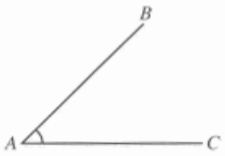
CHAPTER 10

Basic Geometry

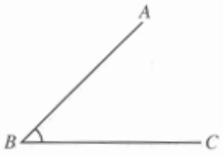
10.1 Angles

An angle is formed when two rays (the **sides**) meet at a common point (the **vertex**).
In the figure, AB and AC are the sides of $\angle BAC$ (or $\angle CAB$) and A is the vertex.

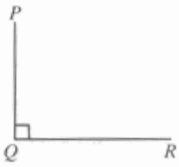


Types of Angles

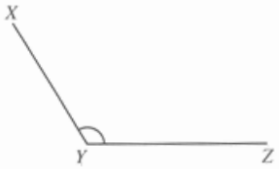
1. $\angle ABC$ is an **acute angle**.
It is less than 90°.



2. $\angle PQR$ is a **right angle**.
It is exactly 90°.

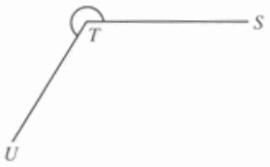


3. $\angle XYZ$ is an **obtuse angle**.
It is more than 90° but less less than 180°.



4. $\angle MNP$ is a straight line.
$\angle MNP$ is a **straight angle**.
It is exactly 180°.

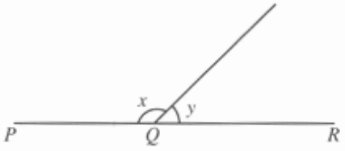
5. $\angle STU$ is a **reflex angle**.
It is more than 180°
but less than 360°.



6. $\angle PQR$ is 90°.
$\angle a$ and $\angle b$ are called
complementary angles.

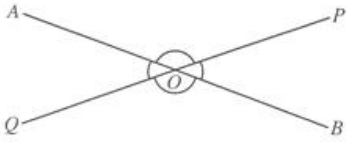
Two angles are complementary if their sum is 90°.

7. $\angle PQR$ is a straight line.
$\angle x$ and $\angle y$ are called **supplementary angles**.



Two angles are supplementary if their sum is 180 °.

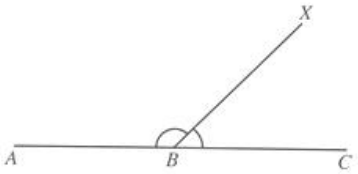
Vertically Opposite Angles



AOB and POQ are straight lines.
$\angle AOP = \angle BOQ$ (vert. opp. $\angle$s)
$\angle AOQ = \angle BOP$ (vert. opp. $\angle$s)

When two straight lines intersect, **vertically opposite angles** are formed. These vertically opposite angles are equal.
(**Abbreviation**: vert. opp. $\angle$s)

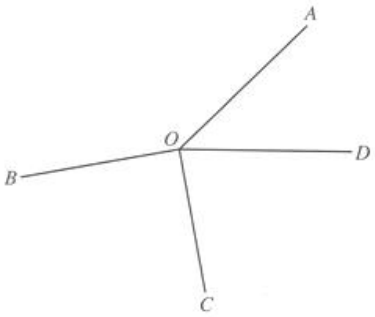
Adjacent Angles on a Straight Line



ABC is a straight line.
$\angle ABX$ and $\angle CBX$ are **adjacent angles**.
The angles share a common vertex B.
They have a common **side** BX.
$\angle ABX + \angle CBX = 180°$ (adj. $\angle$s on a str. line)

The sum of the **adjacent angles** on a straight line is 180°.
(**Abbreviation**: adj. $\angle$s on a str. line)

Angles at a Point



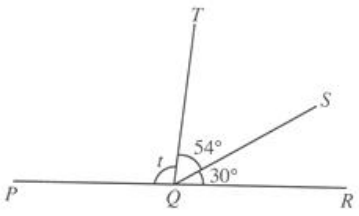
$\angle AOB$, $\angle BOC$, $\angle COD$ and $\angle AOD$ share a common vertex O. The arms of all the angles meet at the point O.
$\angle AOB$, $\angle BOC$, $\angle COD$ and $\angle AOD$ form angles at the point O.

$\angle AOB + \angle BOC + \angle COD + \angle AOD$
$= 360°$ ($\angle$s at a point)

The sum of all the angles at a point is 360°.
(**Abbreviation**: $\angle$s at a point)

Example: In the figure, PQR is a straight line.
Find $\angle t$.

$\angle t + 54° + 30° = 180°$ (adj. $\angle$s on a str. line)
$\angle t = 180° - 84°$
$= 96°$



Practice 10.1

Basic

1. Find the complementary angle for each of the following.

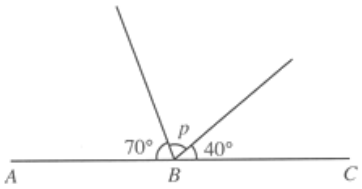
   (a) 25° (b) 45° (c) 67.5° (d) 82.5°

2. Find the supplementary angle for each of the following.

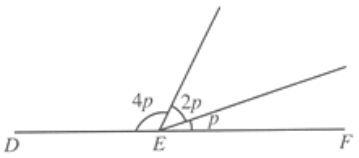
   (a) 15° (b) 70.5° (c) 135° (d) 169.5°

3. Find the value of p in each of the following figures.

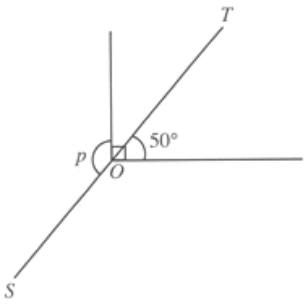
   (a) ABC is a straight line.



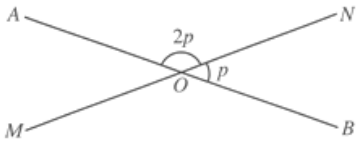
   (b) DEF is a straight line.



   (c) TOS is a straight line.

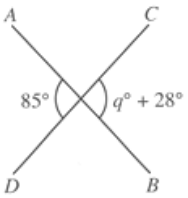


   (d) AOB and MON are straight lines.

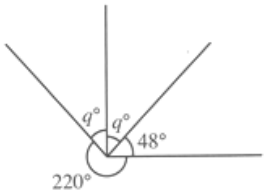


4. Find the value of q in each of the following figures.

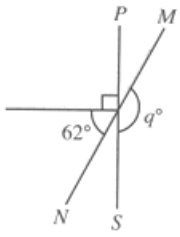
   (a) AB and CD are straight lines.



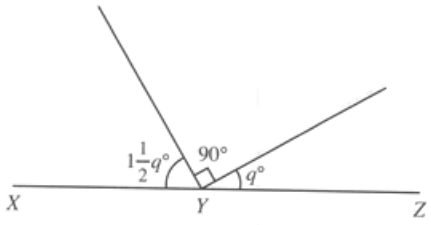
   (b)



(c) PS and MN are straight lines.



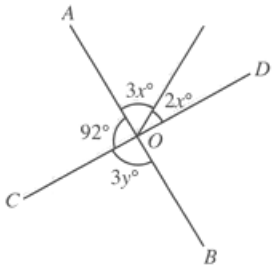
(d) XYZ is a straight line.



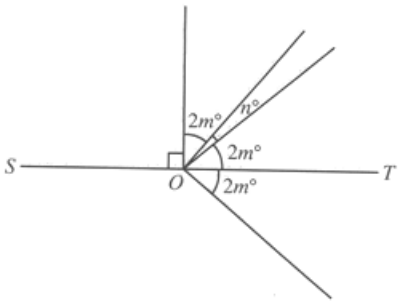
Advanced

5. Given that $\angle A$ and $\angle B$ are supplementary angles and that $5\angle A = 2\angle B$, find $\angle A$ and $\angle B$.

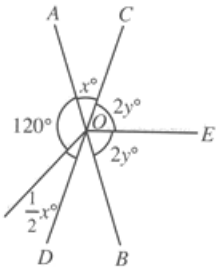
6. AOB and COD are straight lines. Find the values of x and y in the given figure.



7. In the figure, SOT is a straight line.
   (a) Form an equation to relate m and n.
   (b) Given that $2m = 3n$, find the value of n.



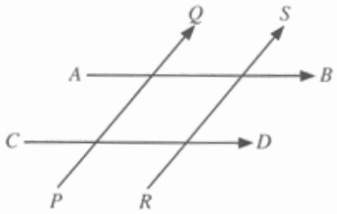
8. In the given figure, AOB and COD are straight lines. Find $\angle DOE$.



10.2 Angles Formed by Parallel Lines and a Transversal

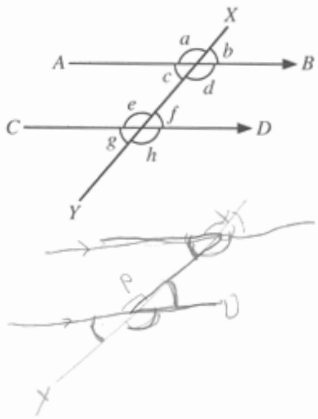
1. **Parallel lines** are straight lines which are drawn equal distance apart and they do not intersect. They are often indicated by arrows. The symbol for 'is parallel to' is '//'.

In the diagram on the right, $AB \,//\, CD$ and $PQ \,//\, RS$.



2. When a pair of parallel lines are intersected by a third straight line, called the **transversal**, the corresponding angles are equal, the alternate angles are equal and the interior angles on the same side of the transversal are supplementary.

In the diagram on the right, $AB \,//\, CD$ and XY is the transversal.



$a = e$
$b = f$
$c = g$
$d = h$

Corresponding angles are equal.
(corr. $\angle$s, $AB \,//\, CD$)

$c = f$
$d = e$

Alternate angles are equal.
(alt. $\angle$s, $AB \,//\, CD$)

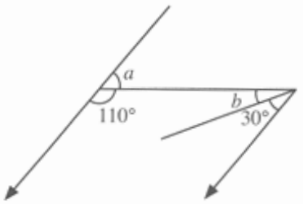
$c + e = 180°$
$d + f = 180°$

Interior angles are supplementary.
(interior $\angle$s, $AB \,//\, CD$)

3. If two straight lines are intersected by a third straight line, and if
either (a) a pair of alternate angles are equal,
or (b) a pair of corresponding angles are equal,
or (c) the interior angles on the same side of the transversal are supplementary,
then the first two straight lines are parallel.

4. Very often, the reasons to support your workings are stated in abbreviated form, within brackets, at different stages of the working.

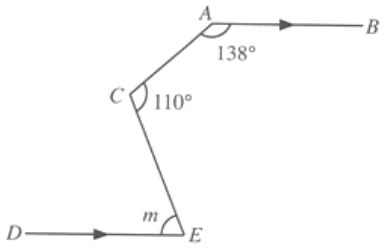
Example: Find the size of $\angle a$ and $\angle b$.



$a + 110° = 180°$ (adj. $\angle$s on a str. line)
$a = 180° - 110°$
$= 70°$
$b + 30° = a$ (alt. $\angle s$)
$b + 30° = 70°$
$b = 70° - 30°$
$= 40°$

5. It may be necessary to construct an additional line to the original diagram before attempting to do the working.

Examples: (a) Find the angle marked m.



Construct $CF \parallel DE$.

$\angle BAC + \angle ACF = 180°$ (int. $\angle$s, $AB \parallel CF$)

$138° + \angle ACF = 180°$

$\angle ACF = 180° - 138°$

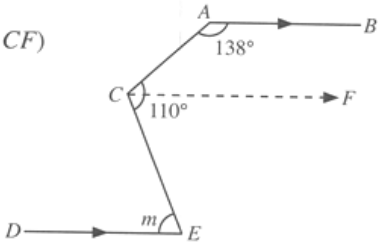
$= 42°$

$\angle FCE = 110° - 42°$

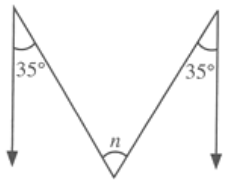
$= 68°$

$m = \angle FCE$ (alt. $\angle$s, $CF \parallel DE$)

$= 68°$



(b) Find the angle marked n.



Construct an additional line parallel to the original pair of parallel lines.

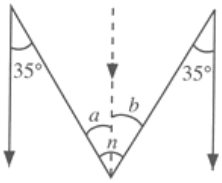
With notations in the figure:

$a = 35°$ (alt. $\angle$s)

$b = 35°$ (alt. $\angle$s)

$n = 35° + 35°$

$= 70°$

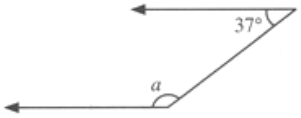


Practice 10.2

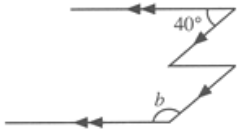
Basic

1. Find the values of the unknowns in each of the following figures.

(a)

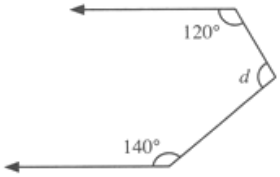


(b)

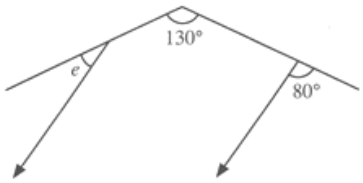


(c)

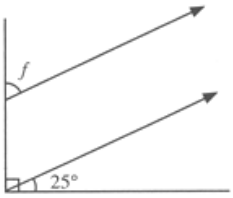
(d)



(e)

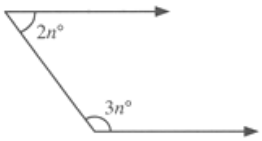


(f)



2. Find the value of n in each of the following figures.

(a)



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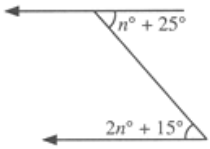
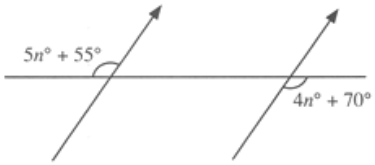
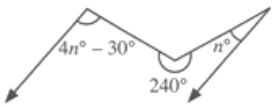


(c)



(d)



Advanced

3. In the figure, $AB \parallel CD$.

(a) Write an equation to connect w, x and y.

(b) Given that $\angle x = 4\angle w$ and $\angle w = 32°$, find $\angle z$.