

CHAPTER

3

Approximation and Estimation

3.1 Approximation

1. To round off a number to a required degree of accuracy is to make an **approximation**.

Examples: Round off 47 268 to the nearest

- (a) 10,
 (b) 100,
 (c) 1 000,
 (d) 10 000.

- (a) $47\,268 = 47\,270$ (correct to nearest 10)
 (b) $47\,268 = 47\,300$ (correct to nearest 100)
 (c) $47\,268 = 47\,000$ (correct to nearest 1 000)
 (d) $47\,268 = 50\,000$ (correct to the nearest 10 000)

Method:

Identify the digit with one place value lower than the required accuracy. If the digit is *less than 5*, change it to 0. If it is *5 or more*, add 1 to the digit denoting the required accuracy.

2. To round off a decimal, use the same procedure for rounding off whole numbers.

Examples: Round off 4.890 45 to

- (a) the nearest whole number,
 (b) 1 decimal place,
 (c) 2 decimal places,
 (d) 3 decimal places,
 (e) 4 decimal places.

- (a) $4.890\,45 = 5$ (correct to the nearest whole number)
 (b) $4.890\,45 = 4.9$ (correct to 1 decimal place)
 (c) $4.890\,45 = 4.89$ (correct to 2 decimal places)
 (d) $4.890\,45 = 4.890$ (correct to 3 decimal places)
 (e) $4.890\,45 = 4.890\,5$ (correct to 4 decimal places)

Practice 3.1

Basic

1. Express 189 765 correct to the nearest

- (a) 10, (b) 100,
 (c) 1 000, (d) 10 000.

2. Express 0.320 48 correct to

(a) 1 decimal place,	(b) 2 decimal places,
(c) 3 decimal places,	(d) 4 decimal places.

3. Express 145.602 correct to

(a) the nearest whole number,	(b) 1 decimal place,
(c) 2 decimal places,	(d) the nearest 10.

4. Express 0.008 03 correct to

(a) 1 decimal place,	(b) 2 decimal places,
(c) 3 decimal places,	(d) 4 decimal places.

5. Round off each of the following.

(a) 13.35 mm to the nearest mm	(b) 145.65 cm to the nearest cm
(c) 240.56 g to the nearest tenth g	(d) \$192.84 to the nearest 10 cents

Advanced

6. The mass of a gold ornament is 15 g when rounded off to the nearest gram. What is the lightest and heaviest possible mass of the ornament, correct to 2 decimal places?

7. The population of a certain city is 3 754 000 rounded off to the nearest 1 000. What is its possible population range?

3.2 Significant Figures

1. Another method of rounding off numbers is using **significant figures**. A reading is more accurate when it is stated to a greater number of significant figures.

2. The following rules apply when rounding off to significant figures.
 - Begin rounding off by considering the digit which is one place value lower than stated in the required accuracy. This procedure is similar to rounding off whole numbers and decimals.
 - All non-zero digits are significant.

Example: 2.345 has 4 significant figures.

- Zeroes between non-zero digits are significant.

Example: 0.160 4 has 4 significant figures.

- Zeroes before non-zero digits are not significant.

Example: 0.000 123 has 3 significant figures.

- Zeroes after the decimal point are significant if they are preceded by non-zero digits.

Example: 2.700 has 4 significant figures.

- Zeroes after non-zero digits in a whole number may or may not be significant depending on how the approximation is done.

Example: 38 400 may have 3, 4 or 5 significant figures.

3. Here are more examples on rounding off to the correct significant figure(s).

123.604 = 100 (correct to 1 significant figure)
 123.604 = 120 (correct to 2 significant figures)
 123.604 = 124 (correct to 3 significant figures)
 123.604 = 123.6 (correct to 4 significant figures)
 123.604 = 123.60 (correct to 5 significant figures)

804 030 = 800 000 (correct to 1 significant figure)
 804 030 = 800 000 (correct to 2 significant figures)
 804 030 = 804 000 (correct to 3 significant figures)
 804 030 = 804 000 (correct to 4 significant figures)
 804 030 = 804 030 (correct to 5 significant figures)

Practice 3.2

Basic

1. Round off the following numbers to the number of significant figures indicated within the brackets.

- | | | | |
|--------------|-----|---------------|-----|
| (a) 14 264 | (1) | (b) 41 624 | (2) |
| (c) 321 370 | (3) | (d) 1 123 701 | (4) |
| (e) 322.008 | (5) | (f) 105.008 | (4) |
| (g) 0.006 95 | (2) | (h) 0.034 595 | (4) |
| (i) 701.999 | (4) | (j) 0.000 567 | (1) |

2. Use a calculator to evaluate the following, correct to 3 significant figures.

(a) $\frac{18.94^2 \times \sqrt{234.9}}{\sqrt[3]{568.6}} = 663.6399256$ (b) $\sqrt{(8.89^2 + 6.28^2 - 2.56^3)}$

Advanced (correct to 3 sf) = 10.0843088
 (correct to 3 sf)

3. A number 456 000 is corrected to n th significant figures. What are the possible values of n ?

4. The value of π is given as 3.142 and $\frac{22}{7}$.

Find the difference in the area of a circle of radius 14 m when you take the different values for π .

3.3 Rounding Off Numbers

In our daily life experiences, there are many instances where rounding off numbers occur. Some of these result in rounding errors which are generally acceptable.

Here are some examples.

Example 1: Dealing with money while paying off bills at the local supermarkets often involves rounding off amounts. A total bill for purchasing some items may total up to \$134.79. With a 7% GST added to that amount, the final amount reads \$144.225 3.... However, the final bill on the payment shown is usually \$144.20. This is because the amount due has been rounded down to the nearest ten cents.

Example 2: When you go to a money changer, the exchange rate for the day differs at different times of the day. The money changer will always take an exchange rate that works to his advantage. The amount offered to the client is often a figure that has been rounded down. S\$100 may give an exchange rate of RMB475.39. It is rare to expect the money changer to give RMB0.39 to the client. The client will probably get RMB475, an amount rounded down to the nearest dollar.

Example 3: A population survey is taken over a period of time. At a given point of time, the figure publicised is usually rounded off. This figure does not take into consideration the births and deaths during and after the period of the survey. Very often, out of convenience, the number quoted is corrected to the nearest thousand.

Example 4: A calculator is pre-programmed to display a certain number of figures on its screen. Take the decimal value of $\frac{2}{3}$ on your calculator. The display is 0.666 666 666. Your calculator value is pre-programmed to give a **truncated** value depending on how many figures your calculator model allows for display. It does not display a rounded off number. It is inconvenient to write a string of digits as displayed on the calculator. Hence, there is a need to express answers correct to significant figures, depending on the accuracy needed.

*What
 are
 words*

Practice 3.3

Basic

- At a supermarket, the selling price for 100 g of tomatoes is listed as 27¢. A customer buys 2.455 kg of tomatoes. How much does the customer have to pay when the bill is rounded off to the nearest 10 cents?
- There were a total of 25 852 spectators at an outdoor concert performance.
 - Express the number of spectators correct to the nearest thousand.
 - Express the number of spectators correct to the nearest ten thousand.
 - Which figure is a better estimation of the number present at the concert? Suggest a reason for your answer.
- Mr Lee wanted to buy a car. A car dealer quoted him a price of about \$130 000. In the end, he paid an actual price of \$134 960.
 - What was the rounding off done to the amount Mr Lee paid for the car?
 - What should he be quoted if the price is rounded off to the nearest thousand dollars?
- A house agent listed the selling price of a bungalow at \$4.25 million.
 - Express the selling price without using a decimal number.
 - Express the selling price correct to the nearest hundred thousand dollars.
- Collin found that the mass of 5 000 similar ball bearings was 2.565 kg.
 - What is the mass of each ball bearing in grams correct to one significant figure?
 - What is the mass of 200 such ball bearings correct to the nearest gram?
- The exchange rate for Singapore dollar to Malaysian ringgit is S\$1 = RM2.653. Mr Lim wants to exchange Singapore dollars for RM1 000. How much Singapore dollars must Mr Lim have for this exchange? Give your answer correct to the nearest cent.

3.4 Estimation

- The symbol ' \approx ' represents 'is approximately equal to'. It is used in statements involving estimation.
- An estimation allows us to check on how reasonable or logical the accuracy of an answer is.

Examples: (a) Find the estimate of 39.8×4.05 .

$$39.8 \times 4.05 \approx 40 \times 4 = 160$$

(The exact answer is 161.19.)

(b) Find the estimate of $4.095 \div 0.508$.

$$4.095 \div 0.508 \approx 4 \div 0.5 = 8$$

(The answer, correct to 3 decimal places, is 8.061.)

(c) Estimate the value of $\sqrt{8.939}$.

$$\sqrt{8.939} \approx \sqrt{9} = 3$$

(The answer, correct to 3 decimal places, is 2.990.)

Practice 3.4

Basic

- Find the estimate for each of the following without using a calculator. (Do not work out the exact answer.)

(a) $7\,216 \div 81$	(b) 6.045×5.089
(c) $\frac{4}{5} \times 0.396 \times 0.99$	(d) $50.47 \div 0.492$
(e) $\sqrt{\left(\frac{3.94 \times 25.2}{1.05}\right)}$	(f) $2.945^2 - 1.822^2$
- Estimate the answers for each of the following. Then, use your calculator to evaluate the answers and correct them to the required accuracy given in brackets.

(a) $\left(18.7 + \frac{0.8}{3.6}\right) \times 2.1$	(2 decimal places)
(b) $\left(32.7 - \frac{3.8}{1.8}\right) \div 0.47$	(3 significant figures)
(c) $\frac{8.4^2 + 3.7^2}{2.1^2}$	(2 significant figures)
(d) $\sqrt{27.2} + \sqrt{50.1}$	(1 significant figure)
(e) $\frac{\sqrt[3]{28.4} \times 2.8}{3.09^2}$	(3 decimal places)

Advanced

3. A Singaporean tourist went for a day tour in Xiamen. The fare for the tour was RMB400. Without using a calculator, estimate the fare in Singapore currency given that the exchange rate is S\$1 = RMB4.72. (RMB is Renminbi for China currency.)
4. Mr Lim is a chef who earns a monthly salary of \$1 954.
 - (a) Without using a calculator, estimate his annual pay which includes a $2\frac{1}{2}$ -month bonus.
 - (b) Calculate his annual pay, correct to the nearest ten dollars.