CHAPTER 14

Volume and Surface Area of Prisms and Cylinders

14.1 Conversion of Units

1. The cubic millimeter (mm^3), cubic centimeter (cm^3), cubic metre (m^3), milliliter (ml) and litre (l) are units of volume.

> ***Examples:*** The volume of a small disc is 157 mm^3.
> The volume of a cuboid is 45 cm^3.
> The volume of airspace in a room is 240 m^3.
> The volume of cough mixture in a bottle is 90 ml.
> The volume of petrol in a fuel tank is 45 l.

2. Remember these equivalents when converting from one unit of volume to another.

$1\ cm^3 = (10 \times 10 \times 10)\ mm^3$
$= 1\ 000\ mm^3$
$1\ l = 1\ 000$ ml

$1\ m^3 = (100 \times 100 \times 100)\ cm^3$
$= 1\ 000\ 000\ cm^3$
$1\ l = 1\ 000\ cm^3$

$m^3 \underset{\times \frac{1}{1\,000\,000}}{\overset{\times 1\,000\,000}{\rightleftarrows}} cm^3 \underset{\times \frac{1}{1\,000}}{\overset{\times 1\,000}{\rightleftarrows}} mm^3$

$l \underset{\times \frac{1}{1\,000}}{\overset{\times 1\,000}{\rightleftarrows}}$ ml

> ***Examples:*** (a) Express 150 mm^3 in cm^3.
>
> $1\ mm^3 = \frac{1}{1\,000}\ cm^3$
> $150\ mm^3 = 150 \times \frac{1}{1\,000}\ cm^3$
> $= 0.15\ cm^3$
>
> (b) Express 250 ml in l.
>
> 1 ml $= \frac{1}{1\,000}\ l$
> 250 ml $= 250 \times \frac{1}{1\,000}\ l$
> $= 0.25\ l$
>
> (c) Express 1.5 m^3 in cm^3.
>
> $1\ m^3 = 1\ 000\ 000\ cm^3$
> $1.5\ m^3 = 1.5 \times 1\ 000\ 000\ cm^3$
> $= 1\ 500\ 000\ cm^3$
>
> (d) Express 1.125 l in ml.
>
> $1\ l = 1\ 000$ ml
> $1.125\ l = 1.125 \times 1\ 000$ ml
> $= 1\ 125$ ml

Practice 14.1

Basic

1. Express (a) 500 mm^3 in cm^3, (b) 5 250 cm^3 in m^3,
   (c) 350 cm^3 in l, (d) 600 ml in l.

2. Express (a) 1.25 m^3 in cm^3, (b) 50 cm^3 in mm^3,
   (c) 1.2 l in cm^3, (d) 0.25 l in ml.

3. Mr Lee was given a bottle containing 67.5 ml of cough syrup. He was told to take 7.5 ml three times a day. How many days will the cough syrup last?

4. A half-litre bottle of orange cordial is diluted in 2.5 l of water to make a drink. The mixture is then poured into cups that hold 150 ml each. How many such cups will the mixture fill?

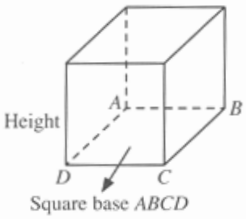
5. The capacity of a water cistern is 6 m^3. How many litres of water does the cistern hold when it is three-quarters full?

6. A healthy person is expected to drink up to about 8 cups of water each day. Each cup has a volume of about 240 cm^3. How many litres is the daily intake?

14.2 Volume and Surface Area of Cubes and Cuboids

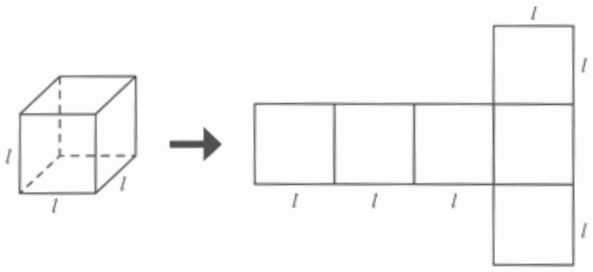
1. You have learnt how to find the volume of a cube by using the formula $V = l^3$, where V is the volume and l is the side of the cube.

Cube



Notice that the volume of the cube is the area of its square base × the height.

2. Drawing the **net** of a solid helps in the finding of its total surface area.
   A net of a cube is shown below.



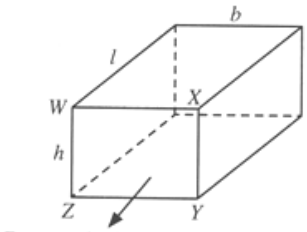
Total surface area of a solid cube = 6 × area of each square side
$= 6l^2$ square units

Example: Find the volume and the total surface area of a 5-cm cube.

Volume of cube = 5^3 cm^3
= 125 cm^3
Total surface area of cube = 6×5^2 cm^2
= 150 cm^2

3. You have also learnt how to find the volume of a cuboid by using the formula $V = lbh$, where V is the volume, l is the length, b is the breadth and h is the height.

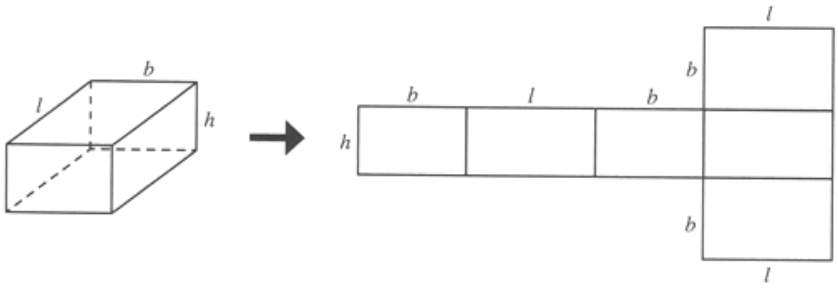
Cuboid



Rectangular base $WXYZ$

Notice that the volume of the cuboid is the area of the rectangular base × the height.

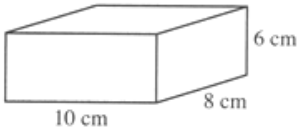
4. A net of a cuboid is shown below.



Total surface area of a solid cuboid = (area of base × 2) + (area of its lateral sides)
= (area of base × 2) + (perimeter of base × height)
$= 2lb + [2(l + b)h]$

Example: Find the volume and total surface area of a cuboid with a base 10 cm by 8 cm and a height of 6 cm.

Volume of cuboid = $10 \times 8 \times 6$ cm^3
= 480 cm^3
Total surface area = $(10 \times 8 \times 2) + [2(10 + 8)6]$ cm^2
= 376 cm^2

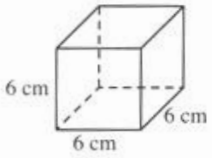


Practice 14.2

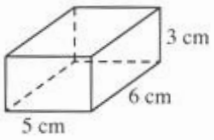
Basic

1. Find the volume and total surface area of each solid.

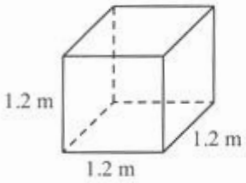
(a) 6-cm cube



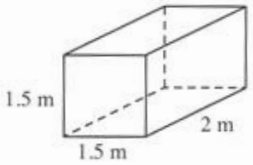
(b) 5 cm by 6 cm by 3 cm cuboid



(c) 1.2-m cube



(d) 1.5 m by 1.5 m by 2 m cuboid



2. The internal dimensions of a box is 20 cm by 10 cm and 8 cm high. How many 2-cm cubes can be packed into the box?

3. A block of wax in the form of a 15-cm cube is melted and reshaped into a rectangular block with a base 20 cm by 10 cm. What is the height of the rectangular block?

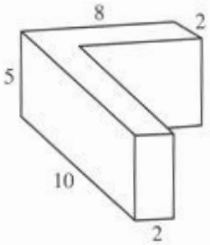
4. Find the total surface area of a cardboard box without a lid when the base is 25 cm by 10 cm and with a height of 8 cm.

5. How many litres of water will fill a swimming pool measuring 20 m by 10 m and with a depth of 1.5 m?

6. A water cistern is 4 m long, 2.5 m wide and 1.5 m high. It contained water to a depth of 0.5 m initially. Find the new depth of water in the cistern when another 5 000 litres of water is pumped into it.

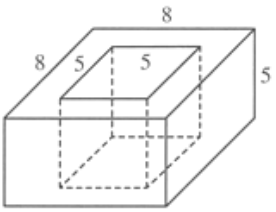
7. Find the total surface area of a metal cube measuring 8 000 cm^3. The cube is then melted down to form a cuboid with a cross-section of 160 cm^2. What is the length of the cuboid?

8. Find the volume of the L-shaped solid. Dimensions are given in cm.



Advanced

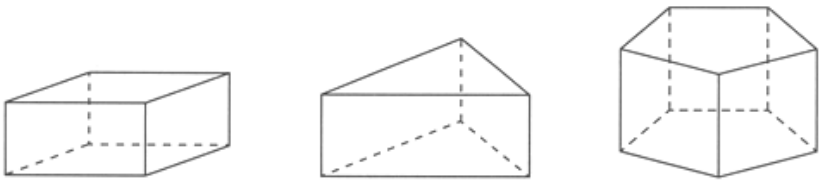
9. Find the volume and total surface area of a wooden structure with a hollow cubical centre as shown. Dimensions are given in cm.



14.3 Volume and Surface Area of Prisms

1. A right prism is a solid of **uniform cross-section**. It has a **flat base** and its **lateral sides** are upright.

Examples:

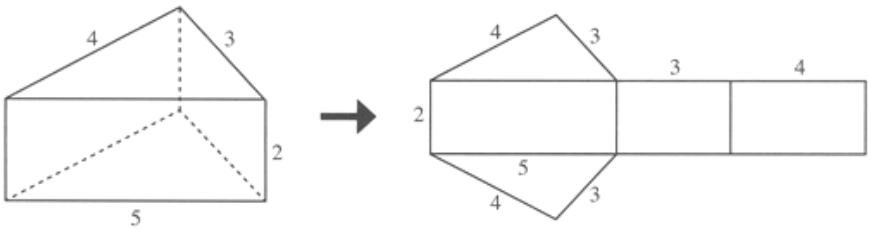


Rectangular prism | Triangular prism | Pentagonal prism

2. Volume of a prism = area of its cross-section × its height.
Total surface area = (area of base × 2) + (area of its lateral sides)
= (area of cross-section × 2) + (perimeter of base × height)

Example: Find the volume and total surface area of the triangular prism with the given dimensions in cm.

$$\text{Volume of prism} = \frac{1}{2} \times 4 \times 3 \times 2 \text{ cm}^3$$
$$= 12 \text{ cm}^3$$



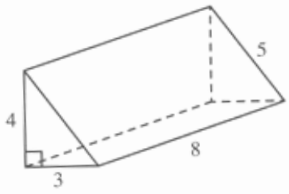
$$\text{Total surface area} = \left(\frac{1}{2} \times 4 \times 3 \times 2\right) + (4 + 3 + 5) \times 2 \text{ cm}^2$$
$$= 12 + 24 \text{ cm}^2$$
$$= 36 \text{ cm}^2$$

Practice 14.3

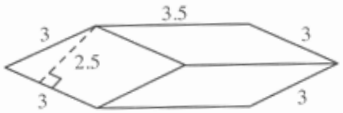
Basic

1. Find the volume and surface area of each solid prism. Dimensions are given in cm.

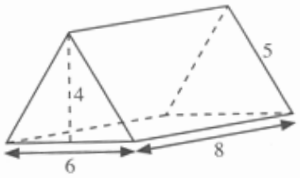
(a)



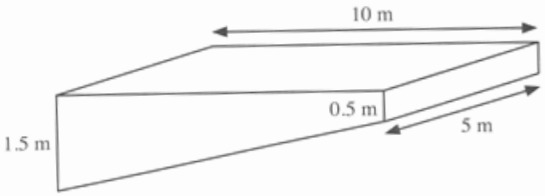
(b)



2. The figure shows a nylon tent consisting of a ground sheet and flaps. Find the total surface area of the nylon sheet required to make this tent. Dimensions are given in m.

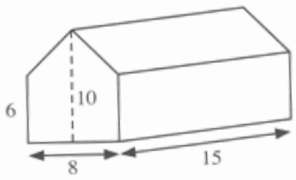


3. The figure shown represents a swimming pool 5 m wide and 10 m long. The shallow end of the pool is 0.5 m and the deeper end is 1.5 m. How many cubic metres of water are needed to fill the pool?

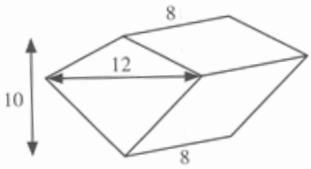


4. Find the volume of each prism. Dimensions are given in cm.

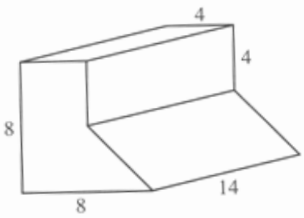
(a)



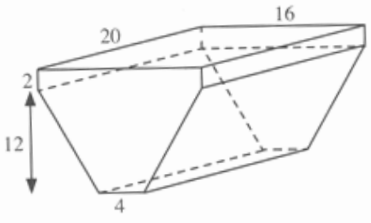
(b)



(c)

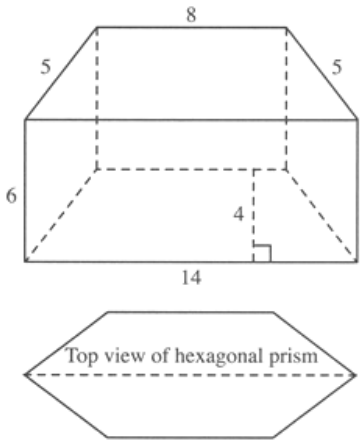


(d)



Advanced

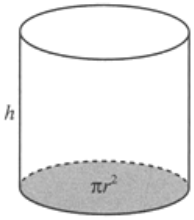
5. Two similar prisms, shown in the given diagram, are placed together to form a hexagonal prism. Find the volume and total surface area of the hexagonal prism. Dimensions are given in cm.



14.4 Volume and Surface Area of Cylinders

1. The cylinder is a special prism with a circular cross-section.
The method of finding the volume of a cylinder is similar to that of a prism.

Cylinder

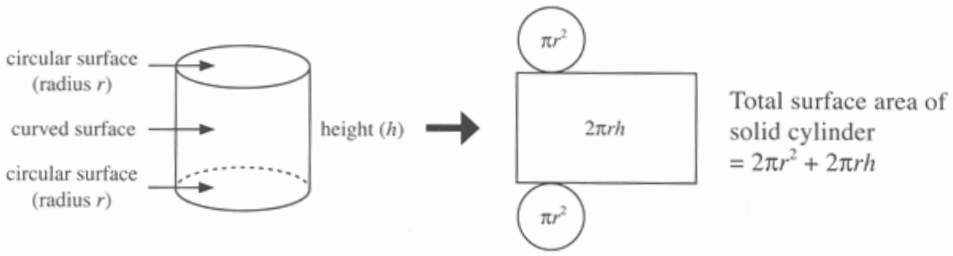


Volume of cylinder = area of base × height
$= \pi r^2 h$

Example: Find the volume of a cylinder with a radius of 3.5 cm and a height of 10 cm.
$\left(\text{Take } \pi = \frac{22}{7}\right)$

Volume of cylinder $= \frac{22}{7} \times 3.5 \times 3.5 \times 10 \text{ cm}^3$
$= 385 \text{ cm}^3$

2. The total surface area of a solid cylinder is the total surface area of two circular surfaces and the area of its curved surface.



Total surface area of solid cylinder
$= 2\pi r^2 + 2\pi rh$

Example: Find the total surface area of a solid cylinder if its radius is 4.2 cm and its height is 12 cm. Correct your answer to one decimal place. (Take $\pi = 3.14$)

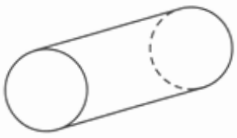
Total surface area of solid cylinder
$= (2 \times 3.14 \times 4.2^2) + (2 \times 3.14 \times 4.2 \times 12)\ \text{cm}^2$
$= 427.3\ \text{cm}^2$

Practice 14.4

Basic

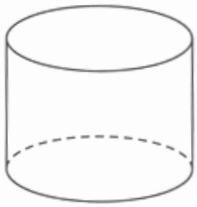
1. Find the volume of each solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a)



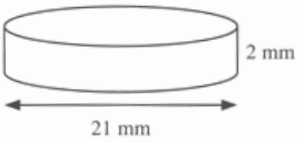
Radius of cylinder = 7 cm
Length of cylinder = 15 cm

(b)

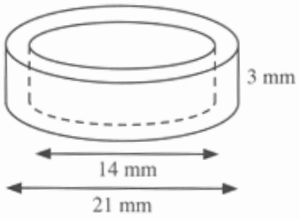


Diameter = 14 cm

(c)



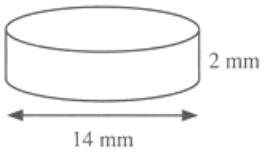
(d)



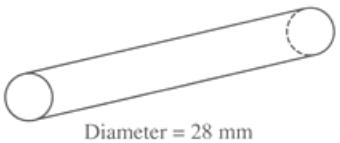
Cylinder with hollow centre

2. Find the total surface area of each solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a)



(b)



3. Find the volume of the material in a 1-m pipe with an internal radius of 2.8 cm and a thickness of 0.25 cm. (Take $\pi = 3.14$)

4. A circular coin has a diameter of 28 mm and a thickness of 1.5 mm. What is the volume of material that is needed to make 1 000 000 such coins? Express your answer in m^3. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Advanced

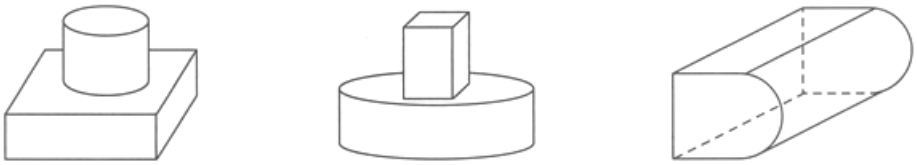
5. A cylindrical drum, measuring 35 cm in radius and 0.5 m in height, is full of water. A tiny leak at the bottom of the drum drains 0.12 litres of water every minute. How long does it take for the water level to fall by 10 cm? $\left(\text{Take } \pi = \frac{22}{7}\right)$

6. The building of 10 large wooden columns of a concert hall was undertaken by a contractor. Each column measures 0.5 m in diameter and 3 m in height. The curved surface of each column is sprayed with two coats of paint at a cost of \$5.25 per m^2 per coat. The building cost of each column, excluding the painting cost, is \$550. Calculate the total cost of building the 10 columns, including the painting cost. (Take $\pi = 3.14$)

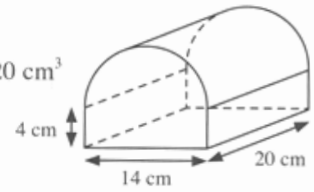
14.5 Volume and Surface Area of Composite Solids

You will need to revise the formulae for the volumes and total surface areas of the various solids learnt in the earlier sections.

The diagrams shown below are some examples of composite solids.



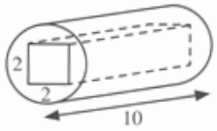
Examples: (a) A solid consists of a half-cylinder mounted on a cuboid. Find the volume and surface area of the solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$



Volume of solid $= \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + 14 \times 4\right) \times 20 \text{ cm}^3$
$= 2\,660 \text{ cm}^3$

Total surface area of solid
$= 2\left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + 14 \times 4\right) + \left(\frac{22}{7} \times 7 + 4 + 14 + 4\right) \times 20 \text{ cm}^2$
$= 1\,146 \text{ cm}^2$

(b) The figure shown consists of a cylinder with a square prism 2 cm by 2 cm by 10 cm carved out in the middle. The radius of the cylinder is 2.5 cm and its length is 10 cm.
Find (a) the area of its cross-section,
(b) the volume of the solid.
(Take $\pi = 3.14$)



(a) Area of cross-section of solid $= (3.14 \times 2.5 \times 2.5) - (2 \times 2) \text{ cm}^2$
$= 15.625 \text{ cm}^2$

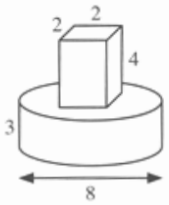
(b) Volume of solid $= 15.625 \times 10 \text{ cm}^3$
$= 156.25 \text{ cm}^3$

Practice 14.5

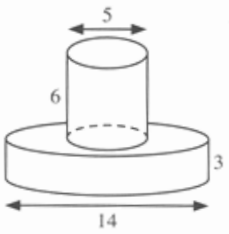
Basic

1. Find the volume of each of the following solids.
Dimensions are given in cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

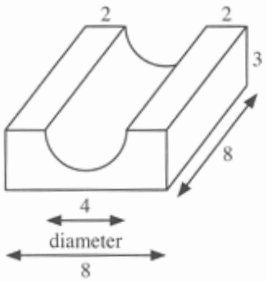
(a)



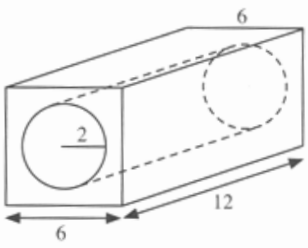
(b)



(c)



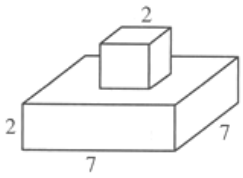
(d)



Solid with cylindrical hollow

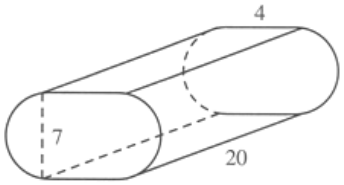
2. Find the surface area of each solid. Dimensions are given in cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a)



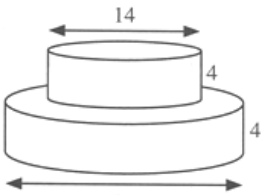
A 2-cm cube mounted on a cuboid 2 cm by 7cm by 7 cm

(b)



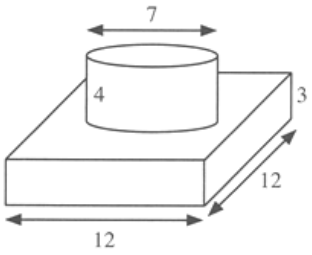
Cross-section with semicircular ends

(c)



A 2-tier cylindrical solid

(d)



A cylinder 4 cm high, diameter 7 cm, mounted on a cuboid 12 cm by 12 cm by 3 cm

Advanced

3. A 5-m solid iron rod has a diameter of 2.5 cm. Calculate the mass of 50 such rods in kg when the mass of 1 cm^3 of iron is 7.2 g. (Take $\pi = 3.14$)

4 A square piece of aluminum, 1 m^2 and 0.5 cm thick, is recast to form a circular disk with a 1-cm thickness.
(a) What is the diameter of the disk?
(b) What is the total surface area of the disk?
(Take $\pi = 3.14$)

5 A road tunnel, half a kilometer in length, is semicircular in cross-section. The width of the road through the tunnel is 21 m. The curved surface of the tunnel is to be cleaned and repainted with two coats of paint. A contractor quotes the cost of cleaning at \$4.50/m^2 and the cost of painting at \$8.20/m^2 per coat. What is the total cost of cleaning and repainting the tunnel?
$\left(\text{Take } \pi = \frac{22}{7}\right)$