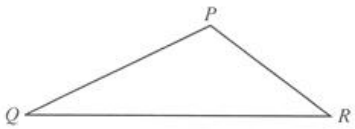
CHAPTER 11

Triangles, Quadrilaterals and Polygons

11.1 Triangles

1. A triangle is a 3-sided plane figure. It has 3 vertices and 3 interior angles.



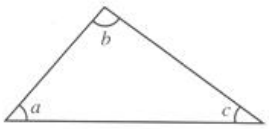
In $\triangle PQR$, the points P, Q and R are the vertices.
$\angle P$, $\angle Q$ and $\angle R$ are the interior angles.
PQ, PR and QR are the sides.

2. Triangles are classified according to the number of equal sides and the type of angles they possess.

Scalene triangle	None of the sides are equal All interior angles are unequal.
Isosceles triangle	2 sides are equal. Base angles are equal.
Equilateral triangle	All sides are equal. All angles are equal. Each interior angle is 60°.
Acute-angled triangle	Each of the interior angles is obtuse.
Obtused-angled triangle	One of the interior angle is obtuse.
Right-angled triangle	One of the interior angle is 90°.

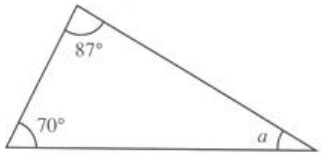
3. The sum of all the angles in a triangle is equal to 180°.

$a + b + c = 180°$ ($\angle$ sum of $\triangle$)



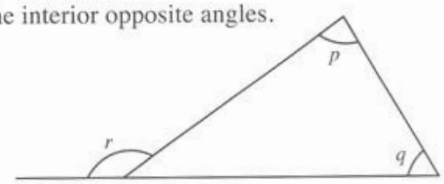
Example: Find the value of angle a in the given figure.

$a + 87° + 70° = 180°$ ($\angle$ sum of $\triangle$)
$a = 180° - 87° - 70°$
$= 23°$



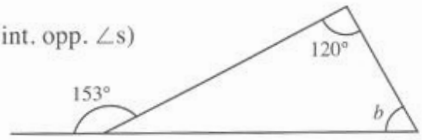
4. The exterior angle of a triangle is equal to the sum of the interior opposite angles.

$r = p + q$ (ext. $\angle$ = sum of int. opp. $\angle$s)



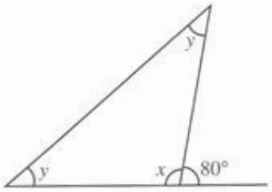
Examples: (a) Find the value of b in the figure.

$153° = b + 120°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$b = 153° - 120°$
$= 33°$



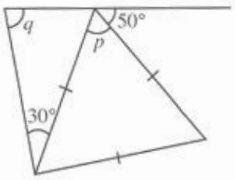
(b) Find the values of x and y in the figure.

$x + 80° = 180°$ (adj. $\angle$s on a str. line)
$x = 180° - 80°$
$= 100°$
$y + y = 80°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$2y = 80°$
$y = 80° \div 2$
$= 40°$



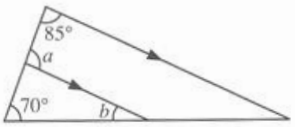
(c) Find the values of p and q in the figure.

$p = 60°$ ($\angle$ in equilateral $\triangle$)
$p + 50° = q + 30°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$60° + 50° = q + 30°$
$q = 110° - 30°$
$= 80°$



(d) Find the values of a and b in the figure.

$a + 85° = 180°$ (int. $\angle$s)
$a = 180° - 85°$
$= 95°$
$a = b + 70°$ (ext. $\angle$ = sum of int. opp. $\angle$s)
$95° = b + 70°$
$b = 95° - 70°$
$= 25°$

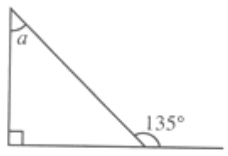


Practice 11.1

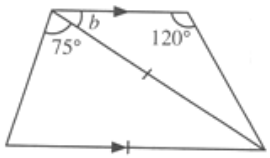
Basic

1. Find the values of the unknowns in each of the following figures.

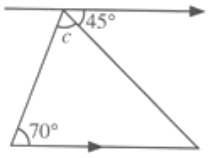
(a)



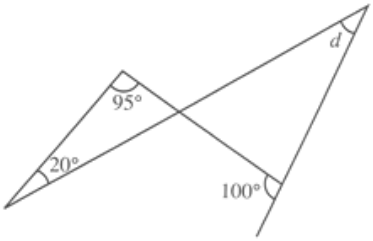
(b)



(c)

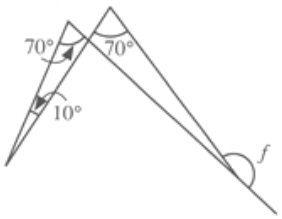


(d)



2. Each of the base angles of an isosceles triangle is twice the third angle. Find the angles of this triangle.

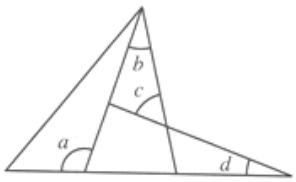
3. Find the value of f in the figure.



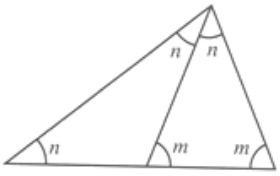
4. In $\triangle ABC$, $\angle A = 80°$ and $\angle ACB = 70°$. BC is produced to D and CK is parallel to AB. Calculate $\angle KCD$.

Advanced

5. Write an equation to relate a, b, c, and d.



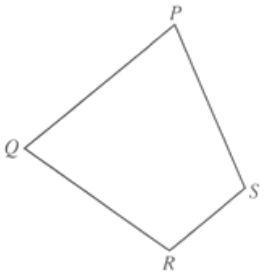
6. Express m as a multiple of n in the given figure. Hence, find the values of m and n.



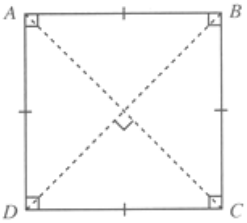
11.2 Quadrilaterals

1. A **quadrilateral** is a four-sided plane figure. It has 4 sides and 4 interior angles.

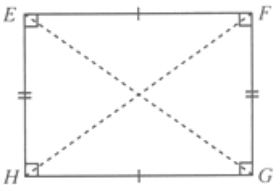
   $PQRS$ is a quadrilateral. PR and QS are the **diagonals**.
   The sum of all its interior angles is 360°.
   $\angle P + \angle Q + \angle R + \angle S = 360°$ ($\angle$ sum of quad.)



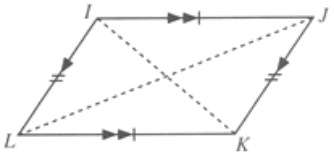
2. $ABCD$ is a **square**. It has 4 equal sides.
   $AB = BC = CD = DA$
   All the angles are right-angled.
   $\angle A = \angle B = \angle C = \angle D = 90°$
   The diagonals are equal and they bisect the opposite angles.
   $AC = BD$, $\angle BAC = \angle DAC$, $\angle ABD = \angle CBD$,
   $\angle DCA = \angle BCA$, $\angle ADB = \angle CDB$
   The diagonals bisect each other at right angles.



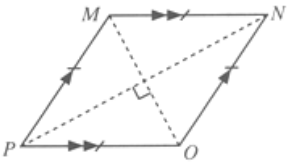
3. $EFGH$ is a **rectangle**. Its opposite sides are equal.
   $EF = GH$, $EH = FG$.
   All the angles are right-angled.
   $\angle E = \angle F = \angle G = \angle H = 90°$
   The diagonals are equal and they bisect each other.
   $EG = FH$



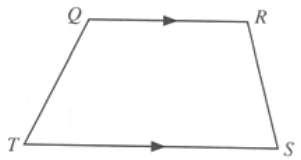
4. $IJKL$ is a **parallelogram**.
   Its opposite sides are equal and parallel.
   $IJ = KL$, $IL = JK$, $IJ \parallel KL$, $IL \parallel JK$
   Its opposite angles are equal.
   $\angle I = \angle K$, $\angle J = \angle L$
   The diagonals bisect each other.



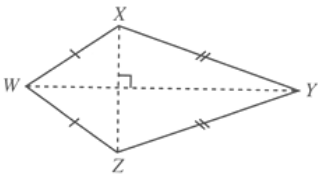
5. $MNOP$ is a **rhombus**. All 4 sides are equal.
   $MN = NO = OP = PM$
   The opposite sides are parallel.
   $MN \parallel PO$, $MP \parallel NO$
   The opposite angles are equal.
   $\angle M = \angle O$, $\angle P = \angle N$
   The diagonals bisect the opposite angles.
   $\angle PMO = \angle NMO$, $\angle POM = \angle NOM$,
   $\angle PNM = \angle PNO$, $\angle NPM = \angle NPO$
   The diagonals bisect each other at right angles.



6. $QRST$ is a **trapezium**. It has a pair of parallel sides.
$QR \parallel ST$
When $QT = RS$, it is an isosceles trapezium with $\angle Q = \angle R$ and $\angle T = \angle S$ and diagonals QS and TR are equal.

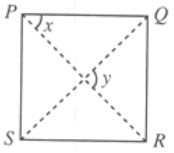


7. $WXYZ$ is a **kite**. It has 2 pairs of equal adjacent sides.
$WX = WZ$, $YX = YZ$
The longer diagonal bisects the opposite angles.
$\angle XWY = \angle ZWY$, $\angle XYW = \angle ZYW$
The diagonals intersect at right angles.
The longer diagonal bisects the shorter diagonal.



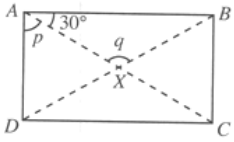
8. Some examples involving the properties of quadrilaterals are shown below.

Examples: (a) $PQRS$ is a square.
Find the values of x and y.



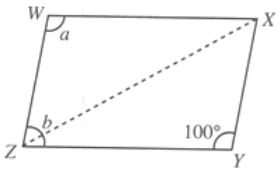
$x = 45°$ (diagonal PR bisects the opp. $\angle$s)
$y = 90°$ (diagonals bisect at right angles)

(b) $ABCD$ is a rectangle.
Find the values of p and q.



$p + 30° = 90°$ ($\angle BAD$ is a right angle)
$p = 90° - 30°$
$= 60°$
$\angle ABX = \angle BAX = 30°$ (base $\angle$s of isos. $\triangle ABX$)
$\angle ABX + \angle BAX + q = 180°$ ($\angle$sum of $\triangle ABX$)
$30° + 30° + q = 180°$
$q = 180° - 60°$
$= 120°$

(c) $WXYZ$ is a rhombus.
Find the values of a and b.

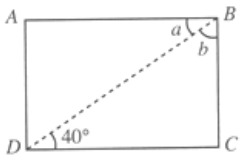


$a = 100°$ (opp. $\angle$s of rhombus are equal)
$b + 100° = 180°$ (int. $\angle$s, $WZ \parallel XY$)
$b = 180° - 100°$
$= 80°$

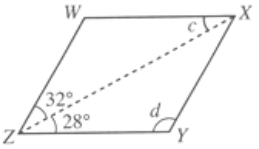
Practice 11.2

Basic

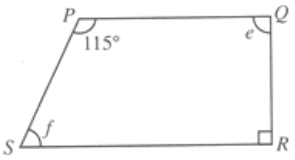
1. $ABCD$ is a rectangle. Find the values of a and b.



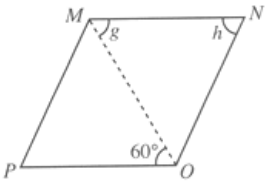
2. $WXYZ$ is a parallelogram. Find the values of c and d.



3. $PQRS$ is a trapezium. Find the values of e and f.

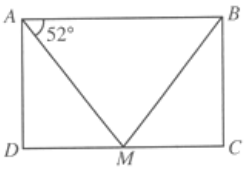


4. $MNOP$ is a rhombus. Find the values of g and h.



5. $ABCD$ is a rectangle. M is the midpoint of DC.
Given that $\angle BAM = 52°$, calculate

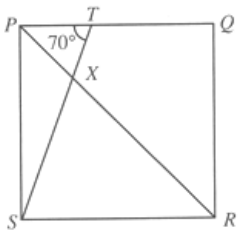
(a) $\angle DAM$, (b) $\angle AMB$.



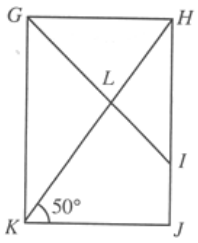
Advanced

6. $PQRS$ is a square and PXR is a straight line.
Given that $\angle PTS = 70°$, calculate

(a) $\angle QTX$, (b) $\angle SXR$.



7. $GHJK$ is a rectangle. $GH = HI$ and $\angle HKJ = 50°$.
HLK and GLI are straight lines. Find $\angle JIG$ and $\angle GLK$.



11.3 Polygons

1. A **polygon** is a plane figure with three or more straight edges. A **regular polygon** is one in which all the sides and interior angles are equal.

2. Special names are given to common polygons according to the number of sides.
Triangles (3 sides); **Quadrilaterals** (4 sides); **Pentagon** (5 sides); **Hexagon** (6 sides); **Heptagon** (7 sides); **Octagon** (8 sides); **Nonagon** (9 sides); **Decagon** (10 sides)

3. The **sum of the interior angles** of a polygon of n sides is $(n - 2) \times 180°$ or $(2n - 4)$ right angles.

Examples: (a) Find the size of an interior angle of a regular hexagon.

Sum of interior angles of a hexagon $= (6 - 2) \times 180°$
$= 4 \times 180°$
$= 720°$

Each interior angle $= 720° \div 6$
$= 120°$

(b) The size of each interior angle of a regular polygon is 135°.
How many sides does the polygon have?

Let n be the number of sides.
$(n - 2) \times 180 = n \times 135$
$180n - 360 = 135n$
$180n - 135n = 360$
$45n = 360$
$n = 360 \div 45$
$= 8$

The polygon has 8 sides.

4. The **sum of the exterior angles** of a polygon is 360°.

Examples: (a) Find the size of an exterior angle of a regular octagon.

Sum of exterior angles = 360°
Each exterior angle = 360° ÷ 8
= 45°

(b) The exterior angle of a regular polygon is 30°.
Find the number of sides this polygon has.

Sum of exterior angles = 360°
Number of sides the polygon has = 360° ÷ 30°
= 12

Practice 11.3

Basic

1. Calculate the sum of the interior angles of a polygon with

   (a) 7 sides, (b) 12 sides, (c) 20 sides.

2. Find the size of each interior angle of a regular polygon with

   (a) 5 sides, (b) 10 sides, (c) 18 sides.

3. Find the number of sides of a regular polygon if each exterior angle of the polygon is

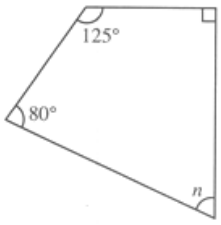
   (a) 15°, (b) 72°.

4. Find the number of sides of a regular polygon if each interior angle of the polygon is

   (a) 120°, (b) 156°.

5. Find the value of n in each of the following figures.

(a)



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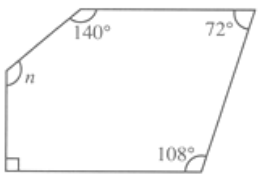
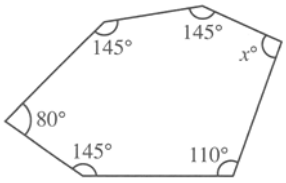
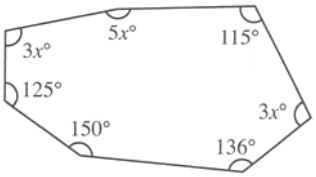


6. Find the value of x in each of the following figures.

(a)



(b)



7. The exterior angles of a triangle are 36°, y° and y°. Calculate each interior angle of the triangle.

8. The interior angles of a polygon are in the ratio 4 : 4 : 3 : 2 : 5.

   (a) Calculate the largest interior angle.

   (b) Find the largest exterior angle of the polygon.

Advanced

9. P, Q, R and S are consecutive vertices of a regular decagon. Find

   (a) $\angle PQR$,

   (b) $\angle PQS$.