CHAPTER 10

Probability

10.1 Introduction to Probability

1. Consider the following statements.
   (a) A high tide at sea is followed by a low tide.
   (b) I get a seven when I roll a normal die.
   (c) There is a 50-50 chance of getting a head when flipping a coin.
   (d) It is likely that I can get an ace card when drawing from a pack of cards.

   For an event that will definitely occur, the number 1 is assigned.
   For an event that is impossible to occur, the number 0 is assigned.
   For an event that is likely to happen, a fraction between 0 and 1 is assigned.

   For event (a), the number 1 is assigned. This event is sure to happen.
   For event (b), the number 0 is assigned. This event can never happen as a normal die only has numbers from 1 to 6.
   For event (c), the fraction $\frac{1}{2}$ is assigned. When flipping a coin, one out of two possible results can occur.
   For event (d), the fraction $\frac{4}{52}$ is assigned. There are 4 aces in a standard pack of 52 cards.

2. **Probability** is assigning a numerical value to the likelihood of an event occurring. Here are some examples involving the likelihood of an event occurring.

   Examples: (a) The probability of getting a tail when flipping a coin is $\frac{1}{2}$.
   (b) The probability of getting an even number when rolling a fair die is $\frac{3}{6}$ or $\frac{1}{2}$.
   (c) The probability of getting a club when drawing a card from a pack of cards is $\frac{13}{52}$ or $\frac{1}{4}$.

3. In an **experiment** on throwing a fair die, the set of **all possible outcomes** is called the **sample space**. An **event** from the sample space is the **favourable outcome** of the experiment.

   Examples: An experiment involves rolling a fair die.

   (a) Write down the sample space and state the total number of possible outcomes.

   The sample space for rolling a fair die is 1, 2, 3, 4, 5 and 6.
   The total number of possible outcomes is 6.

   (b) List the favourable outcomes for rolling a number less than 3. What is the number of favourable outcomes?

   The favourable outcomes for rolling a number less than 3 are 1 and 2.
   The number of favourable outcomes is 2.

4. The following symbols are used in writing probability statements.

$n(S)$	Number of possible outcomes in the sample space
$n(E)$	Number of favourable outcomes
$P(E)$	Probability of a favourable outcome

$$P(E) = \frac{n(E)}{n(S)} \qquad 0 \leq P(E) \leq 1$$

Practice 10.1

Basic

1. A card is drawn at random from a stack of cards numbered 0 to 5.

   (a) List the sample space.

   (b) State the number of possible outcomes.

   (c) List the favourable outcomes of drawing a number less than 3.

   (d) State the number of favourable outcomes in (c).

2. A marble is drawn at random from a bag containing 3 blue marbles and 1 red marble.

   (a) List the sample space.

   (b) State the number of possible outcomes.

   (c) List the favourable outcomes of drawing a blue marble.

   (d) State the number of favourable outcomes in (c).

3. A card is drawn at random from a complete suit of diamond cards within a standard pack of playing cards.

   (a) List the sample space.

   (b) State the number of possible outcomes.

   (c) List the favourable outcomes of drawing a picture card.

   (d) State the number of favourable outcomes in (c).

4. Ten cards marked with vowels in lower case and upper case are placed in a bag. A card is drawn at random from the bag.

   (a) List the sample space.

   (b) State the number of possible outcomes.

   (c) List the favourable outcomes of drawing a card with a lower case vowel.

   (d) State the number of favourable outcomes of drawing a card with an upper case vowel.

5. A bag contains 10 balls numbered 0 to 9. A ball is removed at random from the bag.

   (a) State the number of possible outcomes.

   (b) List the favourable outcomes of removing a ball with a number exceeding 5.

   (c) List the favourable outcomes of removing an even numbered ball.

   (d) State the number of favourable outcomes of removing a prime numbered ball.

10.2 Probability of Single Events

1. An **experimental probability** is derived from repeated trails of an experiment.

2. **Theoretical probability** is given by the following rule.

$$\textbf{Probability of an event occurring} = \frac{\textbf{Number of favourable outcomes}}{\textbf{Number of possible outcomes}}$$

Examples: (a) What is the probability of getting a number greater than 4 when rolling a fair die?

Numbers greater than 4 = 5 and 6
Number of favourable outcomes = 2
Possible outcomes = 1, 2, 3, 4, 5 and 6
Number of possible outcomes = 6

$$\text{Probability of getting a number greater than 4} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

(b) A bag contains 4 red balls and 5 yellow balls. A ball is picked at random from the bag.
(i) What is the probability of picking a yellow ball?
(ii) What is the probability of picking a red ball after a yellow ball is picked and not replaced into the bag?

(i) Number of possible outcomes = 9
Number of favourable outcomes = 5
P(picking a yellow ball) = $\frac{5}{9}$

(ii) Number of possible outcomes after picking a yellow ball without replacement = 8
Number of favourable outcomes = 4
P(picking a red ball) = $\frac{4}{8}$
$= \frac{1}{2}$

(c) Two Joker cards are added to a standard pack of 52 playing cards. A card is then drawn at random from the 54 cards. What is the probability of drawing a diamond?

Number of diamonds = 3
P(drawing a diamond) = $\frac{3}{54}$
$= \frac{1}{18}$

Practice 10.2

Basic

1. A pointer spins randomly on a number board marked with numbers 1 to 8. What is the probability of the pointer landing on numbers greater than 4?

2. A card is drawn at random from 2 red suits of a standard pack of playing cards. Find the probability of drawing

(a) an ace,

(b) a picture card,

(c) a five of spades,

(d) a heart.

3. A card is drawn at random from a standard pack of 52 playing cards. Find the probability of drawing

   (a) a red picture card,

   (b) a black queen,

   (c) a diamond,

   (d) a ten.

4. The chart shows the composition of students in a class.

	Malays	Chinese	Indians	Eurasians
Boys	3	5	3	3
Girls	4	7	2	4

   A student is chosen at random to be the class monitor. What is the probability that the student is

   (a) a boy,

   (b) a girl,

   (c) an Eurasian girl,

   (d) not a Chinese?

5. A head prefect is chosen from the members of prefectorial board. The probability of choosing a boy is $\frac{3}{5}$ and the probability of choosing a girl is $\frac{2}{5}$. How many members are there in the prefectorial board if the number of boys is 5 more than the number of girls?

Advanced

6. There are some black, white and grey cards in a bag. All the cards are of the same size. One card is drawn at random from the bag. The probability of drawing a black card is $\frac{1}{3}$ and the probability of drawing a white card is $\frac{2}{5}$.

   (a) What is the probability of drawing a grey card from the bag?

   (b) If there are 10 black cards, how many white cards are there in the bag?

   (c) 5 grey cards and 3 black cards are removed from the bag. What is the probability of drawing a white card?

7. 4 cards numbered 1, 2, 3 and 4 are placed in a bag. Two cards are drawn at random to obtain a two-digit number. What is the probability that

   (a) the two-digit number is an odd number?

   (b) the two-digit number is divisible by 4?

   (c) the two-digit number is a prime number?