CHAPTER 11

Averages of Statistical Data

11.1 Mean

1. The **arithmetic mean** or mean of a set of numbers is the same as the **average** of all the numbers.

$$\text{Mean} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$$

Example: A student scored 65, 56, 77, 68 and 61 marks for five tests. Find the mean mark for all the tests.

$$\text{Mean} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$$
$$= \frac{65 + 56 + 77 + 68 + 61}{5}$$
$$= 65.4$$

The mean mark for all the tests is 65.4.

2. In general, if the quantities are represented by $x_1, x_2, x_3, \ldots, x_n$, the mean is calculated by using the following formula.

$$\bar{x} = \frac{\Sigma x}{n},$$

where $\bar{x}$ is the mean, Σx is the sum of all quantities and n is the number of quantities. (Σ is the Greek symbol to denote 'the sum of'.)

3. Consider a frequency table from which the mean is to be calculated.

Example: The table shows the number of books read by 40 students during a one-week holiday period. Find the mean number of books read by each student.

Number of books read	0	1	2	3	4	5
Number of students	2	10	14	5	6	3

$$\text{Mean} = \frac{\text{Total number of books read}}{\text{Total number of students}}$$
$$= \frac{(2 \times 0) + (10 \times 1) + (14 \times 2) + (5 \times 3) + (6 \times 4) + (3 \times 5)}{50}$$
$$= 2.3$$

The mean number of books read by each student is 2.3.

4. Replacing the above frequency table with x for the number of books and f for the frequencies, we have the following.

Number of books read	x_1	x_2	x_3	x_4	x_5	x_6
Number of students	f_1	f_2	f_3	f_4	f_5	f_6

In general, for a set of numbers $x_1, x_2, x_3, x_4, x_5, \ldots, x_n$ corresponding to their frequencies $f_1, f_2, f_3, f_4, f_5, \ldots, f_n$, the formula for calculating the mean is given by the following formula.

$$\bar{x} = \frac{\Sigma fx}{\Sigma f},$$

where Σf is the sum of frequencies.

Example: During a fundraising project, the amount of money raised by the participants in a community was recorded as follows.

Amount of money ($)	0	10	20	30	40	50
Number of participants (f)	3	20	48	125	52	112

Calculate the mean amount collected by the participants.

Extend the table to include the following details.

Amount of money ($)	0	10	20	30	40	50	
Number of participants (f)	3	20	48	125	52	112	$\Sigma f = 360$
fx	0	200	960	3750	2080	5600	$\Sigma fx = 12\,590$

Extend the table to include the following details.

Total amount collected $= \Sigma fx$
$= \$12\,590$

Total number of participants $= \Sigma f$
$= 360$

Mean amount collected $= \frac{\Sigma fx}{\Sigma f}$
$= \frac{\$12\,590}{360}$
$= \$34.97$ (2 dec. plc.)

5. Consider another situation which involves the calculation of the arithmetic mean.

Example: During a certain month, the mean savings of three brothers was \$168. The ratio of their savings was 3 : 4 : 5 respectively.

(a) Calculate the amount each of them saved.

(b) The three brothers agreed to spend \$25, \$30 and \$35 respectively from their savings to buy a gift for their mother. What was their new mean savings after spending on the gift?

(a) Total amount saved = \$168 × 3
= \$504

$\frac{3}{12} \times \$504 = \$126$

$\frac{4}{12} \times \$504 = \$168$

$\frac{5}{12} \times \$504 = \$210$

The amount each of them saved was \$126, \$168 and \$210 respectively.

(b) Total amount left after spending on the gift = \$504 – (\$25 + \$30 + \$35)
= \$414

New mean savings = \$414 ÷ 3
= \$138

Their new mean savings after spending on the gift was \$138.

Practice 11.1

Basic

1. Find the mean for each of the following set of numbers.

   (a) 12, 13, 15, 13, 14, 17, 14, 18, 12, 11, 13, 10

   (b) 10.7, 10.8, 11.3, 10.7, 10.8, 10.5, 10.7, 10.8, 10.3, 11.0

2. The mean height of five boys is 156 cm. If another boy of height 162 cm joins the group, what is the mean height of the six boys?

3. The mean of the numbers 10, 13, n, $n + 5$, 14 and 12 is 15.8. What is the value of n?

4. Find, in terms of n, the mean of the following set of numbers.

   $n - 4, n - 2, n, n + 2, n + 4, n + 6$

5. The mean mass of four children is 11.13 kg. The mass of the lightest child is 8.64 kg. The other three children have the same mass. What is the difference in mass between the lightest and heaviest child?

6. Calculate the mean score from the given frequency table, where x is the number and f is the frequency.

x	1	2	3	4	5	6
y	3	4	5	2	5	3

7. The table below shows the frequency distribution of the number of days of urgent leave taken by all the employees in a company during the year.

Number of days of urgent leave taken	0	1	2	3	4
Number of employees	11	2	n	2	3

Given that the mean number of days of urgent leave taken was 1.36, find

(a) the value of n,

(b) the total number of employees in the company.

8. During a particular week, a survey on the amount of money spent by the students in a school canteen was carried out. The table below shows the results.

Amount spent ($)	0	5	10	15	20	25
Number of students	5	22	35	42	24	14

Calculate the mean amount spent by each student.

Advanced

9. The mean of five numbers is 15. The mean of two of the numbers is 18. What is the mean of the remaining three numbers?

11.2 Median

1. When a given set of numbers is arranged in order of magnitude (ascending order), the middle number is called the **median**.

2. The median is a measure of central tendency which is preferred when a set of data shows extreme values.

3. When there an odd number of terms in the set of data, the middle term is the median.

4. When there is an even number of terms in the set of data, the mean of the two middle terms is the median.

Examples: (a) Find the median from the given set of numbers.
2, 3, 2, 3, 4, 3, 4, 1, 10

Arranging in order of magnitude,
1, 2, 2, 3, 3, 3, 4, 4, 10

median = middle term

There is an odd number of terms.
The median is 3.

(b) Find the median from the given set of numbers.
5, 3, 5, 2, 4, 2, 2, 1, 5, 6, 12, 5

Arranging in order of magnitude,
1, 2, 2, 2, 3, 4, 5, 5, 5, 5, 6, 12

median = mean of 6^{th} and 7^{th} term

There is an even number of terms.

$$\text{Median} = \frac{4+5}{2}$$
$$= 4.5$$

(c) In a mental arithmetic test, the marks of 50 students are tabulated as shown. What is the median mark?

Mark	0	1	2	3	4	5
Frequency	2	9	14	13	11	1

There are 50 scores arranged in order of magnitude.
The median is the mean of the 25^{th} score and the 26^{th} score.
The 25^{th} score is 2 marks and the 26^{th} score is 3 marks.

$$\text{Median} = \frac{2+3}{2}$$
$$= 2.5 \text{ marks}$$

Practice 11.2

Basic

1. Find the median for each of the following set of numbers.

(a) 1, 2, 3, 5, 2, 3, 4, 6, 6, 8, 3, 4, 5

(b) 40.2, 40.1, 40.2, 40.0, 41.3, 40.5, 40.6, 40.5, 41.2, 40.9

2. Use the information given in the frequency table to find the median.

Value (x)	1	1.5	2	2.5	3	3.5
Frequency	3	3	5	6	4	1

3. The following table shows the number of goals scored in 20 soccer matches.

3	1	3	2	2
2	3	0	1	3
2	2	3	3	5
3	2	1	3	2

Find the median number of goals scored.

4. The median for the following set of numbers $a, a-2, a-1, a+1, a-3$ and $a+3$ is 15.

(a) Find the value of a.

(b) Hence, find the mean.

Advanced

5. The following table shows the scores when tossing a die during an experiment.

Score	1	2	3	4	5	6
Frequency	15	x	12	20	10	16

What is the smallest possible value of x if the median is 2?

11.3 Mode

The number which occurs most frequently in a given set of data is called the **mode**.

Examples: (a) The following numbers show the shoe sizes of 10 people.

$6, 5\frac{1}{2}, 6\frac{1}{2}, 6, 5, 6, 6, 5, 7, 5\frac{1}{2}$

The shoe size that occurs most frequently is size 6.
The mode of the given set of shoe sizes is size 6.

(b) The following set of numbers shows the number of goals scored by a soccer team during a soccer season. A total of 15 matches were played.

2	1	3	0	4
0	1	1	2	2
2	6	3	4	1

The score of 1 goal occurred 4 times.
The score of 2 goals also occurred 4 times.
The modal scores in this case are 1 and 2 goals.

(c) The following frequency table shows the results of a random survey on the number of times some adults go to the movie theatres during a certain month.

Number of visits	0	1	2	3	4	5
Number of adults	5	6	3	2	1	1

From the information listed in the frequency table, most adults (6 of them) visited the movie theatres once in that month.
The mode of the distribution is 1 visit.

Practice 11.3

Basic

1. Find the mode for each of the following set of numbers.

   (a) 24, 23, 22, 24, 25, 26, 22, 21, 27, 22

   (b) 30.2, 30.1, 30.2, 30.0, 31.3, 30.5, 30.6, 30.5, 31.2, 30.9

2. The following frequency table shows the results of tossing a die 50 times.

Score	1	2	3	4	5	6
Frequency	7	13	3	13	8	6

   (a) What is the mode?

   (b) What is the median?

3. The results of a survey of the number of children per family in a sample of 50 families are shown in the table.

Number of children per family	0	1	2	3	4
Number of families	15	10	19	5	1

(a) Write down the modal number of children per family.

(b) Find the median number of children per family.

(c) Find the mean number of children per family.

4. The following table shows the amount of wages paid to the workers of a certain factory.

Amount ($)	450	600	750	900	1 050	1 200
Number of workers	5	22	19	21	1	2

Find

(a) the modal wage,

(b) the median wage,

(c) the mean wage.

Advanced

5. The following table shows the results of a survey of masses of the members in an Adventure Club.

Mass (kg)	39	40	41	42	43	44	45
Number of members	2	5	9	7	8	4	5

(a) What is the modal mass?

Two members from the modal group left the Adventure Club after the survey.

(b) What is the new modal mass?

(c) What is the median mass of the remaining members?

(d) Find the mean mass of the remaining members.