

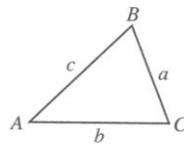
CHAPTER

7

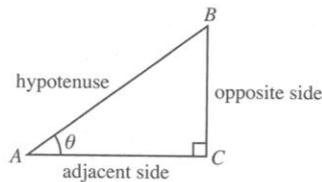
Trigonometric Ratios

7.1 Trigonometric Ratios of Acute Angles

1. Trigonometry is a branch of Mathematics which deals with the relationship between the sides and angles of a triangle.
2. Conventionally, the notations used to refer to the angles of $\triangle ABC$ are $\angle A$, $\angle B$ and $\angle C$ and the sides facing these angles are denoted by a , b and c respectively.



3. If $\triangle ABC$ is right-angled at C , the side facing $\angle C$ is known as the **hypotenuse**. With reference to the angle marked θ , BC is the **opposite side** and AC is the **adjacent side**.

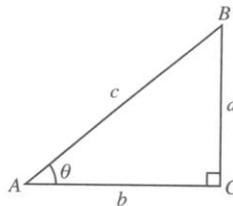


The ratio $\frac{BC}{AC}$ or $\frac{\text{opposite side}}{\text{adjacent side}}$ is called the **tangent** of $\angle A$.

The ratio $\frac{BC}{AB}$ or $\frac{\text{opposite side}}{\text{hypotenuse}}$ is called the **sine** of $\angle A$.

The ratio $\frac{AC}{AB}$ or $\frac{\text{adjacent side}}{\text{hypotenuse}}$ is called the **cosine** of $\angle A$.

4. The abbreviations for tangent, sine and cosine are **tan**, **sin** and **cos** respectively.



With reference to angle θ ,

$$\tan \theta = \frac{a}{b}$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

5. Before using a calculator, ensure that it is set to 'degree' mode. Your calculator is programmed to display the trigonometrical ratio of an angle in decimal form.

Examples: (a) $\tan 30^\circ = 0.5774$

(b) $\sin 55^\circ = 0.8192$

(c) $\cos 72^\circ = 0.3090$

To find the angle which gives the trigonometrical ratio, we need to use the inverse trigonometrical function keys (\tan^{-1} , \sin^{-1} or \cos^{-1}). Unless otherwise stated, angles are given in degrees and corrected to 1 decimal place.

Examples: (a) $\tan \theta = 1.213$

$$\theta = \tan^{-1} 1.213$$

$$= 50.5^\circ$$

(b) $\sin \theta = 0.3546$

$$\theta = \sin^{-1} 0.3546$$

$$= 20.8^\circ$$

(c) $\cos \theta = 0.8990$

$$\theta = \cos^{-1} 0.8990$$

$$= 26.0^\circ$$

(d) $\cos \theta = 0.4385$

$$\theta = \cos^{-1} 0.4385$$

$$= 64.0^\circ$$

6. For problems involving the sides or the angles of a triangle, drawing a sketch and labelling it with the information given often helps in solving the unknown.

Example: The sides AB and AC of $\triangle ABC$ are equal.

- (a) Calculate the size of the included angle, $\angle BAC$, if $AB = 10$ cm and $BC = 15$ cm.

- (b) What is the shortest length from A to BC ?

- (a) Draw the perpendicular line from A to N on BC . The perpendicular line from A to BC bisects BC and $\angle BAC$.

$$\sin x = \frac{7\frac{1}{2}}{10}$$

$$= 0.75$$

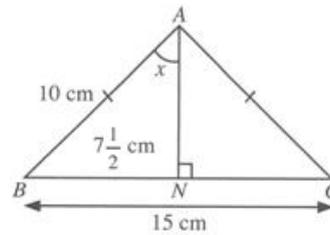
$$x = \sin^{-1} 0.75$$

$$= 48.6^\circ$$

$$\angle BAC = 2x$$

$$= 2(48.6^\circ)$$

$$= 97.2^\circ$$



- (b) AN is the shortest length from A to BC .

$$\tan x = \frac{7\frac{1}{2}}{AN}$$

$$AN = \frac{7\frac{1}{2}}{\tan 48.6^\circ}$$

$$= 6.61 \text{ cm (2 dec. plc.)}$$

Practice 7.1

Basic

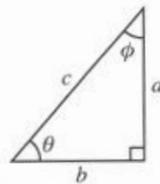
1. Using a calculator, find the value of the following, giving your answers correct to 4 significant figures.

- | | |
|-----------------------|-----------------------|
| (a) $\tan 48^\circ$ | (b) $\sin 38^\circ$ |
| (c) $\cos 75^\circ$ | (d) $\tan 47.3^\circ$ |
| (e) $\sin 18.6^\circ$ | (f) $\cos 63.8^\circ$ |

2. Using a calculator, find the value of the following, giving your answers correct to 1 decimal place.

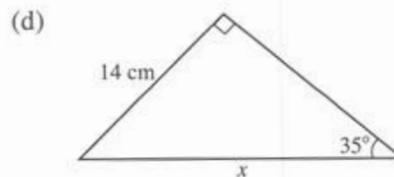
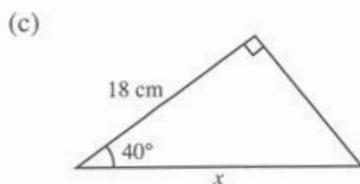
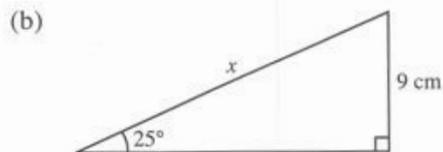
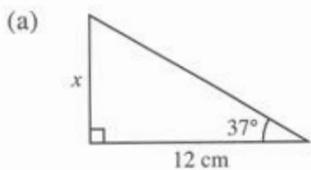
- | | |
|------------------------------------|------------------------------------|
| (a) $\tan \text{_____} = 0.443\ 3$ | (b) $\tan \text{_____} = 2.674\ 5$ |
| (c) $\sin \text{_____} = 0.352\ 1$ | (d) $\sin \text{_____} = 0.877\ 9$ |
| (e) $\cos \text{_____} = 0.677\ 0$ | (f) $\cos \text{_____} = 0.216\ 2$ |

3. With reference to the angles marked θ and ϕ , and sides a , b and c in the figure, express the following in terms of a , b and c .



- | | |
|-------------------|-------------------|
| (a) $\tan \theta$ | (b) $\sin \theta$ |
| (c) $\cos \theta$ | (d) $\tan \phi$ |
| (e) $\sin \phi$ | (f) $\cos \phi$ |

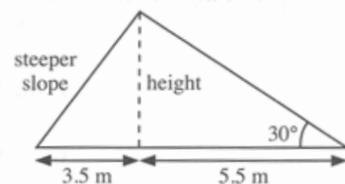
4. Find the value of x in each of the following, giving your answers correct to 4 significant figures.



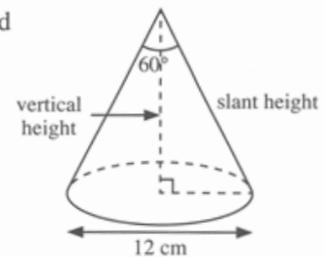
Practice 7.2

Basic

- The angle of elevation of the sun from the tip of the shadow of a pole is 50° . The shadow of the pole is 12 m long. What is the height of the pole?
- A ladder is placed against a wall and it forms an angle of 15° with the wall. The top of the ladder is at a height of 5 m above the ground.
 - What is the distance of the foot of the ladder from the base of the wall?
 - What is the length of the ladder?
- The figure shows the framework of a roof which slopes more on one side than on the other side.
 - Calculate the height of the framework.
 - Find the length of the steeper slope.



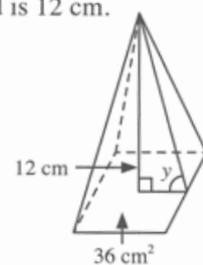
- Give your answers correct to 4 significant figures.
- A cone has a vertical angle of 60° and a base diameter of 12 cm. Find
 - the length of the slant height of the cone,
 - the vertical height of the cone.



- A string, tied to a large balloon, makes an angle of 72° with ground. If the length of the string from the balloon to the ground is 110 m, what is the height of the balloon from the ground? Give your answer correct to 1 decimal place.
- From the top of an animal observatory 15 m above the ground, an observer was able to see a wild boar some distance away from the observatory. If the angle of depression of the wild boar from the observer was 35° , what was the distance of the wild boar from the foot of the observatory?

Advanced

- The base of a square pyramid has an area of 36 cm^2 . The height of the pyramid is 12 cm. Calculate the angle at which the slant height meets the base. (Hint: y is the required angle.)



- A clock tower stands on top of a building. At a distance of 60 m away from the foot of the building, the angles of elevation of the top of the clock tower and the top of the building are 55° and 50° respectively. What is the height of the clock tower?