CHAPTER 2 Linear Equations and Linear Graphs

2.1 Linear Equations with Two Variables

1. The following equations are some examples of equations involving one variable, i.e. one unknown value. These are **linear equations with one variable**.

> *Examples:* $2a = 5$ $\quad b + 3 = 6$ $\quad 2 + 3c = 2\frac{2}{3}$ $\quad 5.2d - 4 = 9$ $\quad \frac{3}{5}e = 1\frac{2}{7}$

- In the equation $b + 3 = 6$, $b = 6 - 3$
  $= 3$
  There is only one solution for the variable b, $b = 3$.

2. The following equations are examples of equations involving two variables, i.e. two unknown values. These are **linear equations with two variables**.

> *Examples:* $a + b = 3$ $\quad 2c - d = 7$ $\quad e = 3f + 4$ $\quad \frac{2}{3}g = 3 + 2h$ $\quad 5j - 4 = k$

- In the equation $2c - d = 7$, the value of c will depend on the value of d and vice versa.
  When $c = 3$, substitute $c = 3$ into the equation $2c - d = 7$.
  $2(3) - d = 7$
  $6 - d = 7$
  $-d = 7 - 6$
  $= 1$
  $d = -1$
- When $d = 5$, substitute $d = 5$ into the equation $2c - d = 7$.
  $2c - 5 = 7$
  $2c = 7 + 5$
  $= 12$
  $c = 12 \div 2$
  $= 6$

3. In the equation $e = 3f + 4$, we can rewrite the equation to express f in terms of e, i.e. making f the subject of the equation.

> *Examples:* (a) Given that $e = 3f + 4$, express f in terms of e.
>
> Rewriting the equation,
> $3f + 4 = e$
> $3f = e - 4$
> $f = \frac{e - 4}{3}$

(b) Given that $2y + 3x = 8$, express x in terms of y.

$$2y + 3x = 8$$
$$3x = 8 - 2y$$
$$x = \frac{8 - 2y}{3}$$

Practice 2.1

Basic

1. Given that $y = x + 3$,

   (a) express x in terms of y,

   (b) find the value of x when $y = 5$.

2. Given that $2y = x - 6$,

   (a) express y in terms of x,

   (b) express x in terms of y,

   (c) find the value of y when $x = 8$.

3. Given that $\frac{1}{2}y = x + 5$,

   (a) express y in terms of x,

   (b) find the value of y when $x = -4$,

   (c) find the value of x when $y = 9$.

4. Given that $2.5x + 0.6y = 6.8$, find

   (a) the value of x when $y = 3$,

   (b) the value of y when $x = 4$.

5. Given that $3x + 4y = 20$, find

   (a) the value of x when $y = \frac{1}{2}x$,

   (b) the value of y when $x = 2y$.

Advanced

6. Given that $\frac{4}{7}x - \frac{2}{3}y = 2$,

   (a) rewrite the equation using only whole numbers,

   (b) express y in terms of x,

   (c) find the value of x when $y = 1\frac{1}{2}x$.

7. Given that $0.25x + 2.5y = 1.2$,

   (a) express y in terms of x,

   (b) find the value of $\frac{x}{y}$ when $y = 2$,

   (c) find the value of $2xy$ when $x = 4$.

2.2 Solving Simultaneous Equations

1. Consider two equations $x + y = 8$ and $x - y = 2$. The values of x and y which satisfy both equations at the same time are when $x = 5$ and $y = 3$. In this situation, $x = 5$ and $y = 3$ are the solutions to the two given equations. Equations which have a common solution are classified as **simultaneous equations**. Hence, $x + y = 8$ and $x - y = 2$ are examples of simultaneous equations.

2. Two methods of solving simultaneous linear equations involving two variables are
   (a) substitution method,
   (b) elimination method.

3. The **substitution method** involves selecting one equation and then expressing one variable in terms of the other variable before substituting it into the second equation.

Examples: (a) Solve the simultaneous equations.

$x + y = 17$ .... (1)
$2x - y = 13$ .... (2)

From (1), $x = 17 - y$ — Express x in terms of y.

Substituting x in terms of y into (2),

$$2(17 - y) - y = 13$$
$$34 - 2y - y = 13$$
$$-3y = 13 - 34$$
$$= -21$$
$$y = -21 \div (-3)$$
$$= 7$$

Substituting $y = 7$ into $x = 17 - y$,

$$x = 17 - 7$$
$$= 10$$

The solutions are $x = 10$ and $y = 7$.

(b) Solve the simultaneous equations.

$3x + 2y = 17$ .... (1)

$2x + 5y = 26$ .... (2)

From (1), $y = \frac{17 - 3x}{2}$

Express y in terms of x.

Substituting y in terms of x into (2),

$2x + \frac{5(17 - 3x)}{2} = 26$

Multiply throughout by 2.

$4x + 85 - 15x = 52$

$-11x = 52 - 85$

$= -33$

$x = -33 \div (-11)$

$= 3$

Substituting $x = 3$ into $y = \frac{17 - 3x}{2}$,

$y = \frac{17 - 3(3)}{2}$

$= 4$

The solutions are $x = 3$ and $y = 4$.

4. In the **elimination method**, the process involves removing one of the variables either by addition or subtraction.

Examples: (a) Solve the simultaneous equations.

$2x + y = 16$ .... (1)

$x - y = 11$ .... (2)

(1) – (2),

Subtract to eliminate y.

$(2x + y) - (x + y) = 16 - 11$

$2x + y - x - y = 5$

$x = 5$

Substituting $x = 5$ into (2),

$5 + y = 11$

$y = 11 - 5$

$= 6$

The solutions are $x = 5$ and $y = 6$.

(b) Solve the simultaneous equations.

$a - 2b = 3$ .... (1)

$3a + b = 72$ .... (2)

(2) × 2,

$6a + 2b = 144$ .... (3)

(1) + (3),

Add to eliminate b.

$(a - 2b) + (6a + 2b) = 3 + 144$

$a - 2b + 6a + 2b = 147$

$7a = 147$

$a = 147 \div 7$

$= 21$

Substituting $a = 21$ into (2),
$3(21) + b = 72$
$b = 72 - 63$
$= 9$
The solutions are $a = 21$ and $b = 9$.

(c) Solve the simultaneous equations.
$\frac{1}{3}x - y = 2$ .... (1)
$x + \frac{1}{2}y = 13$ .... (2)

(1) × 3,
$x - 3y = 6$ .... (3)
(2) – (3),
$\left(x + \frac{1}{2}y\right) - (x - 3y) = 13 - 6$
$x + \frac{1}{2}y - x + 3y = 7$
$3\frac{1}{2}y = 7$
$y = 7 \div 3\frac{1}{2}$
$= 2$
Substituting $y = 2$ into (3),
$x - 3(2) = 6$
$x = 6 + 6$
$= 12$
The solutions are $x = 12$ and $y = 2$.

Subtract to eliminate x.

Practice 2.2

Basic

1. Solve the following pairs of simultaneous equations using the substitution method.

(a) $a + b = 9$
$2a - b = 6$

(b) $5a + b = 9$
$2a - b = 16$

(c) $7a - b = 2$
$6a - b = 0$

(d) $12a - 8b = 4$
$10b = 3 - a$

2. Solve the following pairs of simultaneous equations using the elimination method.

(a) $x + y = 13$
$x - y = 1$

(b) $3x + y = 11$
$x + y = 7$

(c) $3x + 5y = 21$
$x + 2y = 7$

(d) $2x + 5y = 8$
$3x + 4y = 5$

3. Use any method to solve the following pairs of simultaneous equations.

(a) $0.2a + 0.2b = 1$
$0.3a + 0.5b = 2.1$

(b) $c = 2d + 1$
$3c = 5(d + 1)$

(c) $2e + 5f = 1\frac{1}{2}$
$9e - 7f = \frac{17}{20}$

(d) $\frac{m}{3} + \frac{n}{2} = 6$
$\frac{m}{4} + \frac{n}{3} = 4\frac{1}{4}$

(e) $\frac{g+1}{2} = 8 - \frac{h-1}{3}$
$\frac{g-1}{3} = 9 - \frac{h+1}{2}$

(f) $3p - 1 = 10p + q = 1.5p$

Advanced

4. Find the values of a and b, given that $x = 2$ and $y = -3$ are the solutions to the simultaneous equations $ax + by = 1$ and $bx + ay = -9$.

2.3 Solving Simultaneous Equations by Graphical Method

1. Recall the linear graph representing an equation of the form $y = mx + c$, where the straight line graph cuts the y-axis at the point $(0, c)$ and m is the gradient. It is possible to solve two simultaneous equations graphically. The ordered pair at the point of intersection of the graphs representing the equations offers the solution to the pair of simultaneous equations.

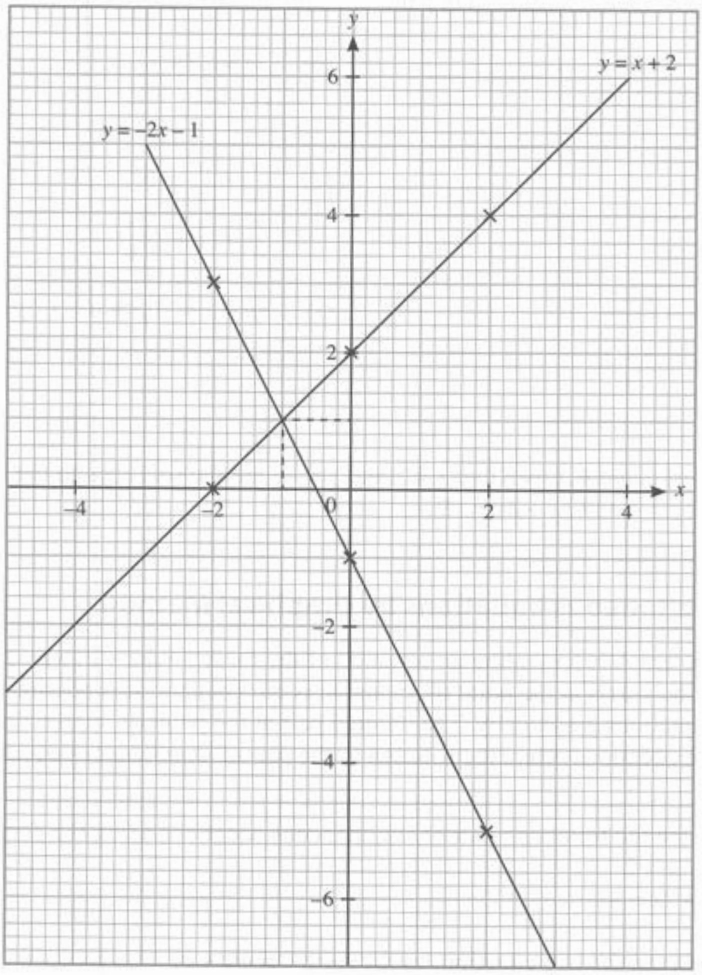
Example: Solve the following pair of simultaneous equations using the graphical method.
$y - x = 2$
$y + 2x = -1$

Rewrite the two equations in the form $y = mx + c$.
The equations become $y = x + 2$ and $y = -2x - 1$.
Draw up the table of ordered pairs for each equation using 3 suitable values of x.

x	-2	0	2
$y = x + 2$	0	2	4

x	-2	0	2
$y = -2x - 1$	3	-1	-5

Draw the graphs of $y = x + 2$ and $y = -2x - 1$.



The two lines intersect at the point (–1, 1).
Hence, the solutions to the simultaneous equations are $x = -1$ and $y = 1$.

Practice 2.3

Basic

1. (a) Complete the following tables.

$y = -2x + 1$

x	–2	0	2
y			

$y = x + 4$

x	–2	0	2
y			

(b) On the same axes, draw the graphs of $y = -2x + 1$ and $y = x + 4$ using a scale of 2 cm to represent 2 units on the x-axis and 1 cm to represent 1 unit on the y-axis. Hence, find the solutions to the pair of simultaneous equations.

2. Solve the following pair of simultaneous equations using the graphical method for values of x from -4 to 4. Use a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis.

   $y = x - 3$ and $y = -x + 1$

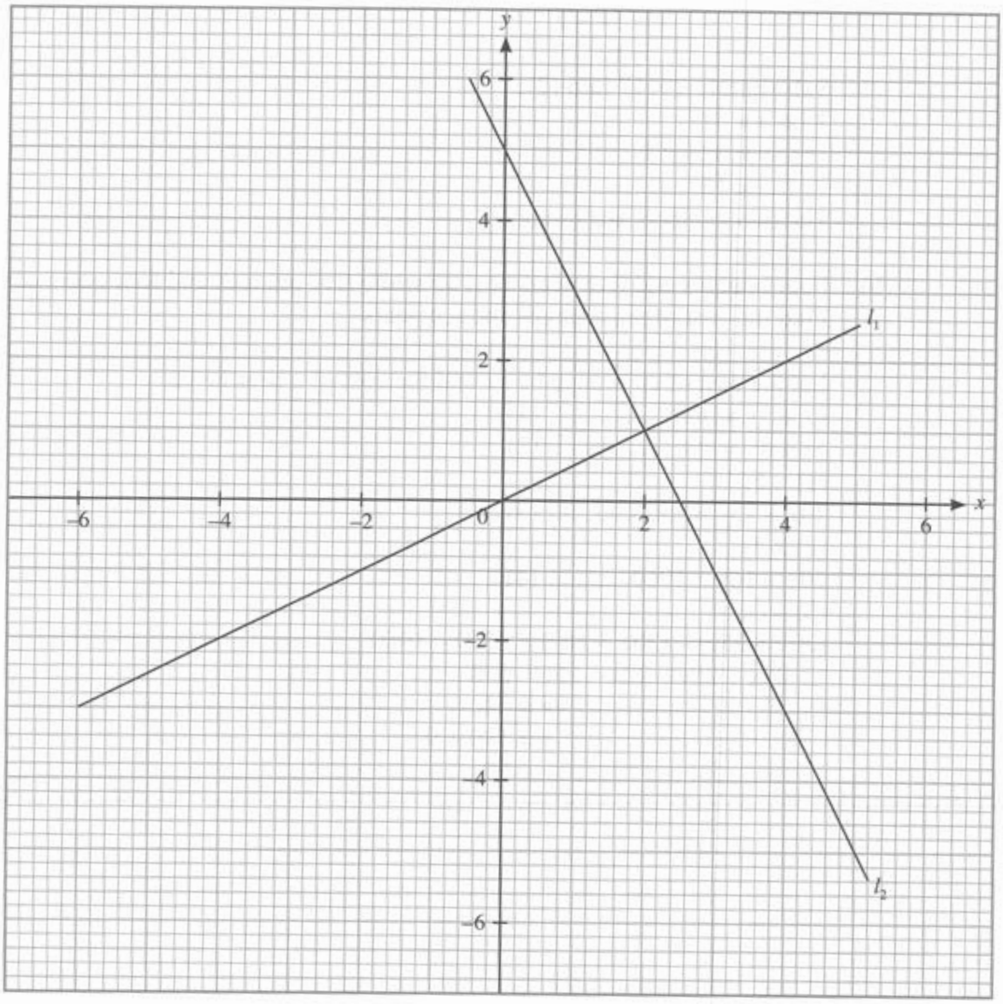
3. Solve the following pair of simultaneous equations using the graphical method for values of x from -4 to 4. Use a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis.

   $6x - 2y = 0$ and $y + x = 4$

4. On the same axes, draw the graphs of $2a + b = 3$ and $a - 2b = 4$ for $-3 \leq a \leq 3$. Hence, find the solution to the pair of simultaneous equations. Use a scale of 1 cm to represent 1 unit on both axes. (Hint: Make the a-axis the horizontal axis.)

Advanced

5. The graphs of two linear equations l_1 and l_2 are shown. Use the graphical information to determine the equations of the lines.



2.4 Solving Problems Using Simultaneous Equations

The following steps may be taken to solve a problem which involves the use of simultaneous equations.

- Study the question carefully to identify the unknowns.
- Identify the constants, if any.
- Define what each variable represents and take note of the unit used.
- Write expressions for the unknowns in terms of the variable.
- Form simultaneous equations to relate the facts of the problem.
- Solve the simultaneous equations.
- Check your answers by substituting the values representing the variables in the problem.

Examples: (a) The difference between two numbers is 2 and their sum is 32. What are the two numbers?

Let the larger number be x and the smaller number be y.

$x - y = 2$ .... (1)

$x + y = 32$ .... (2)

(1) + (2),

$(x - y) + (x + y) = 2 + 32$

$2x = 34$

$x = 34 \div 2$

$= 17$

Substituting $x = 17$ into (2),

$17 + y = 32$

$y = 32 - 17$

$= 15$

The two numbers are 15 and 17.

(b) Four oranges and five pears cost \$5 while five oranges and four pears cost \$4.90. Find the cost of ten oranges and ten pears.

Let the cost of an orange be x¢ and the cost of a pear be y¢.

$4x + 5y = 500$ .... (1)

$5x + 4y = 490$ .... (2)

(1) × 5,

$20x + 25y = 2500$ .... (3)

(2) × 4,

$20x + 16y = 1960$ .... (4)

(3) – (4),

$(20x + 25y) - (20x + 16y) = 2500 - 1960$

$20x + 25y - 20x - 16y = 540$

$9y = 540$

$y = 540 \div 9$

$= 60$

Substituting $y = 60$ into (1),

$$4x + 5(60) = 500$$
$$4x = 500 - 300$$
$$= 200$$
$$x = 200 \div 4$$
$$= 50$$
$$10x + 10y = 10(50) + 10(60)$$
$$= 500 + 600$$
$$= 1100$$

1100¢ = \$11

The cost of ten oranges and ten pears is \$11.

Practice 2.4

Basic

1. The cost of 3 kg of Grade A coffee and 2 kg of Grade B coffee is \$91. The cost of 6 kg of Grade A coffee and 3 kg of Grade B coffee is \$168. Calculate the cost per kilogram of each grade of coffee.

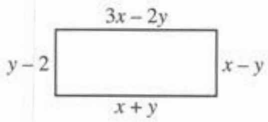
2. The sum of two numbers is 45 and their difference is 1. Find the product of the two numbers.

3. John is three years older than his sister, Jane. In three years' time, Jane's age will be $\frac{3}{4}$ of John's age. Find their present ages.

4. The denominator of a fraction is 4 more than its numerator. If 2 is subtracted from both the numerator and the denominator, the new fraction becomes $\frac{1}{5}$. Find the original fraction.

5. The sum of two numbers is 11. Twice the larger number added to the smaller number is 17. Find the larger number.

6. The figure shows a rectangle. Dimensions are given in cm.



   (a) Calculate its length and breadth.

   (b) Hence, calculate its perimeter.

Advanced

7. The difference between a two-digit number and the number formed by reversing the digits is 27. The sum of thrice the tens' digit and twice the ones' digit is 19. Find the number.

8. A sum of money is shared by Amy and Alice. Amy received \$400 more than Alice. If Alice's share is $\frac{1}{3}$ of Amy's share, find the original sum of money.