6 Number Patterns

Key Notes

A **number sequence** is a series of numbers which follow a specific rule. The numbers in the sequence are called **terms**.

6.1 General Term of a Number Sequence

We can represent the terms of a sequence by:

T_1: 1st term

T_2: 2nd term

T_3: 3rd term

⋮

$\boldsymbol{T_n}$**: *n*th term**

When the terms in a sequence follow a particular rule, the general term in the sequence, T_n, can be represented by a formula.

For example,

In the sequence 3, 5, 7, 9, …

> **Note:**
> We observe that every term is 2 more than its previous term. That is the rule of the sequence.

$$T_1 = 3 + 2 \times 0 = 3$$

$$T_2 = 3 + 2 \times 1 = 5$$

$$T_3 = 3 + 2 \times 2 = 7$$

$$T_4 = 3 + 2 \times 3 = 9$$

⋮

$$\begin{aligned} T_n &= 3 + 2 \times (n - 1) \\ &= 3 + 2n - 2 \\ &= 2n + 1 \end{aligned}$$

6.2 Square and Cube Numbers

Some common number sequences are square numbers and cube numbers.

For example,

64, 81, 100, 121, 144, …
This sequence is made up of square numbers.

> **Note:**
> $T_1 = 64 = 8^2$, $T_2 = 81 = 9^2$, $T_3 = 100 = 10^2$, $T_4 = 121 = 11^2$, $T_5 = 144 = 12^2$

125, 216, 343, 512, 729, …
This sequence is made up of cube numbers.

> **Note:**
> $T_1 = 125 = 5^3$, $T_2 = 216 = 6^3$, $T_3 = 343 = 7^3$, $T_4 = 512 = 8^3$, $T_5 = 729 = 9^3$

6.3 Two-Layer Patterns

When the terms in a sequence do not have a constant difference, we can consider the difference between the differences of the terms instead.

For example,
In the sequence 5, 7, 11, 17, 25, …

$T_1 = 5 = 1(1)^2 + (-1)(1) + 5$
$T_2 = 7 = 1(2)^2 + (-1)(2) + 5$
$T_3 = 11 = 1(3)^2 + (-1)(3) + 5$
$T_4 = 17 = 1(4)^2 + (-1)(4) + 5$
$T_5 = 25 = 1(5)^2 + (-1)(5) + 5$

$\therefore$ General term, $T_n = 1(n)^2 + (-1)(n) + 5$
$= n^2 - n + 5$

> **Note:**
> 5, 7, 11, 17, 25, …
> +2 +4 +6 +8
> +2 +2 +2

> **Note:**
> Try to find a formula where there is an increasing n.