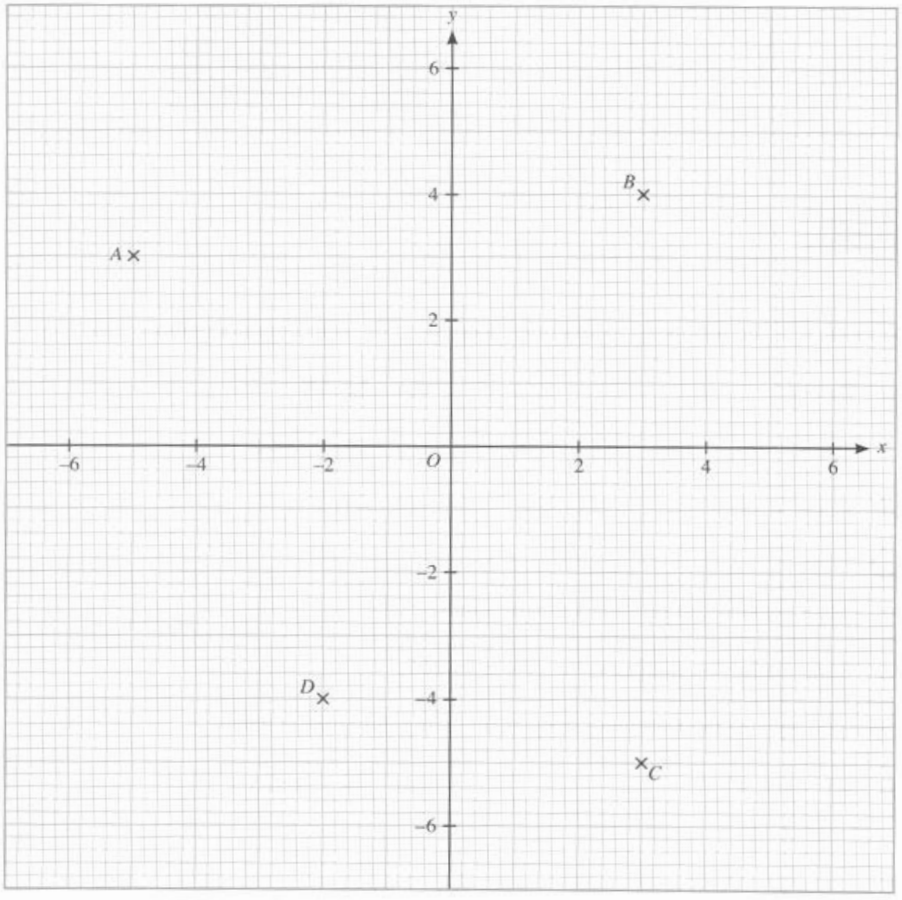
CHAPTER 6

Functions and Linear Graphs

6.1 Cartesian Coordinates

The figure shows a coordinate plane. The **horizontal axis** is known as the ***x*-axis** and the **vertical axis** is known as the ***y*-axis**. The intersection of the two axes forms the point O which is called the **origin**. All points on the coordinate plane are represented by **ordered pairs** and written in the form (x, y). Thus, O is represented by $(0, 0)$.



A is the point $(-5, 3)$. B is the point $(3, 4)$. C is the point $(3, -5)$. D is the point $(-2, -4)$. For each point, there is the ***x*-coordinate** and the ***y*-coordinate**. For point C, the x-coordinate is 3 and the y-coordinate is -5. The x and y values of each point are called the **Cartesian coordinates** of the point.

Practice 6.1

Basic

1. For each part of this question, plot the points on the coordinate plane using a graph paper. Join the points in alphabetical order with straight lines and identify the geometrical figure you have drawn.

   (a) $A(-2, -2), B(0, 2), C(2, -2)$

   (b) $D(4, 4), E(4, 0), F(0, 0), G(4, 0)$

   (c) $H(3, 2), I(6, 2), J(3, -2), K(0, -2)$

   (d) $L(-1, 6), M(-3, 4), N(-1, -1), P(1, 4)$

   (e) $Q(4, 4), R(6, 4), S(6, -1), T(4, -2)$

Advanced

2. $ABCD$ is a parallelogram. The coordinates of three of the vertices are $A(2, 2)$, $B(-4, 2)$ and $C(-3, -2)$. Mark the points on a graph paper. What are the coordinates of the point D? State the coordinates of the intersection of the two diagonals. State the coordinates of the mid point of CD. (Hint: You may want to use a scale of 1 cm to represent 1 unit on each axis.)

3. The vertices of $\triangle ABC$ are $A(4, 6)$, $B(2, 2)$ and $C(6, 2)$. Draw this triangle on a graph paper. Mark the mid points of AB, AC and BC and name these points X, Y and Z respectively. What are the coordinates of X, Y and Z? What type of triangle is $\triangle XYZ$?
   (Hint: You may want to use a scale of 1 cm to represent 1 unit on each axis.)

6.2 Functions

A function links two variables x and y such that each x value is matched with a y value. The linear equation $y = x + 1$ is a function as each value of x matches a value of y.

Below are examples of ordered pairs which satisfy the equation $y = x + 1$.

Examples: (a) When $x = 1, y = 1 + 1 = 2$. The ordered pair is $(1, 2)$.

(b) When $x = 0, y = 0 + 1 = 1$. The ordered pair is $(0, 1)$.

(c) When $x = -1, y = -1 + 1 = 0$. The ordered pair is $(-1, 0)$.

Practice 6.2

Basic

1. The equation of a function is $y = x + 3$. Find the value of y when

   (a) $x = 2$, (b) $x = -4$.

2. The equation of a function is $y = 2x + 5$. Find the value of y when

   (a) $x = 1$, (b) $x = -3$.

3. The equation of a function is $y = -3x - 2$. Find the value of y when

   (a) $x = 3$, (b) $x = -4$.

4. The equation of a function is $y = 3x + 5$. Find the value of x when

   (a) $y = 17$, (b) $y = -10$.

5. The equation of a function is $y = 7 - 2x$. Find the value of x when

   (a) $y = 9$, (b) $y = -11$.

Advanced

6. Given that the equation of a function is $y = \frac{2}{3}x - 4$, find the value of y when

   (a) $x = 6$, (b) $x = 2\frac{1}{2}$.

7. Given that the equation of a function is $y = 1\frac{3}{4}x + 1\frac{2}{3}$, find the value of x when

   (a) $y = 24$, (b) $y = -1\frac{1}{2}$.

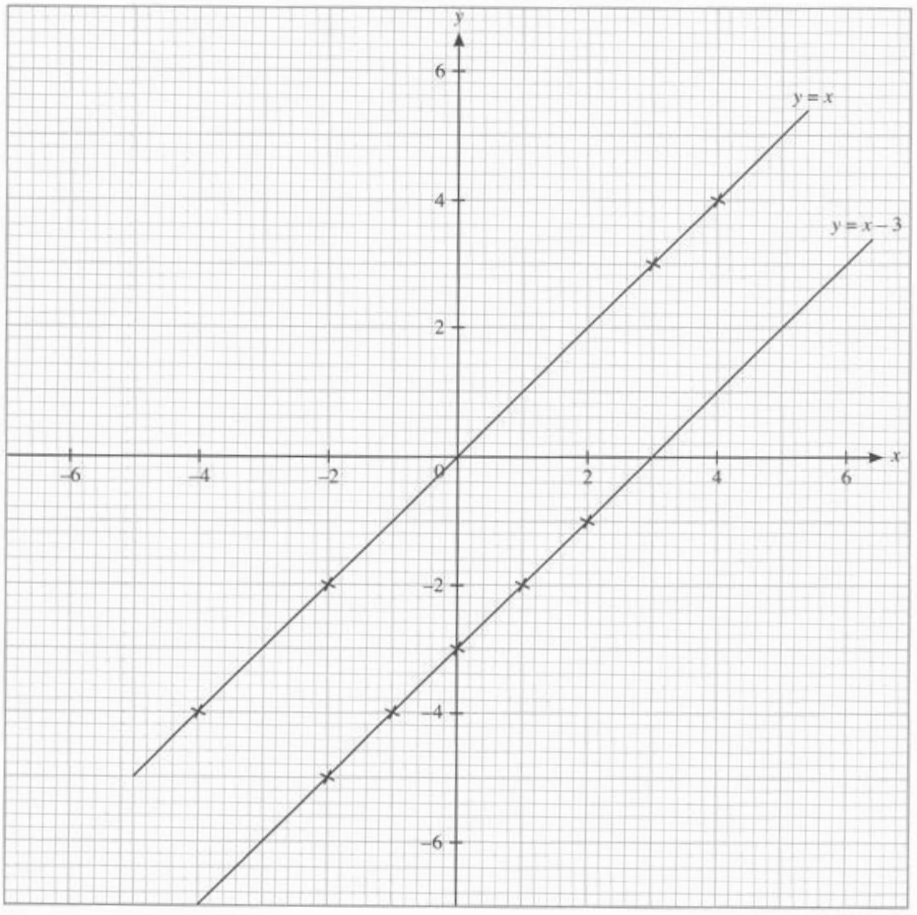
6.3 Linear Graphs

6.3.1 Graphs of Linear Functions

Consider the following points $(-4, -4)$, $(-2, -2)$, $(3, 3)$ and $(4, 4)$. For each point, the x-coordinate is equal to the y-coordinate. The equation that relates the values of x and y in the ordered pairs is $y = x$. When these points are represented on a coordinate plane, they lie along a straight line. The straight line which joins these points is a **linear graph** representing the linear equation $y = x$. The linear graph is shown below.

The table shows five ordered pairs which satisfy the equation $y = x - 3$. The linear graph of $y = x - 3$ is also shown below.

x	-2	-1	0	1	2
$y = x - 3$	-5	-4	-3	-2	-1



The graph of $y = x$ cuts the y-axis at the origin $(0, 0)$ while the graph of $y = x - 3$ cuts the y-axis at $(0, -3)$. The two lines are parallel to each other.

Practice 6.3.1

Basic

1. Draw a table of ordered pairs for each of the following equations. Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw on a graph paper, the graphs of the following functions for values of x from 0 to 4. What do you notice about the lines drawn?

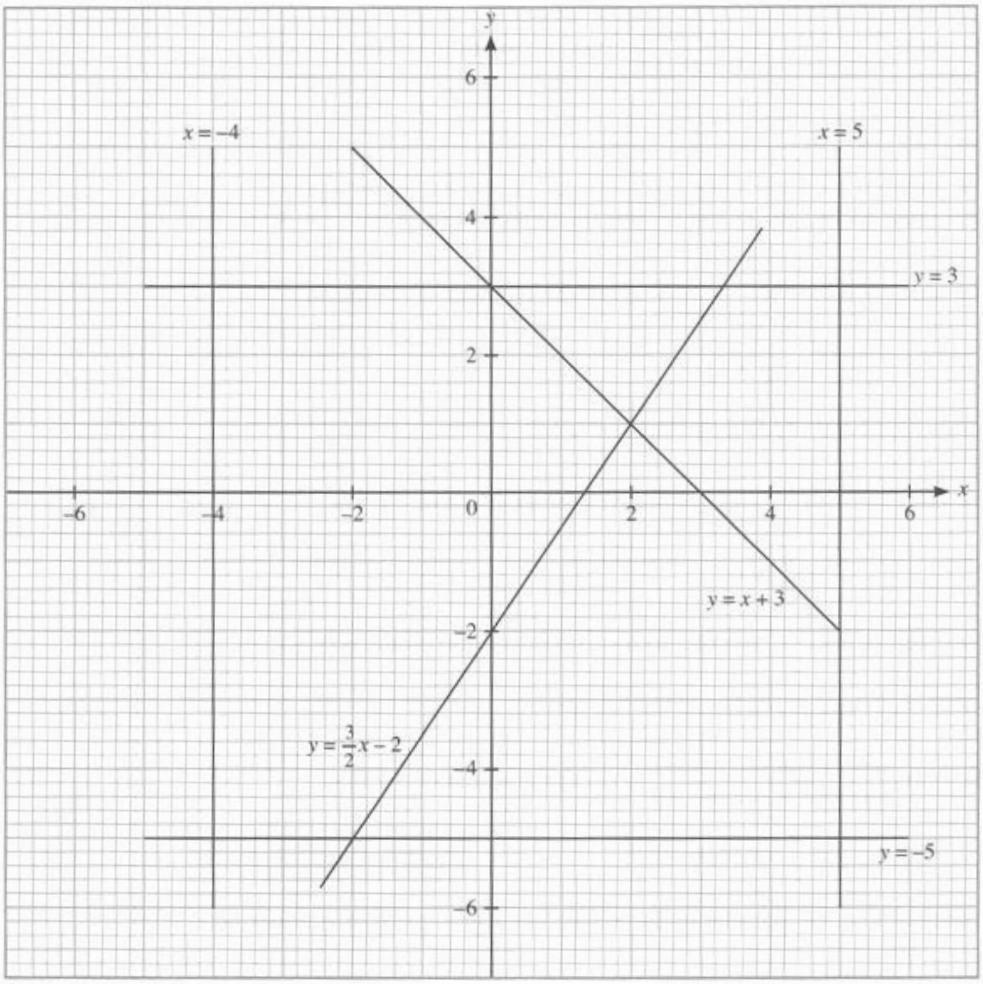
   (a) $y = 2x$ (b) $y = 2x + 1$ (c) $y = 2x + 2$

2. Draw a table of ordered pairs for each of the following equations. Using a scale of 1 cm to represent 1 unit on both axes, draw on a graph paper, the graphs of the following functions for values of x from -3 to 3. What do you notice about the lines drawn?

   (a) $y = -x$ (b) $y = -x + 3$ (c) $y = -x - 1$

6.3.2 Gradient of Linear Graphs

Study the various graphs drawn below.



1. The ***y*-axis** is represented by the equation $x = 0$ and the ***x*-axis** is represented by the equation $y = 0$. Lines that satisfy the equation $y = c$ are parallel to the x-axis and lines that satisfy the equation $x = c$ are parallel to the y-axis.

2. A linear graph in the form of $y = mx + c$ has the following characteristics.
   - It is a straight line graph which cuts the y-axis at the point $(0, c)$.
   - The value m is called the **gradient** of the line and the value c, the ***y*-intercept**.
   - When m is a positive value, the line slopes upwards from left to right.
   - When m is a negative value, the line slopes downwards from left to right.

3. The graph of $y = \frac{3}{2}x - 2$ has a gradient of $\frac{3}{2}$ and its y-intercept is -2.
These two points, $(-2, -5)$ and $(0, -2)$ lie on the graph of $y = \frac{3}{2}x - 2$.

The ratio $\frac{\textbf{difference in } y\textbf{-coordinates}}{\textbf{difference in } x\textbf{-coordinates}}$ gives the **gradient** of the graph.

i.e. $\frac{-5 - (-2)}{-2 - 0} = \frac{-3}{-2} = \frac{3}{2}$

4. The graph of $y = -x + 3$ has a gradient of -1 and its y-intercept is $+3$.
These two points, $(0, 3)$ and $(5, -2)$ lie on the graph of $y = -x + 3$.

The ratio $\frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}$ gives the gradient of the graph.

i.e. $\frac{3 - (-2)}{0 - 5} = \frac{5}{-5} = -1$

5. Points $(2, 3)$ and $(4, 3)$ lie on the line $y = 3$.

The ratio $\frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}$ gives the gradient of the line.

i.e. $\frac{3 - 3}{2 - 4} = \frac{0}{-2} = 0$

A horizontal line, i.e. parallel to the x-axis, has the gradient 0.

6. Points $(5, 3)$ and $(5, 0)$ lie on the line $x = 5$.

The ratio $\frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}$ gives the gradient of the line.

i.e. $\frac{3 - 0}{5 - 5} = \frac{3}{0}$ = undefined

Division by 0 is undefined.

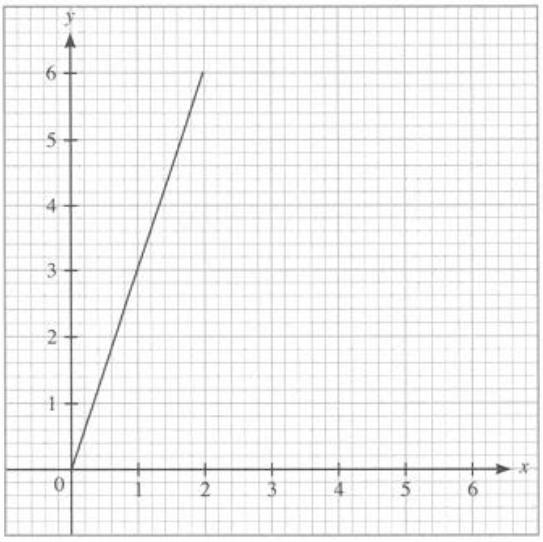
A vertical line, i.e. parallel to the y-axis, has a gradient that is undefined.

Practice 6.3.2

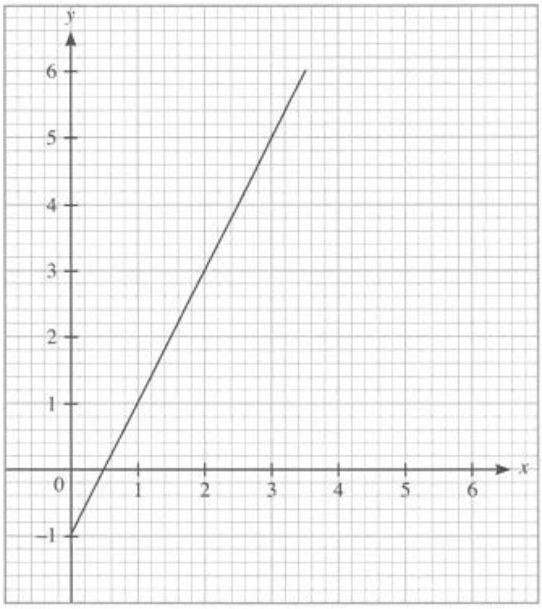
Basic

1. Given that each of the following linear graphs is in the form $y = mx + c$, find the gradient m, the y-intercept c, and the equation of each of the graphs.

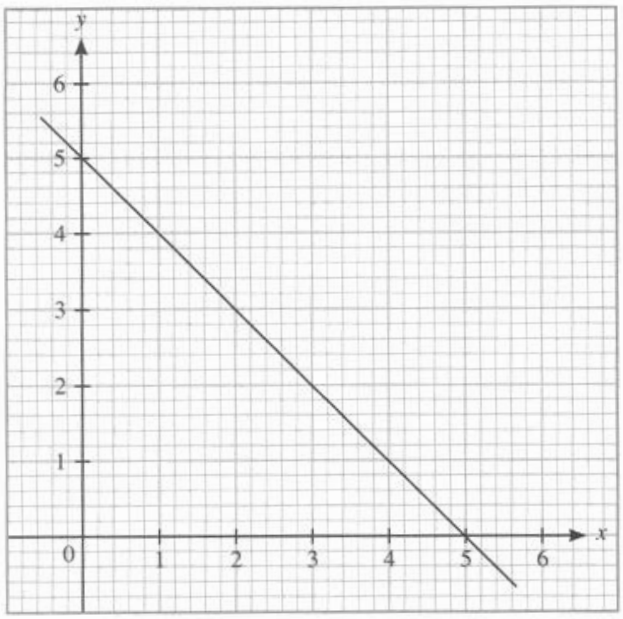
(a)



(b)



(c)



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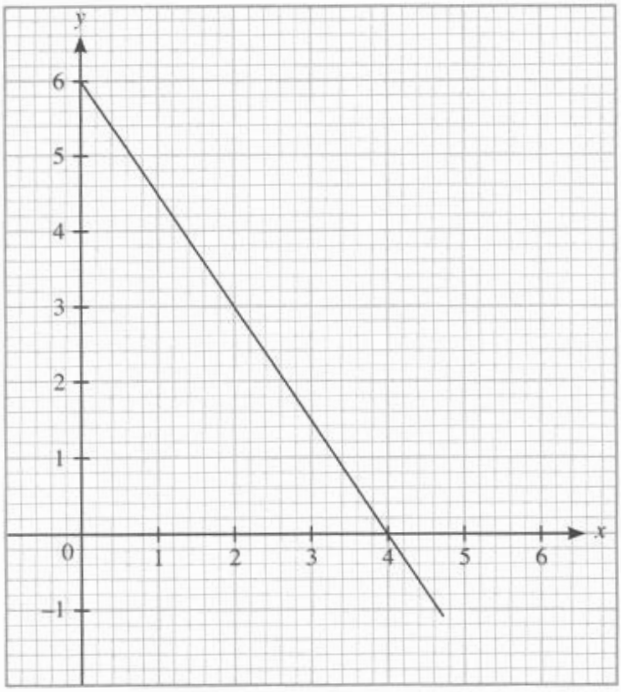
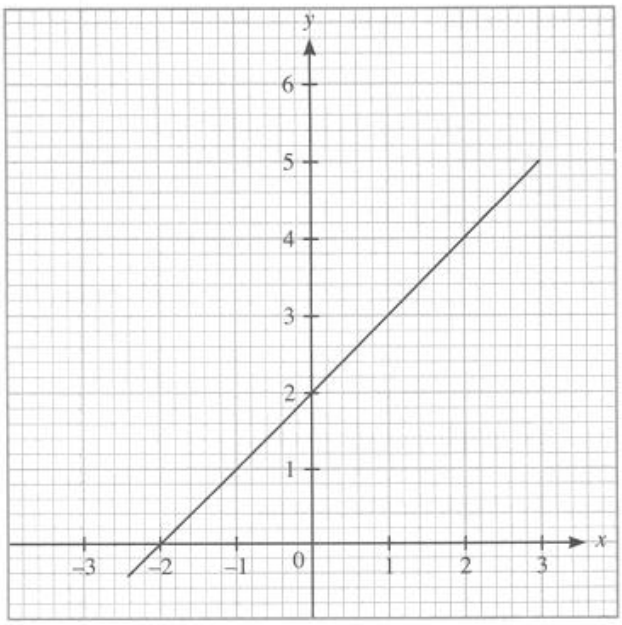
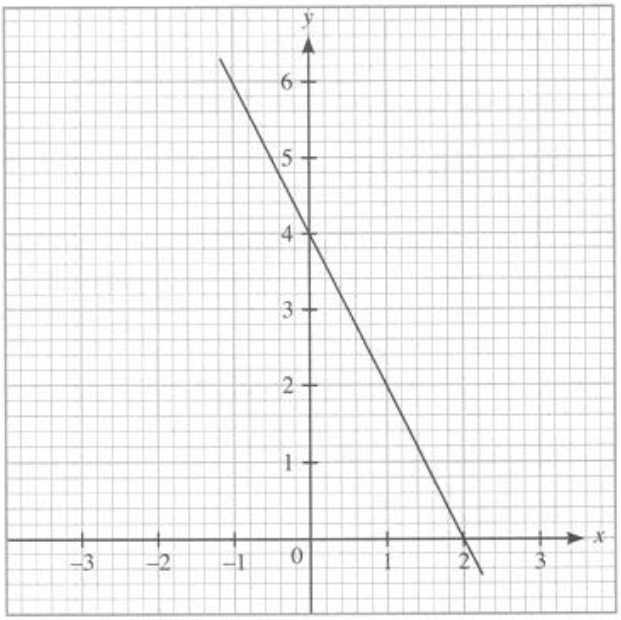


(e)



(f)



2. Each of the following linear equations represents a linear graph. Without drawing the graph, state the gradient and y-intercept of each line.

   (a) $y = 3x + 7$

   (b) $y = -4x - 3$

   (c) $y = 2\frac{3}{4}x + 1\frac{1}{2}$

   (d) $y = -1\frac{1}{2}x - 5$

3. Point $(2, q)$ lies on the line $y = \frac{1}{3}x - 2$. Without drawing the line on a graph paper, find

   (a) the value of q,

   (b) the coordinates of the point where the graph cuts the y-axis,

   (c) the coordinates of the point where the graph cuts the x-axis.

Advanced

4. The equation of a function is $2y = -3x + 5$. Without drawing the graph,

   (a) find the gradient of the line representing $2y = -3x + 5$,

   (b) find the coordinates of the point where the line $2y = -3x + 5$ cuts the y-axis,

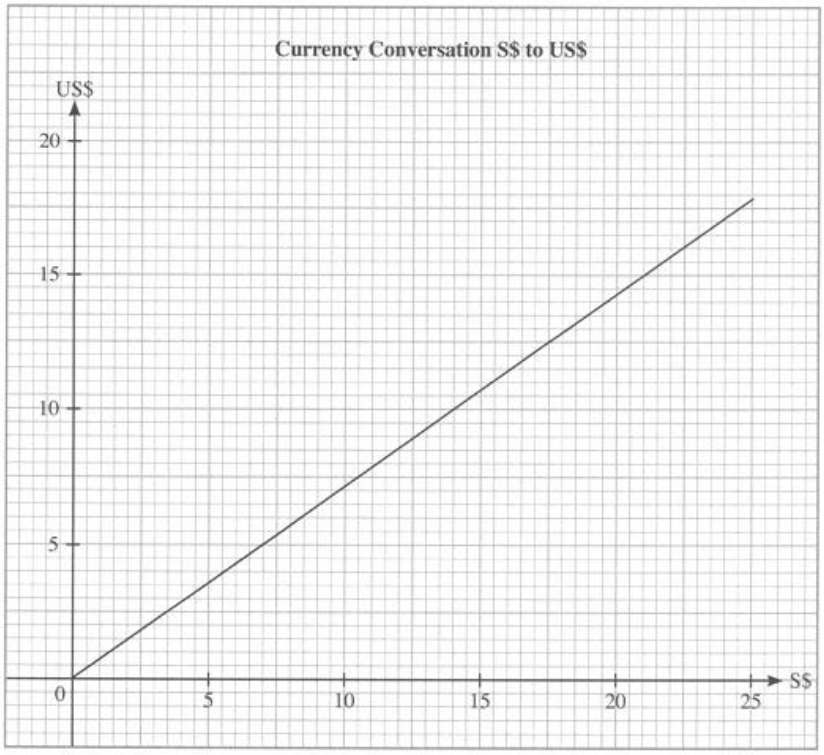
   (c) show that the point $(-3, 7)$ lies on the line $2y = -3x + 5$.

5. The graphs of $2y + 3x = 6$ and $y = 6$ intersect at point A. The graph of $2y + 3x = 6$ cuts the x-axis and the y-axis at points B and C respectively. What are the coordinates of points A, B and C?

6.4 Applying Linear Graphs

Currency conversions, tracking moving vehicles, temperature conversions, population growths and performance tests are some examples of everyday situations which involve the use of linear graphs.

Examples: (a) The graph shows the conversion from Singapore dollars (S$) to US$ on a certain day.

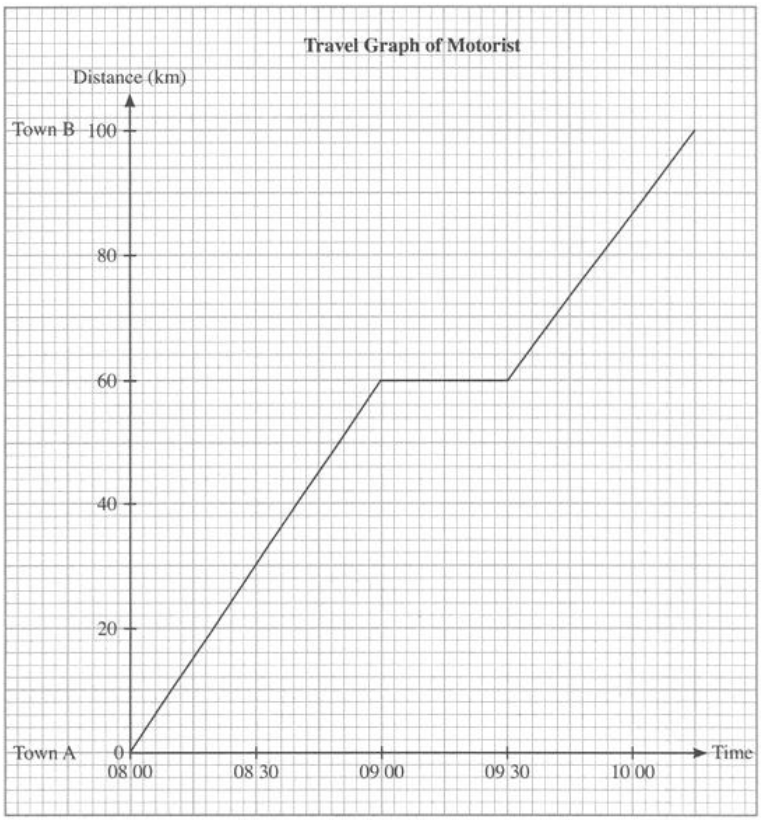


Using the graph, we can determine the exchange rate for the day.

In this instance, US$10 = S$14. The exchange rate was US$1 = S$1.40.

From the graph, US$17 = S$23.80,
S$16.80 = US$12.

(b) A motorist left Town A for Town B, 100 km away. He started at 08 00. After travelling for an hour at an average speed of 60 km/h, he stopped to rest for 30 min. He then continued his journey to Town B at an average speed of 50 km/h. The travel graph for the journey is shown below.



From the graph, we can tell that he had travelled 60 km before resting from 09 00 to 09 30 hours.

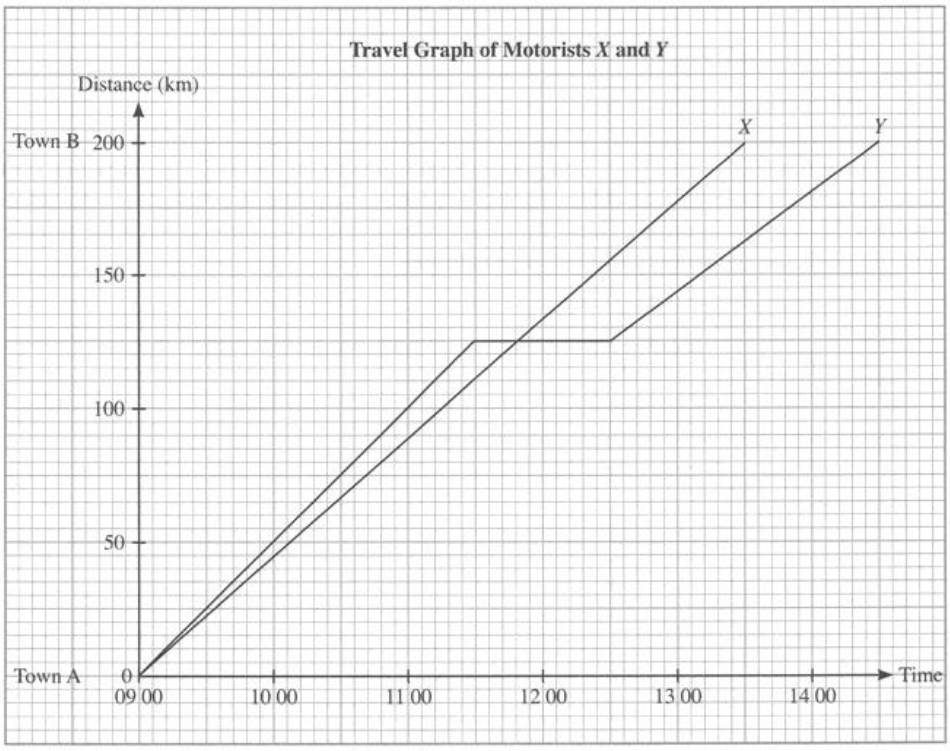
At 10 00 hours, he was still 15 km away from his destination.

He arrived at Town B before 10 30 hours.

Practice 6.4

Basic

1. The travel graphs below show the journeys of two motorists, X and Y, both leaving Town A at 09 00 hours and arriving at Town B at different times. Town A and Town B are 200 km apart.



Study the graphs carefully and answer the following questions.

(a) Motorist X travelled without taking any rest. How much earlier did he arrive at Town B compared to Motorist Y?

(b) What was the average speed of Motorist X for the whole journey?

(c) What was the average speed of Motorist Y before taking a rest at 11 30 hours?

(d) How far was Motorist X away from Town B when he passed Motorist Y?

2. A mathematics test paper has a maximum score of 150 marks.
Draw a graph to convert the scores out of 150 to percentages. Use a scale of 1 cm to represent 10 marks on the vertical axis and 1 cm to represent 10% on the horizontal axis. From your graph, find the following.

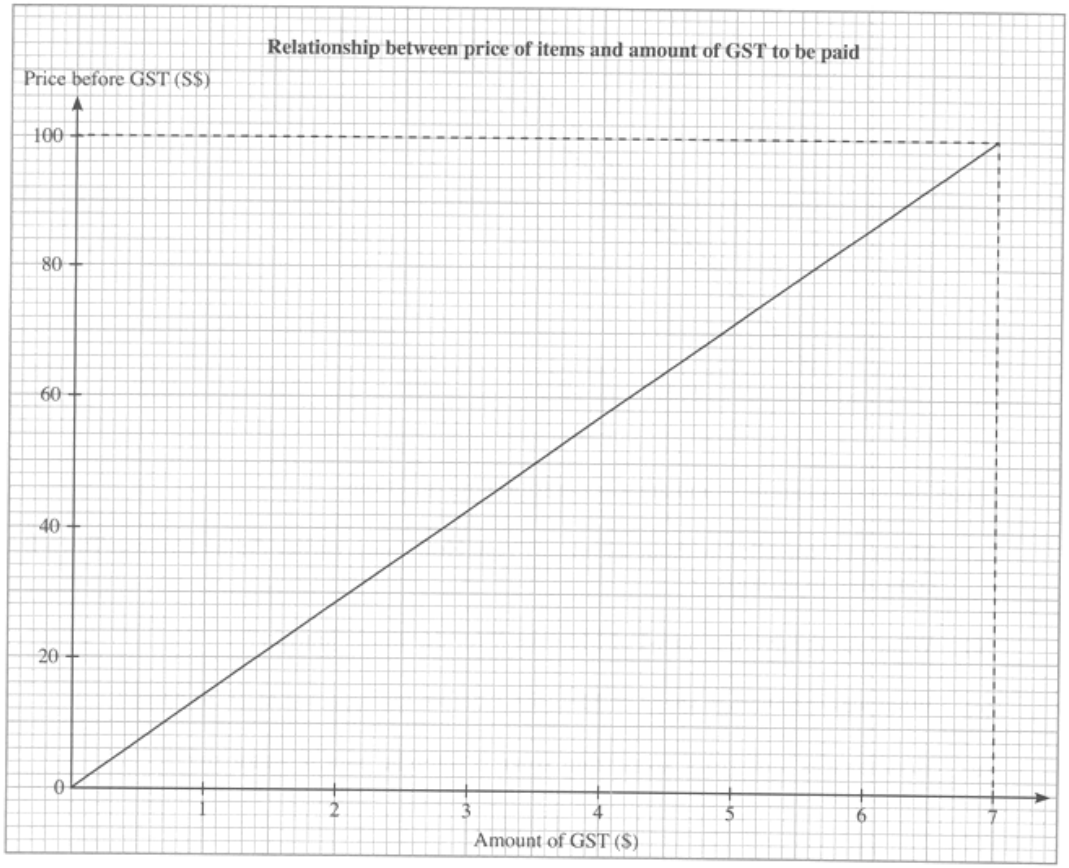
   (a) The percentages that correspond to 42, 60 and 132 marks

   (b) The marks that correspond to 30%, 62% and 96%

   (c) The minimum marks a student has to score to get 90%

   (d) The marks a student must obtain if the passing grade is 60%

3. Singapore has a GST rate of 7% on goods. Sam wants to buy a DVD player which costs $80 before GST. In order to find the amount of GST he is required to pay for the various items he bought, he decides to draw a linear graph showing the relationship between the price of items before GST and the amount of GST he has to pay.



Study the graph carefully and answer the following questions.

   (a) Determine the price of the DVD player inclusive of GST.

   (b) What is the total price he must pay for a thumb drive if it costs $45 before GST?

   (c) In the end, Sam bought several items and the total price before GST was $267.50. What was the amount of GST he paid?