\cos \angle XAC = \frac{25^2 + 28.2843^2 - 5^2}{2(25)(28.2843)}$ [1]

$= 0.989\ 950$

$\angle XAC = \cos^{-1}(0.989\ 950)$

$\approx 8.1299°$

$=$ **8.1°** (1 d.p.) [1]

> **Must-Know Concept:**
> Cosine Rule:
> $a^2 = b^2 + c^2 - 2bc \cos A$
> $\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

4. (a) Since the area of $\triangle ABC$ is 2760 m²,

$\frac{1}{2}(56)(99) \sin \angle BAC = 2760$ [1]

$2772 \sin \angle BAC = 2760$

$\sin \angle BAC = \frac{2760}{2772}$

$\angle BAC = \sin^{-1}\left(\frac{2760}{2772}\right)$

$= 84.6668°$

$=$ **84.7°** (1 d.p.) [1]

> **Must-Know Concept:**
> Area of a Triangle $= \frac{1}{2}ab \sin C$

(b) Using Cosine Rule,

$BC^2 = 56^2 + 99^2 - 2(56)(99)\cos 84.6668°$ [1]

$= 11\ 906.398$ m

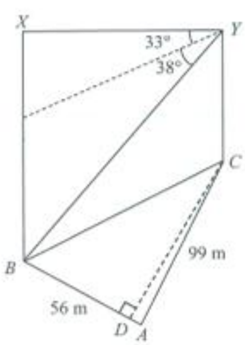
$BC = \sqrt{11\ 906.398}$

$= 109.116$

$=$ **109 m** (3 s.f.) [1]

> **Must-Know Concept:**
> Cosine Rule:
> $a^2 = b^2 + c^2 - 2bc \cos A$

(c) Let the shortest distance from C to AB be CD.



Method 1

$\frac{1}{2}(56)(CD) = 2760$ [1]

$28(CD) = 2760$

$CD = \frac{2760}{28}$

$= 98.5714$

$=$ **98.6 m** (3 s.f.) [1]

> **Must-Know Concept:**
> Since the area of $\triangle ABC$ is 2760 m²,
> $\frac{1}{2}(BA)(CD) = 2760$

Method 2

Since CAD is a right-angled triangle,

$\sin 84.6668° = \frac{CD}{99}$ [1]

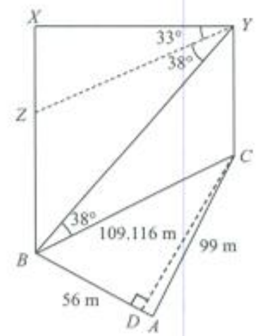
$CD = 99 \sin 84.6668°$

$= 98.5714$

$=$ **98.6 m** (3 s.f.) [1]

> **Must-Know Concept:**
> $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
> In $\triangle CAD$, if we take $\angle CAD$ as the reference angle, CD is the opposite side while AC is the hypotenuse.

(d)



$\tan 38° = \frac{YC}{109.116}$

$YC = 109.116 \tan 38°$

$= 85.2508$ m [1]

$\tan 33° = \frac{XZ}{109.116}$

$XZ = 109.116 \tan 33°$

$= 70.8608$ m [1]

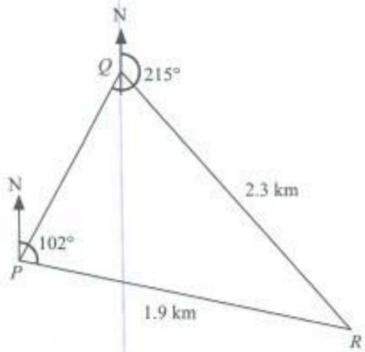
$XB = 85.2508 + 70.8608$

$= 156.1116$

$=$ **156 m** (3 s.f.) [1]

> **Must-Know Concept:**
> Since the angle of depression of B from Y is 38°, the angle of elevation of Y from B is also 38°.
> $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

5. (a)



Bearing of Q from $P = 215° - 180°$
$= 035°$

$\angle QPR = 102° - 35°$
$= \mathbf{67°}$ [1]

Must-Know Concept:
$\angle QPR$ = Bearing of R from P – Bearing of Q from P

(b) Using Sine Rule,

$$\frac{2.3}{\sin 67°} = \frac{1.9}{\sin \angle PQR}$$ [1]

$2.3 \sin \angle PQR = 1.9 \sin 67°$

$$\sin \angle PQR = \frac{1.9 \sin 67°}{2.3}$$

$$\angle PQR = \sin^{-1}\left(\frac{1.9 \sin 67°}{2.3}\right)$$

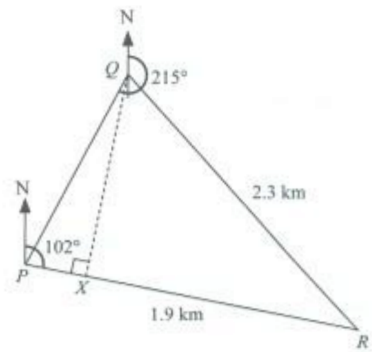
$\approx 49.501°$ [1]

$\angle PRQ = 180° - 67° - 49.501°$ ($\angle$ sum of $\triangle$)
$= 73.499°$

Area of triangle PQR
$= \frac{1}{2}(1.9)(2.3) \sin 73.499°$ [1]
≈ 2.0950
$= \mathbf{2.10\ km^2}$ (3 s.f.) [1]

Must-Know Concept:
Sine Rule:
$\frac{a}{\sin A} = \frac{b}{\sin B}$
Area of a Triangle $= \frac{1}{2}ab \sin C$

(c) Let the shortest distance from Q to PR be QX.



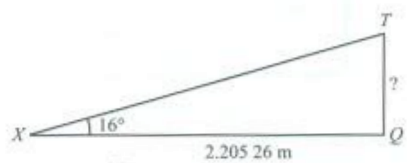
$\frac{1}{2}(1.9)(QX) = 2.0950$ [1]

$0.95QX = 2.0950$

$$QX = \frac{2.0950}{0.95}$$

$\approx 2.205\ 26$ km [1]

Let the point on the top of the tower be T.



$$\tan 16° = \frac{QT}{2.205\ 26}$$ [1]

$QT = 2.205\ 26 \tan 16°$
$\approx 0.632\ 348$ km
$= 632.348$ m
$= \mathbf{632\ m}$ (3 s.f.) [1]

Must-Know Concept:
The nearer Kathy stands from the tower, the greater the angle of elevation.
Therefore, to find the greatest angle of elevation, we find the shortest distance from Q to PR.

(d) $\tan 67° = \frac{2.205\ 26}{PX}$ [1]

$PX \tan 67° = 2.205\ 26$

$PX = \frac{2.205\ 26}{\tan 67°}$

$= 0.936\ 08$ km

Time taken for Kathy to walk

$= \frac{0.936\ 08}{4.8}$ [1]

$= 0.195\ 017$ h

$= 11.701\ 02$ min

$=$ **11 min 42 s** (nearest second) [1]

Must-Know Concept:
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
Time taken $= \frac{\text{Distance}}{\text{Speed}}$

6. (a) (i) Bearing of B from A

$= 180° + (90° - 83°)$

$=$ **187°** [1]

Must-Know Concept:
For bearings, always start by facing north and move in a **clockwise** direction to determine the required angle.
Any two North lines are parallel to each other.
Therefore, the concept of alternate angles is applicable.

(ii) Using Sine Rule,

$\frac{46}{\sin 83°} = \frac{32}{\sin \angle BAC}$ [1]

$46 \sin \angle BAC = 32 \sin 83°$

$\sin \angle BAC = \frac{32 \sin 83°}{46}$

$\angle BAC = \sin^{-1}\left(\frac{32 \sin 83°}{46}\right)$

$\approx 43.6671°$ [1]

$\angle BAC = 180° - 83° - 43.6671°$ ($\angle$ sum of $\triangle$)

$= 53.333°$

$=$ **53.3°** (1 d.p.) [1]

Must-Know Concept:
Sine Rule:
$\frac{a}{\sin A} = \frac{b}{\sin B}$

(iii) In a triangle, the largest angle is the angle facing the longest side.

$\therefore$ Largest angle in $\triangle ADC$ is $\angle CAD$

Using Cosine Rule,

$\cos \angle CAD = \frac{115^2 + 46^2 - 123^2}{2(115)(46)}$ [1]

$= \frac{53}{2645}$

$\angle CAD = \cos^{-1}\left(\frac{53}{2645}\right)$ [1]

$\approx 88.8518°$

$=$ **88.9°** (1 d.p.) [1]

Must-Know Concept:
In a triangle, the largest angle is the angle facing the longest side.
Cosine Rule:
$a^2 = b^2 + c^2 - 2bc \cos A$
$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(iv) Area of quadrilateral $ABCD$

$=$ Area of $\triangle ABC$ + Area of $\triangle ACD$

$= \frac{1}{2}(32)(46) \sin 53.333° +$

$\frac{1}{2}(46)(115) \sin 88.8518°$ [1]

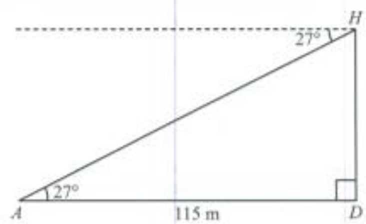
$= 590.3601 + 2644.4689$

$= 3234.829$

$=$ **3230 m²** (3 s.f.) [1]

Must-Know Concept:
Area of a Triangle $= \frac{1}{2}ab \sin C$

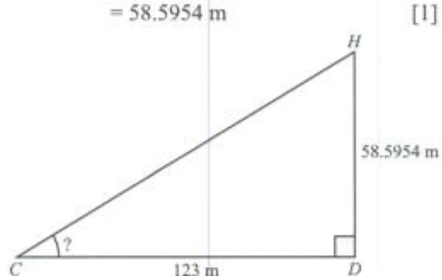
(b)



$\tan 27° = \frac{DH}{115}$

$DH = 115 \tan 27°$

$= 58.5954$ m [1]



$\tan \angle HCD = \frac{58.5954}{123}$ [1]

$\angle HCD = \tan{-1}\left(\frac{58.5954}{123}\right)$

$= 25.4724°$

$= \mathbf{25.5°}$ (1 d.p.) [1]

Must-Know Concept:
Note that the angle of depression of A from H is the same as the angle of elevation of H from A.
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

7. (a) (i) Using Cosine Rule,

$SQ^2 = 16^2 + 19.5^2 - 2(16)(19.5)\cos 124°$ [1]

≈ 985.186

$SQ = \sqrt{985.186}$

≈ 31.3877

$= \mathbf{31.4\ m}$ (3 s.f.) [1]

Must-Know Concept:
Cosine Rule:
$a^2 = b^2 + c^2 - 2bc\cos A$

(ii) Using Sine Rule,

$\frac{16}{\sin\angle PQS} = \frac{31.3877}{\sin 124°}$ [1]

$31.3877 \sin\angle PQS = 16\sin 124°$

$\sin\angle PQS = \frac{16\sin 124°}{31.3877}$

$\angle PQS = \sin^{-1}\left(\frac{16\sin 124°}{31.3877}\right)$

$\approx 24.9992°$ [1]

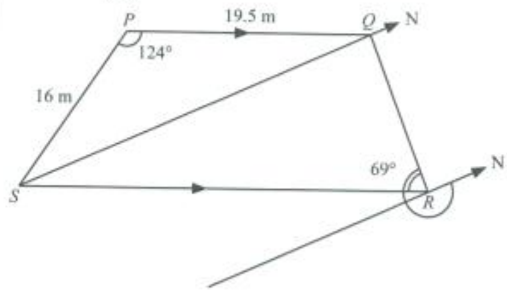
$\angle QSR = \angle PQS$ (alternate ∠s, $PQ // SR$)

$= 24.9992°$

$= \mathbf{25.0°}$ (1 d.p.) [1]

Must-Know Concept:
Sine Rule:
$\frac{a}{\sin A} = \frac{b}{\sin B}$
When there are parallel lines, look out for alternate, corresponding and interior angles.

(iii)



$\angle SQR = 180° - 69° - 24.9992°$ (∠ sum of △)

$= 86.0008°$ [1]

Bearing of Q from R

$= 360° - 86.0008°$ (∠s at a point)

$= \mathbf{274.0°}$ (1 d.p.) [1]

Must-Know Concept:
For bearings, always start by facing north and move in a **clockwise** direction to determine the required angle
The two North lines are parallel to each other.
Therefore, the concept of alternate angles is applicable.

(iv) Using Sine Rule,

$\frac{QR}{\sin 24.9992°} = \frac{31.3877}{\sin 69°}$ [1]

$QR\sin 69° = 31.3877\sin 24.9992°$

$QR = \frac{31.3877\sin 24.9992°}{\sin 69°}$

≈ 14.2083

$= \mathbf{14.2\ m}$ (3 s.f.) [1]

Must-Know Concept:
Sine Rule:
$\frac{a}{\sin A} = \frac{b}{\sin B}$

(v) Area of $PQRS$

= Area of $\triangle PQS$ + Area of $\triangle QRS$

$= \frac{1}{2}(16)(19.5)\sin 124° +$

$\frac{1}{2}(31.3877)(14.2083)\sin 86.0008°$ [1]

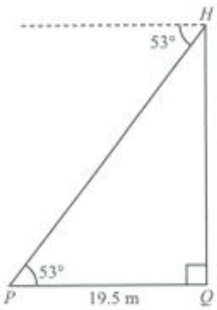
$\approx 129.3299 + 222.4399$

$= 351.7698$

$= \mathbf{352\ m^2}$ (3 s.f.) [1]

Must-Know Concept:
Area of a Triangle $= \frac{1}{2}ab\sin C$

(b)

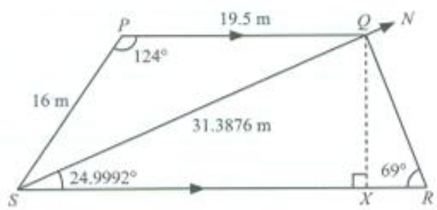


$\tan 53° = \frac{HQ}{19.5}$

$HQ = 19.5 \tan 53°$

≈ 25.8774 m [1]

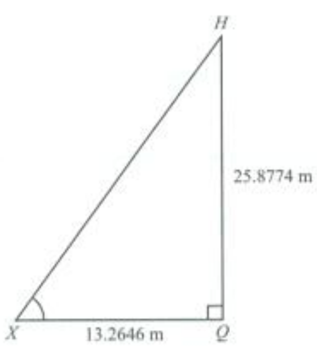
Let the shortest distance from Q to SR be QX.



$\sin 24.9992° = \frac{QX}{31.3877}$

$QX = 31.3877 \sin 24.9992°$

≈ 13.2646 m [1]



$\tan \angle HXQ = \frac{25.8774}{13.2646}$

$\angle HXQ = \tan^{-1}\left(\frac{25.8773}{13.2646}\right)$

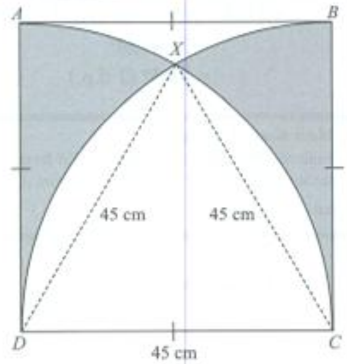
$\approx 62.8606°$

$= \mathbf{62.9°}$ (1 d.p.) [1]

Must-Know Concept:
To find the greatest angle of elevation, we find the shortest distance from Q to SR first.

2.5 Mensuration

1. (a)



Since $DX = CX = DC = 45$ cm (radii of circle), $\triangle DXC$ is an equilateral triangle.

$\angle XDC = \frac{\pi}{3}$ rad [1]

$\angle ADX = \frac{\pi}{2} - \frac{\pi}{3}$

$= \frac{\pi}{6}$ **rad** (shown) [1]

Must-Know Concept:
π rad = 180°
$\frac{\pi}{2}$ rad = 90°

(b) Area of segment DX

$= \frac{1}{2}(45)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(45)(45)\sin\left(\frac{\pi}{3}\right)$ [1]

≈ 183.437 cm²

Area of sector $ADX = \frac{1}{2}(45)^2\left(\frac{\pi}{6}\right)$ [1]

≈ 530.144 cm²

Area of the shaded regions

$= 2(530.144 - 183.437)$

$= 693.414$

$=$ **693 cm²** (3 s.f.) [1]

Must-Know Concept:
Area of a Sector $= \frac{1}{2}r^2\theta$, where is the angle in radians.
Area of a Triangle $= \frac{1}{2}ab \sin C$

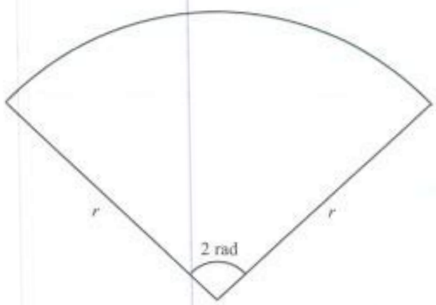
2. (a) (i) Length of each piece of the wire
$= 48 \div 2$
$= 24$ cm
Breadth of the rectangle
$= \frac{24 - 2x}{2}$
$= \mathbf{(12 - x)}$ **cm** [1]

> **Must-Know Concept:**
> Perimeter of a Rectangle = 2(Length) + 2(Breadth)

(ii) Area of the rectangle
$= x(12 - x)$ cm^2
$= \mathbf{(12x - x^2)}$ **cm²** [1]

> **Must-Know Concept:**
> Area of a Rectangle = Length × Breadth

(b) Let the radius of the sector be r.



Since the perimeter of the sector is 24 cm,
$r + r + r(2) = 24$ cm [1]
$4r = 24$ cm
$r = 6$ cm [1]
Area of the sector $= \frac{1}{2}(6)^2(2)$
$= \mathbf{36}$ **cm²** [1]

> **Must-Know Concept:**
> Arc Length = $r\theta$, where θ is the angle in radians.
> Area of a Sector = $\frac{1}{2}r^2\theta$, where θ is the angle in radians.

(c) $A = (12x - x^2) + 36$
$\mathbf{A = 36 + 12x - x^2}$ (shown) [1]

> **Must-Know Concept:**
> Sum up the answers of (a)(ii) and (b).

3. (a) Let the radius of the circle be r.
$\angle TXO = 90°$ (tan ⊥ rad)
By Pythagoras' Theorem,
$(r + 12)^2 = r^2 + 24^2$ [1]
$r^2 + 2(r)(12) + 12^2 = r^2 + 576$
$r^2 + 24r + 144 = r^2 + 576$
$24r = 432$
$r = \frac{432}{24}$
$= \mathbf{18}$ **cm** [1]

> **Must-Know Concept:**
> The Pythagoras' Theorem is applicable for a right-angled triangle.
> $c^2 = a^2 + b^2$

(b) Since TOX is a right-angled triangle,
$\tan \angle TOX = \frac{24}{18}$
$\angle TOX = \tan^{-1}\left(\frac{24}{18}\right)$
$\approx 0.927\ 295$
$= \mathbf{0.9273}$ **rad (4 s.f.)** (shown) [1]

> **Must-Know Concept:**
> $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
> In $\triangle TOX$, if we take $\angle TOX$ as the reference angle, TX is the opposite side while OX is the adjacent side.

(c) Area of TYX
= Area of $\triangle TXO$ – Area of sector OXY
$= \frac{1}{2}(24)(18) - \frac{1}{2}(18)^2(0.927\ 295)$ [1]
≈ 65.778 cm^2
Area of segment XZ
= Area of sector OXZ – Area of $\triangle OXZ$
$= \frac{1}{2}(18)^2(\pi - 0.927\ 295) -$
$\frac{1}{2}(18)^2 \sin(\pi - 0.927\ 295)$ [1]
≈ 229.116 cm^2
Total area of the shaded regions
$= 65.778 + 229.116$
$= 294.894$
$= \mathbf{295}$ **cm²** (3 s.f.) [1]

> **Must-Know Concept:**
> π rad = 180°
> Area of a Sector = $\frac{1}{2}r^2\theta$, where θ is the angle in radians.
> Area of a Triangle = $\frac{1}{2}ab \sin C$

(d) Major arc length $XY = 18(2\pi - 0.927\ 295)$
$= 96.406$ cm [1]

Let the radius of the cone be R.

$2\pi R = 96.406$

$R = \frac{96.406}{2\pi}$

$= 15.3435$ cm [1]

Total surface area of the cone
$= \pi(15.3435)^2 + \pi(15.3435)(18)$
$= 1607.258$
$=$ **1610 cm²** (3 s.f.) [1]

> **Must-Know Concept:**
> Since the major sector YOX was folded to form the right circular cone,
> Circumference of base of cone = Major arc length XY
> Slant height of cone = OY
> Total Surface Area of a Cone = $\pi r^2 + \pi rl$

4. (a) Capacity of container A
$= \pi(x)^2(4) + \pi\left(\frac{x}{2}\right)^2(12 - 4)$ [1]
$= 4\pi x^2 + \pi\left(\frac{x^2}{4}\right)(8)$
$= 4\pi x^2 + 2\pi x^2$
$= 6\pi x^2$ cm³

Time taken to fill container A to a height of 4 cm $= \frac{4\pi x^2}{6\pi x^2} \times 18$
$=$ **12 s** [1]

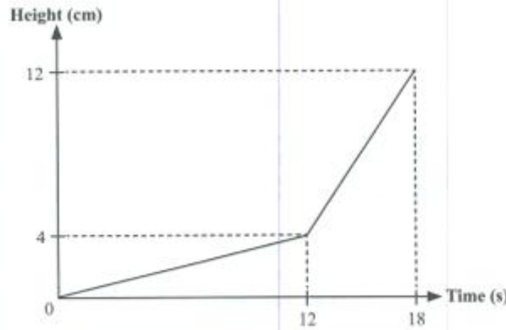
> **Must-Know Concept:**
> Capacity of a Cylinder = $\pi r^2 h$

(b) Capacity of container B
$= \frac{2}{3}\pi(6)^3 + \frac{1}{3}\pi(6)^2(6)$
$= 144\pi + 72\pi$
$= 216\pi$ cm³ [1]

Time taken to fill container B to a height of 6 cm $= \frac{144\pi}{216\pi} \times 18$
$=$ **12 s** (shown) [1]

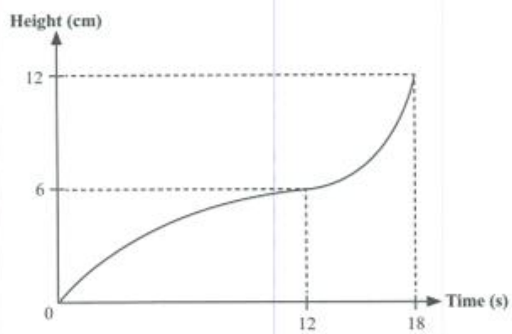
> **Must-Know Concept:**
> Volume of a Hemisphere = $\frac{2}{3}\pi r^3$
> Volume of a Cone = $\frac{1}{3}\pi r^2 h$

(c) (i)



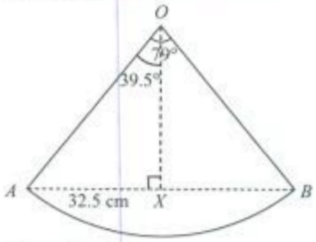
> **Must-Know Concept:**
> As water is poured into each cylindrical component of container A, the height of water increases at a constant rate.
> *Marking scheme:*
> *Correct shape for first line of graph [1]*
> *Correct shape for second line of graph [1]*
> *(Deduct 1 mark if '12' is not labelled on the horizontal axis)*

(ii)



> **Must-Know Concept:**
> As water is poured into the hemispherical component, the height of water increases at a decreasing rate.
> As water is poured into the conical component, the height of water increases at an increasing rate.
> *Marking scheme:*
> *Correct shape for first portion of graph [1]*
> *Correct shape for second portion of graph [1]*

5. (a) Let the midpoint of AB be X.



$AX = 65 \div 2$
$= 32.5$ cm
$\angle AOX = 79° \div 2$
$= 39.5°$

Since AOX is a right-angled triangle,

$\sin 39.5° = \frac{32.5}{OA}$ [1]

$OA \sin 39.5° = 32.5$

$OA = \frac{32.5}{\sin 39.5°}$
$= 51.0943$ cm
$=$ **51.1 cm** (3 s.f.) [1]

> **Must-Know Concept:**
> $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
> In $\triangle AOX$, if we take $\angle AOX$ as the reference angle, AX is the opposite side while OA is the hypotenuse.

(b) Area of the rectangle portion of the table top
$= 65 \times 48$ [1]
$= 3120$ cm^2

Area of the segment portion of the table top
$=$ Area of sector AOB – Area of $\triangle AOB$
$= \frac{79°}{360°} \times \pi(51.0943)^2 - \frac{1}{2}(51.0943)^2 \sin(79°)$ [1]
$= 518.448$ cm^2

Total area of the table top $= 3120 + 518.448$
$= 3638.448$ cm^2

Volume of wood used to make the table top
$= 3638.448 \times 3.2$ [1]
$= 11\,643.03$ cm^3
$=$ **11 600 cm³** (3 s.f.) [1]

> **Must-Know Concept:**
> Area of a Sector $= \frac{\theta}{360°} \times \pi r^2$, where θ is the angle in degrees.
> Area of a Triangle $= \frac{1}{2}ab \sin C$
> Area of a Rectangle = Length × Breadth
> Note that the table top has the shape of a prism.
> Volume of a Prism = Cross-sectional area × Length

2.6 Properties of Circle

1. (a) $PS = QR$ (opposite sides of a parallelogram) [1]

Since $RS = PQ$ (opposite sides of a parallelogram)
and X and Y are the midpoints of PQ and SR,
$SY = QX$ [1]

$\angle PSY = \angle RQX$ (opposite angles of a parallelogram) [1]

$\therefore$ **$\triangle PSY$ is congruent to $\triangle RQX$ (SAS).** (shown)

> **Must-Know Concept:**
> To prove that two triangles are congruent using the SAS rule, we show that:
> – 2 corresponding lengths of the two triangles are equal
> – The angles subtended between the 2 corresponding lengths are equal
> *Marking scheme:*
> *Deduct 1 mark if the case of congruency (SAS) is not stated.*

(b) $\angle PSY = 180° - 75°$ (interior $\angle$s, $SR \parallel PQ$)
$= 105°$

Since $\angle PSY > 90°$, S will lie **within the circle** by the concept of 'right angle in a semicircle'.

> **Must-Know Concept:**
> If $\angle PSY < 90°$, S will lie outside the circle.
> If $\angle PSY = 90°$, S will lie on the circumference of the circle.
> *Marking scheme:*
> *Gave the correct answer ('within the circle') [1]*
> *Showed that $\angle PSY > 90$ and stated the correct property of circles (angle in a semicircle) [1]*

2. (a) (i) $\angle TEO = \angle TBO = 90°$ (tan ⊥ rad) [1]

$\angle EOB = 360° - 41° - 90° - 90°$ ($\angle$s sum of quad)
$=$ **139°** [1]

> **Must-Know Concept:**
> When a tangent to a circle meets its radius, there is a right angle.
> Angles in a quadrilateral sum up to 360°.

(ii) Reflex $\angle EOB = 360° - 139°$ ($\angle$s at a point)
$= 221°$ [1]

$\angle EAB = \frac{221°}{2}$ ($\angle$ at centre = 2 $\angle$s at circumference)
$= \mathbf{110.5°}$ [1]

Must-Know Concept:
Angles at a point sum up to 360°
An angle at the centre of a circle is twice the corresponding angle at its circumference.

(iii) $\angle EDB = 180° - 110.5°$ ($\angle$s in opposite segments)
$= \mathbf{69.5°}$ [1]

Must-Know Concept:
Since A, B, D and E lie on the circumference of the circle, $ABDE$ is a cyclic quadrilateral.
Angles in opposite segments of a circle sum up to 180°.

(iv) $\angle DBC = 90° - 53°$
$= 37°$
$\angle DCB = 90°$ ($\angle$ in a semicircle) [1]
$\angle BDC = 180° - 37° - 90°$ ($\angle$ sum of $\triangle$)
$= \mathbf{53°}$ [1]

Must-Know Concept:
When there is a semicircle, look out for a right-angle.
Angles in a triangle sum up to 180°.

(b) (i) $\angle ETO = \frac{41°}{2}$
$= 20.5°$
Since $\triangle ETO$ is a right-angled triangle,
$\tan 20.5° = \frac{8}{TE}$ [1]
$TE \tan 20.5° = 8$
$TE = \frac{8}{\tan 20.5°}$
≈ 21.3970
$= \mathbf{21.4\ cm}$ (3 s.f.) [1]

(ii) Area of $TEOB$
$= 2 \times$ Area of $\triangle TOE$
$= 2 \times \frac{1}{2} \times 8 \times 21.3970$ [1]
$= 171.176\ cm^2$
Area of sector EOB
$= \frac{139°}{360°} \times \pi(8)^2$ [1]
$= 77.6322\ cm^2$

Area of the shaded region
$= 171.176 - 77.6322$
$= 93.5438$
$= \mathbf{93.5\ cm^2}$ (3 s.f.) [1]

Must-Know Concept:
Note that $TEOB$ is made up of two congruent triangles, $\triangle TEO$ and $\triangle TBO$.
Area of a Triangle $= \frac{1}{2} \times$ Base $\times$ Height
Area of a Sector $= \frac{\theta}{360°} \times \pi r^2$, where θ is the angle in degrees

3. (a) In $\triangle ABC$ and $\triangle EDC$,
$\angle ABC = \angle EDC$ ($\angle$s in the same segment) [1]
$BC = DC$ (given) [1]
$\angle BCA = \angle DCE$ (vertically opposite $\angle$s) [1]
$\therefore$ **$\triangle ABC$ is congruent to $\triangle EDC$ (ASA).** [1]

Must-Know Concept:
To prove that two triangles are congruent using the ASA rule, we show that:
- 1 pair of corresponding lengths of the two triangles is equal
- 2 pairs of corresponding angles of the two triangles are equal

(b) (i) $\angle AOE = 2(51°)$ ($\angle$ at centre = 2 $\angle$s at circumference)
$= \mathbf{102°}$ [1]

Must-Know Concept:
An angle at the centre of a circle is twice the corresponding angle at its circumference.

(ii) $\angle TEO = 90°$ (tan $\perp$ rad)
$\angle AEO = \frac{180° - 102°}{2}$ (base $\angle$s of isosceles $\triangle$)
$= 39°$ [1]
$\angle AET = 90° - 39°$
$= \mathbf{51°}$ [1]

Must-Know Concept:
When a tangent to a circle meets its radius, there is a right angle.
Since $OA = OE$ (radii of circle), $\triangle AOE$ is an isosceles $\triangle$.

(iii) Method 1

$\angle BCA = 180° - 72°$ (adjacent $\angle$s on a straight line)

$= 108°$

$\angle BAC = 180° - 108° - 51°$ ($\angle$s sum of $\triangle$)

$= 21°$ [1]

$\angle CED = \angle BAC$ ($\angle$s in the same segment $\triangle$)

$= \mathbf{21°}$ [1]

Method 2

$\angle BAC = 72° - 51°$ (ext. $\angle$ of $\triangle$) [1]

$= 21°$

$\angle CED = \angle BAC$ ($\angle$s in the same segment)

$= \mathbf{21°}$ [1]

> **Must-Know Concept:**
> Angles in the same segment of a circle are equal.
> An exterior angle of a triangle is equal to the sum of the interior opposite angles.

(iv) Method 1

$\angle CEA = \frac{180° - 72°}{2}$ (base $\angle$s of isosceles $\triangle$)

$= 54°$ [1]

$\angle DBE = 180° - 51° - 21° - 54°$ ($\angle$s in opposite segments)

$= \mathbf{54°}$ [1]

Method 2

$\angle BCD = \angle ACE = 72°$ [1]

$\angle DBE = \frac{180° - 72°}{2}$ (base $\angle$s of isosceles $\triangle$)

$= \mathbf{54°}$ [1]

> **Must-Know Concept:**
> Since $AC = EC$ (corresponding lengths of $\triangle ABC$ and $\triangle EDC$), $\triangle ACE$ is isosceles.
> Angles in opposite segments of a circle sum up to 180°.

(c) Yes. Since $\angle DBE = \angle CEA = 54°$, BD is parallel to AE by the concept of alternate angles. [1]

> **Must-Know Concept:**
> If a pair of lines satisfies the concepts of alternate, corresponding or interior angles, they are parallel.

4. (a) In $\triangle AXE$ and $\triangle CXB$

$\angle AXE = \angle CXB$ (common $\angle$) [1]

Let $\angle XAE$ be $x°$.

$\angle BCE = 180° - x°$ ($\angle$s in opposite segments)

$\angle XCB = 180° - (180° - x°)$ (adjacent $\angle$s on a straight line)

$= 180° - 180° + x°$

$= x°$

$\therefore \angle XAE = \angle XCB$ [1]

Hence, $\angle AEX = \angle CBX$ ($\angle$ sum of $\triangle$)

$\therefore$ **$\triangle AXE$ is similar to $\triangle CXB$ (3 pairs of corresponding $\angle$s are equal)** (shown) [1]

> **Must-Know Concept:**
> If all the corresponding angles in two triangles are equal, the two triangles are similar.

(b) $\angle EDC = 180° - y° - y°$ ($\angle$ sum of $\triangle$)

$= 180° - 2y°$ [1]

$\angle EAD = 180° - (180° - 2y°)$ ($\angle$s in opposite segments)

$= 180° - 180° + 2y°$

$= 2y°$ [1]

$\angle EAX = \angle BCX$ (Corresponding angles of similar triangles)

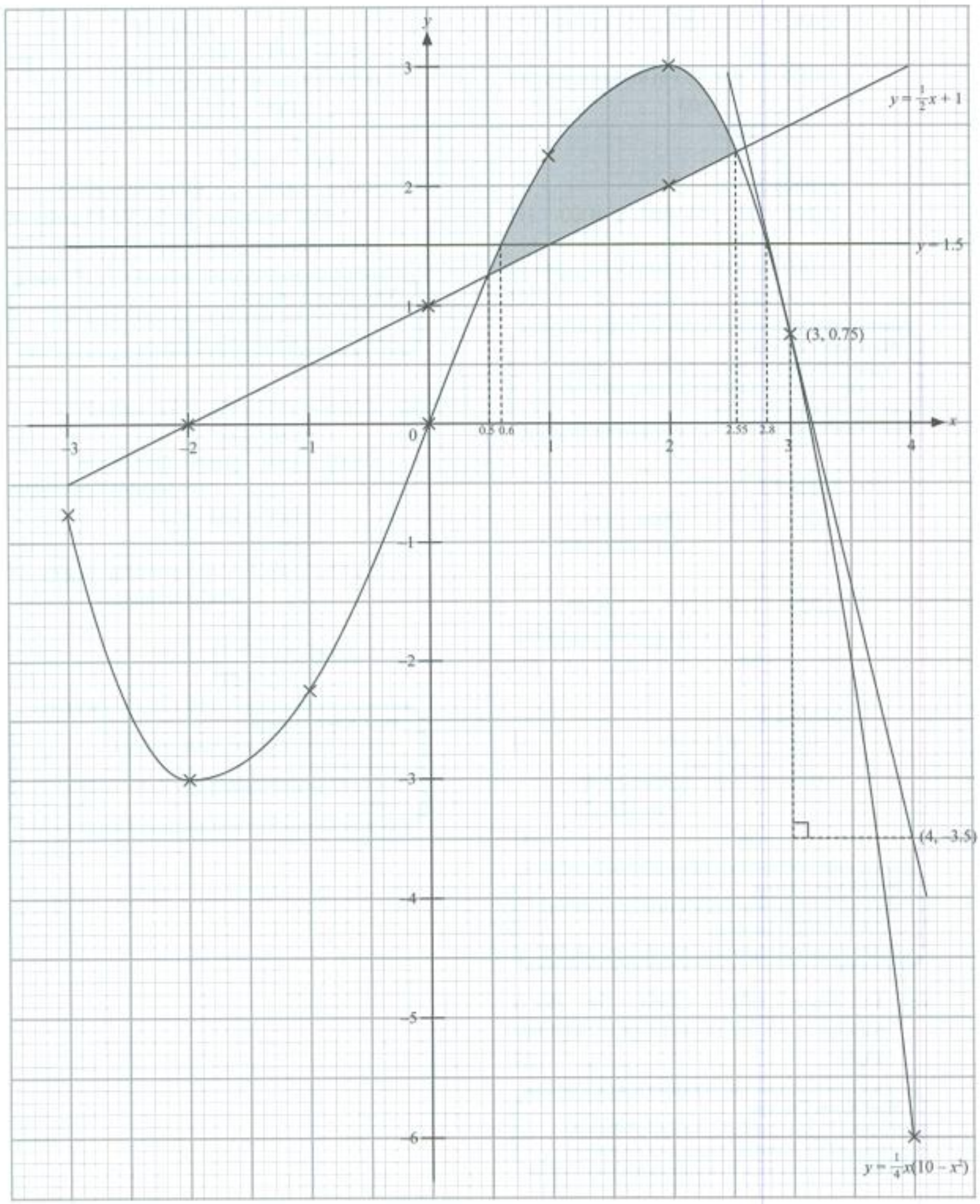
$= (3y + 5)°$

$\therefore \angle BAC = (3y + 5)° - 2y°$

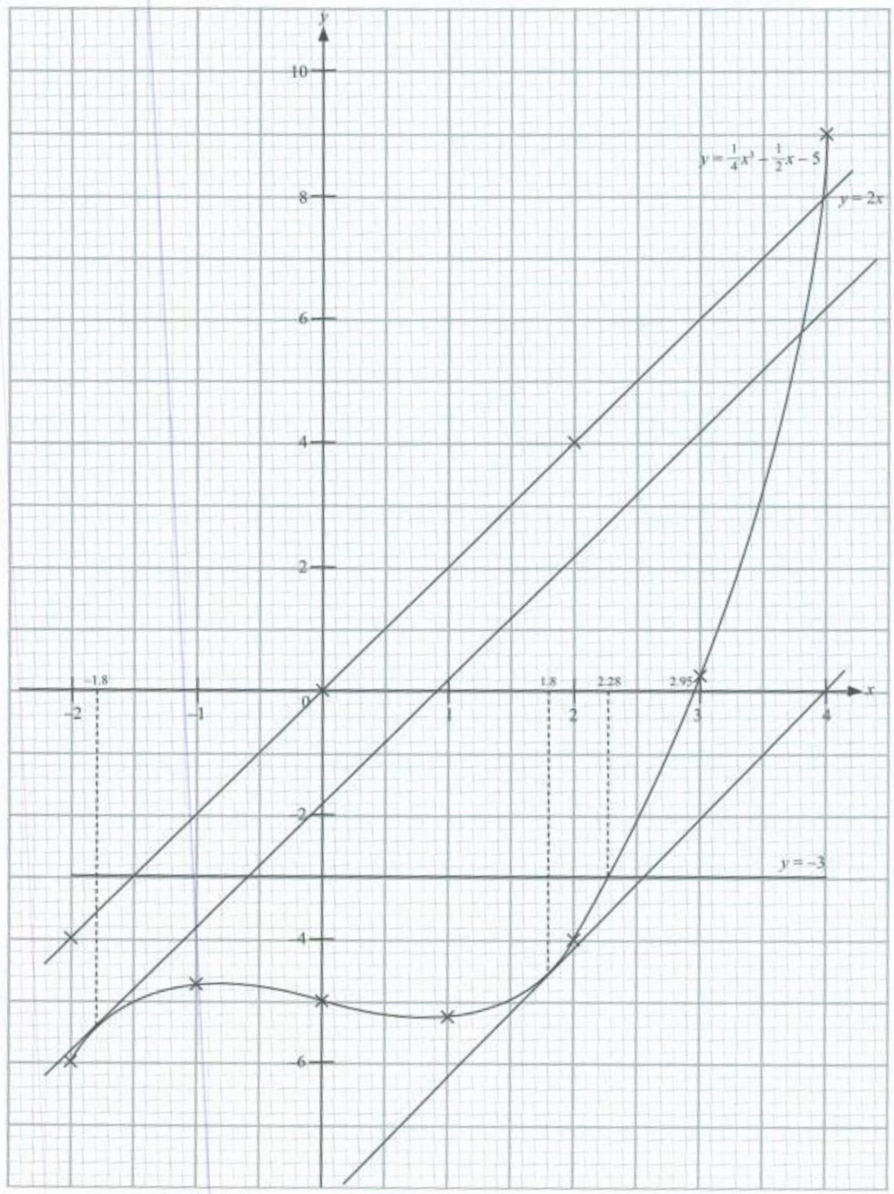
$= \mathbf{(y + 5)°}$ [1]

> **Must-Know Concept:**
> Note that both $ABCE$ and $ACDE$ are cyclic quadrilaterals.
> Angles in opposite segments of a circle sum up to 180°.

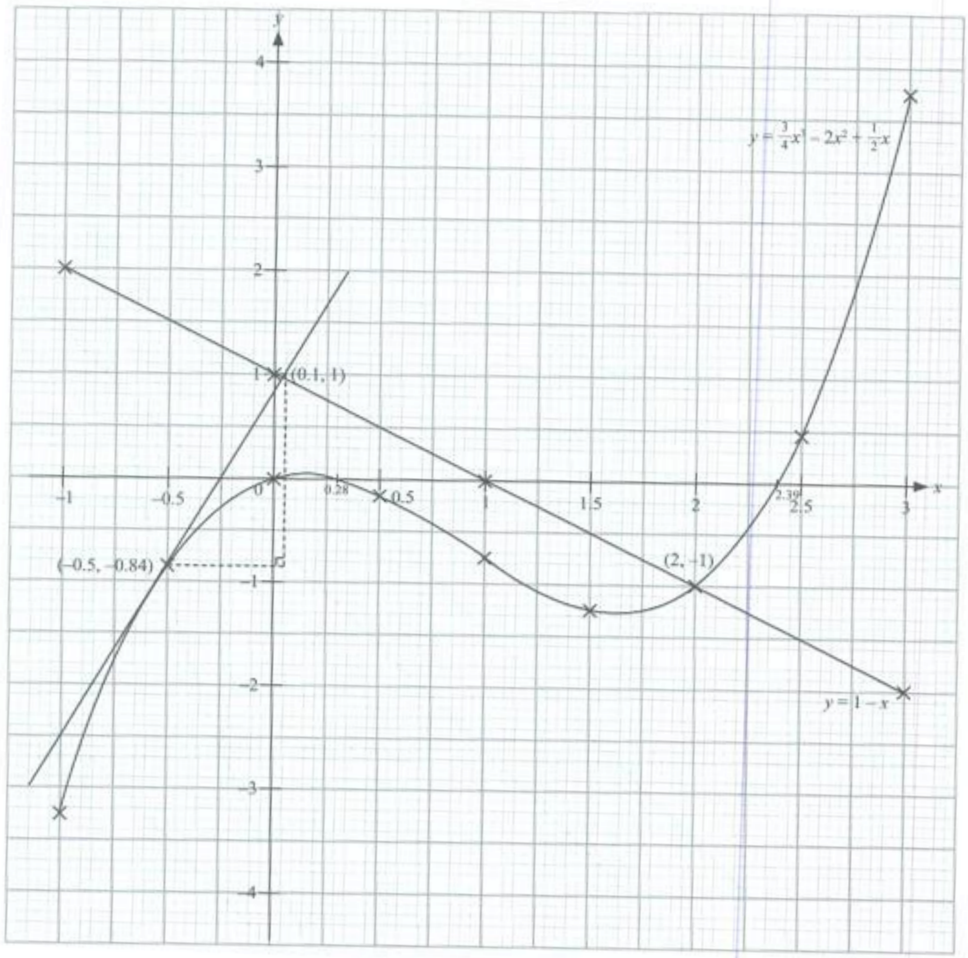
Appendix 1


y
x
0
3
2
1
-1
-2
-3
-4
-5
-6
-3
-2
-1
1
2
3
4
0.5
0.6
2.55
2.8
$y = \frac{1}{2}x + 1$
$y = 1.5$
(3, 0.75)
(4, −3.5)
$y = \frac{1}{4}x(10 - x^2)$

Appendix 2


y
10
8
6
4
2
0
-2
-4
-6
x
-2
-1
1
2
3
4
-1.8
1.8
2.28
2.95
$y = \frac{1}{4}x^3 + \frac{1}{2}x - 5$
$y = 2x$
$y = -3$

Appendix 3


y
x
4
3
2
1
0
-1
-2
-3
-4
-1
-0.5
0.28
0.5
1
1.5
2
2.39
2.5
3
$y = \frac{3}{4}x^3 - 2x^2 + \frac{1}{2}x$
(0.1, 1)
(–0.5, –0.84)
(2, –1)
$y = 1 - x$