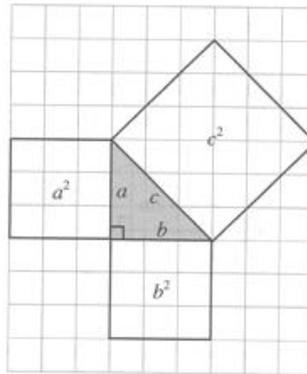# Pythagoras' Theorem

8.1 Pythagoras' Theorem

1. The figure shows a right-angled triangle where the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. In the figure, c is the hypotenuse and a and b are the other two sides of the right-angled triangle.


Thus, $c^2 = a^2 + b^2$. This statement is referred to as the **Pythagoras' Theorem**.

In any right-angled triangle, where $c^2 = a^2 + b^2$, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
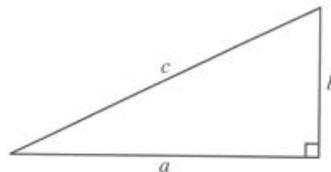
2. The triangle with sides 3, 4 and 5 units is a right-angled triangle since the square on the longest side is equal to the sum of the squares on the other two sides, i.e. $5^2 = 3^2 + 4^2$. The set of numbers 3, 4 and 5 is known as Pythagorean triples. Another set of Pythagorean triples is 6, 8 and 10.
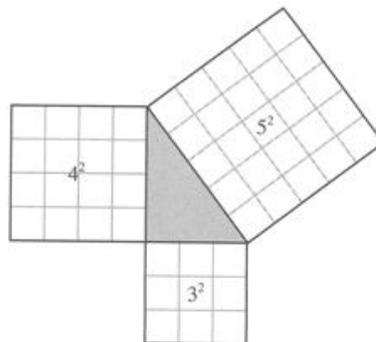
3. If two sides of a right-angled triangle are given, the third side can be calculated by using Pythagoras' Theorem.

Examples: (a) $\triangle ABC$ is right-angled at B. If $AB = 18$ cm and $AC = 30$ cm, find the length of BC.
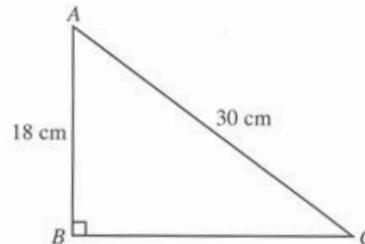
By Pythagoras' Theorem,
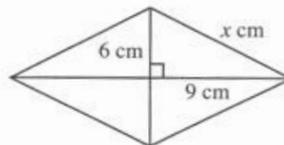$$AB^2 + BC^2 = AC^2$$
$$BC^2 = AC^2 - AB^2$$
$$= 30^2 - 18^2$$
$$= 576$$
$$BC = \sqrt{576}$$
$$= 24 \text{ cm}$$

(b) The diagonals of a rhombus are 12 cm and 18 cm. Find the perimeter of the rhombus, giving your answer correct to 2 decimal places.

The diagonals of a rhombus bisect each other at right angles.
Let each side of the rhombus be x cm.
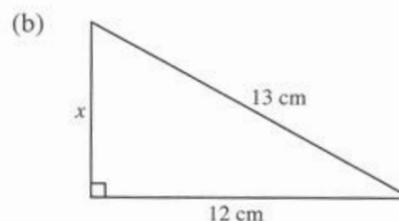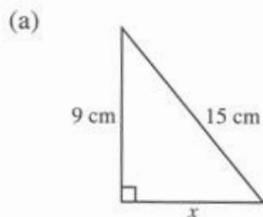By Pythagoras' Theorem,
$$x^2 = 6^2 + 9^2$$
$$= 117$$
$$x = \sqrt{117}$$
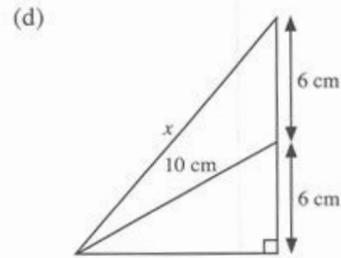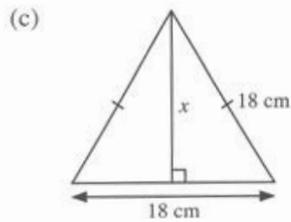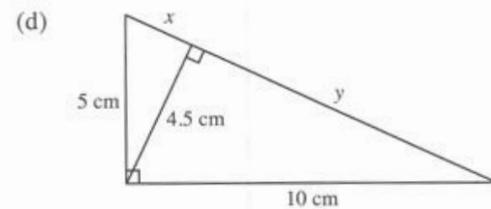Perimeter of the rhombus $= 4 \times \sqrt{117}$
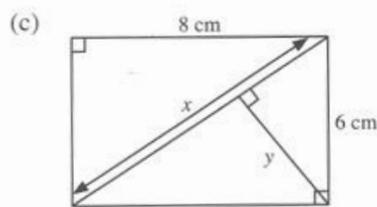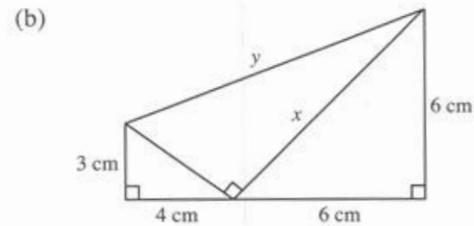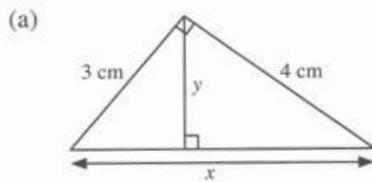$$= 43.27 \text{ cm} \quad \text{(2 dec. plc.)}$$

Practice 8.1

Basic

1. Find the value of x in each of the following triangles. Give your answers correct to 2 decimal places whenever necessary.

(a)

(b)

(c)

(d)

2.   Find the values of x and y in each of the following. Give your answers correct to 1 decimal place.

(a)

(b)

(c)

(d)

Advanced

3.   The sides of a right-angled triangle are $(4x - 3)$ cm, $3x$ cm and $(x + 3)$ cm.

(a)   Find the value of x.

(b)   Hence, find the area of the triangle.

8.2   Application of Pythagoras' Theorem

In the fields of civil engineering, navigation and structural design, architects, draughtsman and engineers often use Pythagoras' Theorem in their work. More examples on the use of this theorem are found in a variety of situations in the following questions.

Examples: (a)   Calculate the area of a square plot of land which has a diagonal of 75 m.

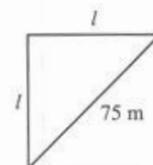Let the sides of the square be l m.
By Pythagoras' Theorem,
$l^2 + l^2 = 75^2$
$\quad 2l^2 = 5625$
$\quad\; l^2 = 5625 \div 2$
$\quad\quad = 2812.5$
The area of the square plot of land is 2812.5 m².

(b)   A TV-antenna on a roof top is supported by a cable with one end secured to the top of the antenna and the other end bolted to the roof top. The height of the antenna is 4.5 m and the distance from the foot of the antenna to the bolted end of the cable is 2 m. Calculate the length of the cable.

Let the length of the cable be c m.
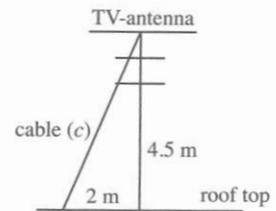By Pythagoras' Theorem,
$c^2 = 4.5^2 + 2^2$
$\quad = 24.25$
$c = \sqrt{24.25}$
$\quad = 4.92$
The length of the cable is 4.92 m.

TV-antenna

cable (c)   4.5 m

2 m   roof top

(c)   The figure shows the cross-section of a shed. The top of the roof is 7 m above the ground. The end walls are 5 m high and 8 m apart. Find the length of the sloping roof.

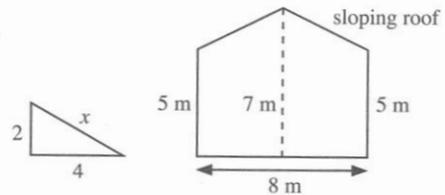Let the length of the sloping roof be x m.
By Pythagoras' Theorem,
$x^2 = 2^2 + 4^2$
$\quad = 20$
$x = \sqrt{20}$
$\quad = 4.47$
The length of the sloping roof is 4.47 m.

sloping roof
5 m   7 m   5 m
2   x
4
8 m

(d)   A patrol boat left a jetty and headed 3.5 km north before turning east and travelled another 4.2 km to meet a distressed boat. How far was the patrol boat from the jetty when it met the distressed boat?

Let the distance of the patrol boat from the jetty be x m.
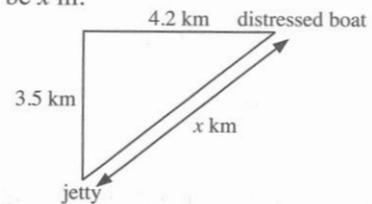By Pythagoras' Theorem,
$x^2 = 3.5^2 + 4.2^2$
$\quad = 29.89$
$x = \sqrt{29.89}$
$\quad = 5.47$
The patrol boat was 5.47 km from the jetty.

4.2 km   distressed boat
3.5 km
x km
jetty

**Practice 8.2**

Basic

Give your answers correct to 4 significant figures whenever necessary.

1.   A rectangular field measures 45 m by 30 m. Calculate the length of its diagonal.

2.   Each side of a rhombus is 10 cm. One of its diagonal is 16 cm. What is the length of the other diagonal?

3.   Calculate the length of the largest square that can be cut from a circular plate of radius 50 cm.

4. The height of a window above the ground is 15 m. One end of a ladder just reaches the top of the window when the foot of the ladder is placed 3 m away from the wall. Find the length of the ladder.

5. A flag pole 12 m tall is supported by three ropes, each of which is secured at the top and to a peg on the ground 3 m from the base of the pole. Calculate the total length of the rope needed if an additional length of 2 m is allowed for tying knots.
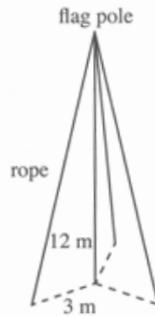
6. A picture frame is made for a square poster with a diagonal of 50 cm. What is the perimeter of the wooden frame that surrounds the poster if the width of the frame is 2 cm?
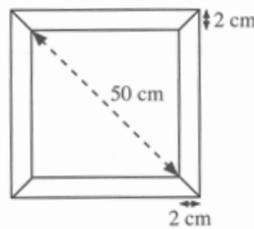

7. The figure shows the entrance to a garage. Calculate the height of the garage.
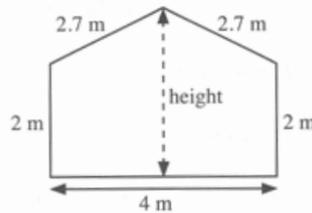
8. An aircraft took off from a runway and climbed to a height of 135 m above the ground after it has flown 400 m in the air. Calculate the horizontal distance the aircraft had flown over the runway.

Advanced

9. A 'wired model' in the shape of a cuboid measures 80 cm by 40 cm by 40 cm. What is the length of the wire that joins the opposite corners of the model?
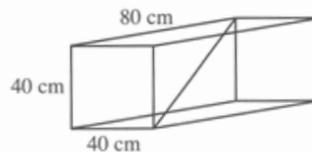
10. A is the point $(3, 3)$ and B is the point $(10, 8)$. Calculate the distance AB.
    (Hint: Sketch the points on a number plane.)

8.3 Converse of Pythagoras' Theorem

If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides of the triangle, then the triangle is a right-angled triangle.
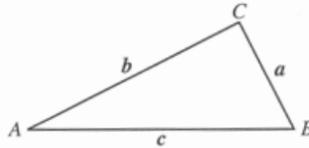
In $\triangle ABC$, if $c^2 = a^2 + b^2$, then $\triangle ABC$ is right-angled at C. The right angle at C faces the longest side c.

Example: Show that the triangle with sides 7 cm, 24 cm and 25 cm is a right-angled triangle.

The longest side is 25 cm.
$$25^2 = 625$$
$$7^2 + 24^2 = 49 + 576$$
$$= 625$$
Since $25^2 = 7^2 + 24^2$, by Pythagoras' Theorem, the triangle with sides 7 cm, 24 cm and 25 cm is a right-angled triangle.

Practice 8.3

Basic

1. Show that each of the following is a right-angled triangle.

    (a) $\triangle ABC$ with $AB = 15$ cm, $BC = 20$ cm and $AC = 25$ cm

    (b) $\triangle PQR$ with $PQ = 40$ cm, $QR = 9$ cm and $PR = 41$ cm

    (c) $\triangle XYZ$ with $YZ = 30$ mm, $XZ = 40$ mm and $XY = 50$ mm

2. Show that the triangle with sides 15 cm, 17 cm and 23 cm is not a right-angled triangle.

Advanced

3. $STUV$ is a rectangle and $SV = 40$ cm, $UV = 30$ cm and $UX = 24$ cm.

    (a) Show that $\triangle TXU$ is right-angled.

    (b) Hence, calculate TX.