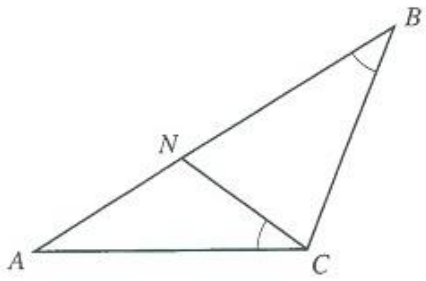
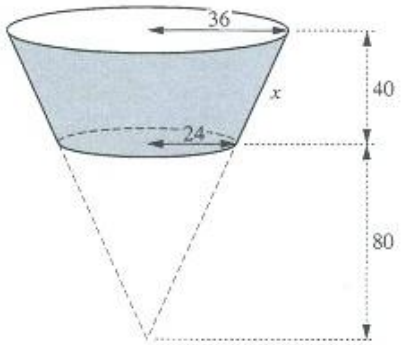
Topic 12 Congruence and Similarity

1. In the diagram, ABC is a triangle.
N is the point on AB such that $A\hat{B}C = A\hat{C}N$.



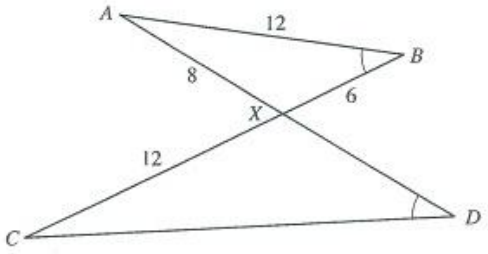
(a) Show that triangles ABC and ACN are similar. [1]
(b) Given that $AN = 4$ m and $NB = 5$ m, find AC. [2]
(N2011/P1/Q10)

2. The diagram shows a pot which is part of a right circular cone of height 120 cm.
The open end of the pot is a circle of radius 36 cm.
The base of the pot is a circle of radius 24 cm.
The height of the pot is 40 cm.
The slant height of the pot is x cm.
[You may ignore any holes in the base of the pot.]



(a) Show that $x = 41.8$, correct to 3 significant figures. [2]
(b) Calculate the total surface area of the outside of the pot. [3]
(c) Another pot is to be made with a volume twice the volume of this pot.
Given that the two pots are geometrically similar, find the height of the larger pot. [2]
(N2011/P2/Q7b)

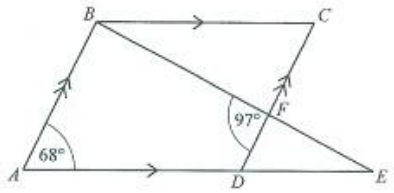
3. In the diagram, triangle ABX is similar to triangle CDX.
Angle ABX = angle CDX.
All measurements are in centimetres.



Calculate
(a) CD, [1]
(b) AD. [2]
(c) The area of triangle ABX is 21.3 cm^2.
Calculate the area of triangle CDX. [2]
(N2012/P1/Q23)

4. Two bottles are geometrically similar.
The smaller bottle has a capacity of 1 litre and the larger bottle has a capacity of 2 litres.
Calculate the height of the smaller bottle as a percentage of the height of the larger bottle.
[2]
(N2014/P1/Q8)

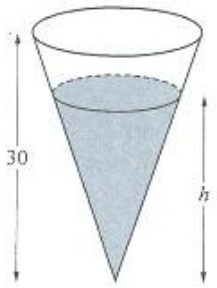
5.



The diagram shows a parallelogram $ABCD$ with AD produced to E.
F is the point of intersection of CD and BE.
Angle $BAD = 68°$ and angle $BFD = 97°$.
(a) Find angle ABF. [1]
(b) Show that triangles BCF and EDF are similar. [2]
(c) State another triangle that is similar to BCF and EDF. [1]
(d) The ratio $AD : DE = 3 : 2$.
(i) Find the ratio $AB : CF$. [1]
(ii) Given that the area of triangle $EDF = 9.72$ cm^2, find the total area of the shape $ABCFE$. [3]
(N2014/P2/Q4)

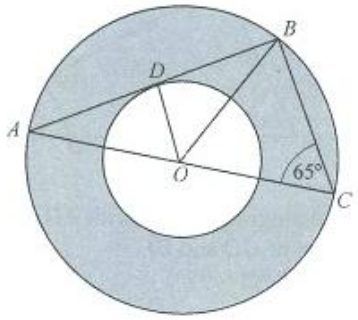
6. Two bottles of water are geometrically similar.
The larger bottle holds 2 litres and the smaller bottle holds 1.25 litres.
The height of the larger bottle is 33.5 cm.
(a) Calculate the height of the smaller bottle. [2]
(b) The ratio surface area of larger bottle : surface area of smaller bottle can be written in the form $k : 1$.
Find the value of k.
Give your answer to 2 decimal places. [2]
(N2015/P1/Q17)

7. The diagram shows a cone of height 30 cm.



The volume of the liquid in the cone is half the volume of the cone.
Calculate the depth, h centimetres, of the liquid. [2]
(N2016/P1/Q7)

8.



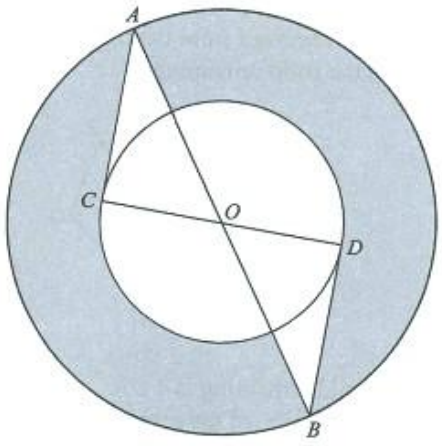
The diagram shows two concentric circles, centre O.
A, B and C are points on the larger circle and D is a point on the smaller circle.
ADB is a tangent to the smaller circle and angle $BCO = 65°$.
(a) Show that triangles ABC and ADO are similar.
Give a reason for each statement you make. [2]
(b) Find the ratio area of triangle BOC : area of triangle ADO. [2]
(c) The radius of the smaller circle is 3 cm.
Find the shaded area. [4]
(N2016/P2/Q6)

9.

A company manufactures three sizes of the same brand of shampoo.
The bottles are all geometrically similar.
The height of the 250 m*l* bottle is 18.4 cm.
Calculate the height of the 75 m*l* bottle.

[3]
(N2017/P1/Q23b)

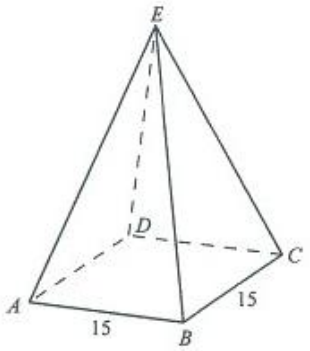
10.



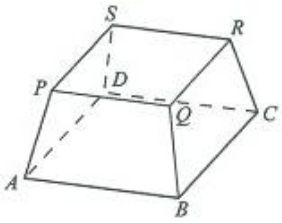
AB is a diameter of the large circle, centre *O*.
CD is a diameter of the small circle, centre *O*.
AC and *BD* are tangents to the small circle.
Show that triangle *OAC* is congruent to triangle *OBD*.
Give a reason for each statement you make.

[3]
(N2018/P2/Q7a)

11.

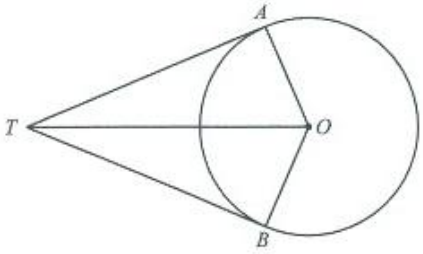


The diagram shows a pyramid $ABCDE$.
The base of the pyramid is a square of side 15 cm.
E is vertically above the centre of the square base.
The vertical height of the pyramid is 20 cm.
A smaller, similar pyramid is removed from the top of the original pyramid.
The diagram below shows the solid remaining.



The vertical height of the solid remaining is 4 cm.
Calculate the area of the top, $PQRS$, of the remaining solid. [2]
(N2018/P2/Q8d)

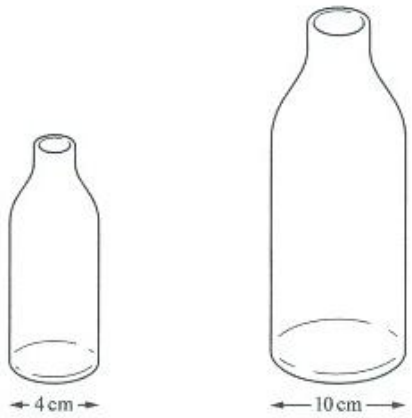
12.



(a) TA and TB are tangents to a circle, centre O.
Show that triangle OAT and triangle OBT are congruent.
Give a reason for each statement you make. [3]

(b) Point P lies on the circumference of the circle so that triangle APT is congruent to triangle BPT.
On the diagram , mark the two possible positions for the point P. [1]
(N2019/P1/Q16)

13.



The diagram shows two geometrically similar bottles.
The diameter of the base of the smaller bottle is 4 cm.
The diameter of the base of the larger bottle is 10 cm.

(a) The height of the larger bottle is 30 cm.
Calculate the height of the smaller bottle. [2]

(b) The smaller bottle has a capacity of 125 m*l*.
The manufacturer makes another geometrically similar bottle that holds 500 m*l*.
Calculate the diameter of the base of this bottle. [2]

(N2019/P1/Q20)