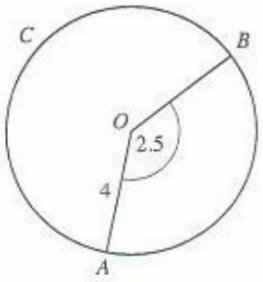
Topic 15 Mensuration

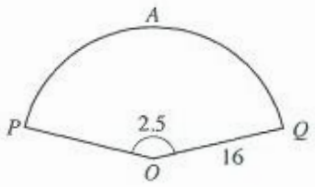
1. A, B and C lie on a circle with centre O and radius 4 cm.
$A\hat{O}B = 2.5$ radians.



(a) Find the area of the minor sector AOB. [1]
(b) (i) Write down an expression, in terms of π, for the reflex angle AOB. [1]
(ii) Find an expression, in terms of π, for the length of the arc ACB. [1]
(N2011/P1/Q15)

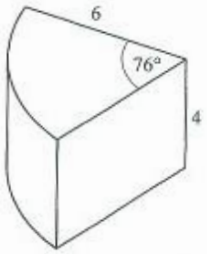
2. A cylindrical container has a radius of 6.8 cm and a capacity of 1.5 litres.
Calculate the height of the container. [2]
(N2012/P1/Q6)

3. $OPAQ$ is a sector of a circle, centre O, of radius 16 cm.
The angle at the centre is 2.5 radians.



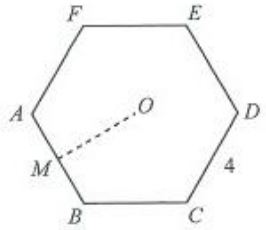
(a) Calculate the length of the arc PAQ. [1]
(b) The sector is formed into a cone by joining the two radii, OP and OQ, together.
Calculate the radius of the base of the cone. [2]
(c) Change 2.5 radians to degrees. [1]
(N2012/P1/Q21)

4.



The cross-section of a solid is the sector of a circle of radius 6 cm and angle 76°.
The height of the solid is 4 cm.
Calculate the volume of the solid. [3]
(N2012/P2/Q5b)

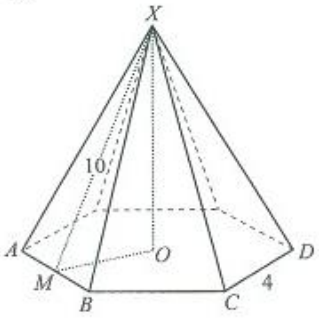
5.



A regular hexagon, $ABCDEF$, has sides of length 4 cm.
M is the midpoint of AB and O is the centre of the hexagon.

(a) Show that the area of the hexagon $ABCDEF$ is 41.6 cm^2, correct to 3 significant figures. [3]

Hexagon $ABCDEF$ forms the base of a pyramid.
The vertex, X, is directly above O.
The slant height, MX, of the pyramid is 10 cm.



(b) Calculate the total surface area of the pyramid. [2]

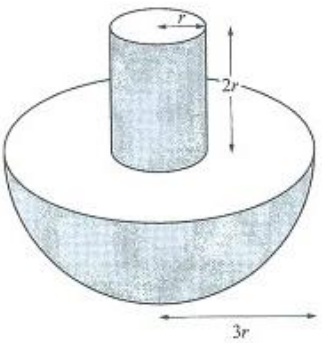
(c) Calculate the height, OX, of the pyramid. [2]

(d) Calculate the volume of the pyramid. [2]

(e) Another similar pyramid is made, with a hexagonal base of side 9 cm. Find the volume of this pyramid. [2]

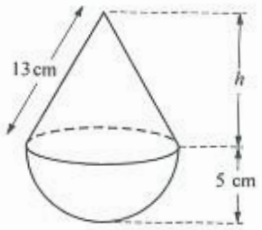
(N2012/P2/Q9)

6. This solid is made from a cylinder and a hemisphere. The cylinder has radius r and height $2r$. The hemisphere has radius $3r$.
Find an expression, in terms of π and r, for the **total surface area** of the solid. [3]



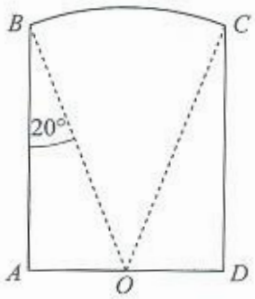
(N2013/P1/Q6)

7. The diagram shows a solid made from a cone and a hemisphere.



(a) Show that the height, h, of the cone is 12 cm. [1]
(b) Calculate the volume of the solid. [2]
(N2013/P1/Q16)

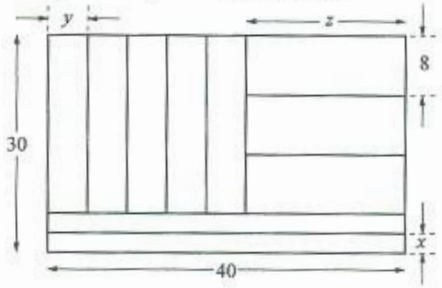
8.



1.20

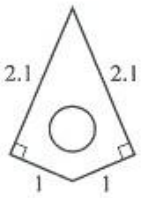
The diagram shows a symmetrical window frame.
OBC is a sector of the circle, centre O.
AD = 1.20 m, angle ABO = 20° and AB is perpendicular to AD.
(a) Calculate
(i) the radius of the sector OBC, [2]
(ii) the angle BOC in radians, [2]
(iii) the total perimeter of the window frame. [4]
(b) A company manufactures windows.
The cost of manufacture is $78.50 per square metre of window.
Work out the cost of manufacturing this window. [4]
(N2013/P2/Q8)

9. A rectangle measures 40 cm by 30 cm.
It is divided into ten rectangles of **equal area** as shown.



Find the lengths labeled x, y and z. [3]
(N2014/P1/Q12)

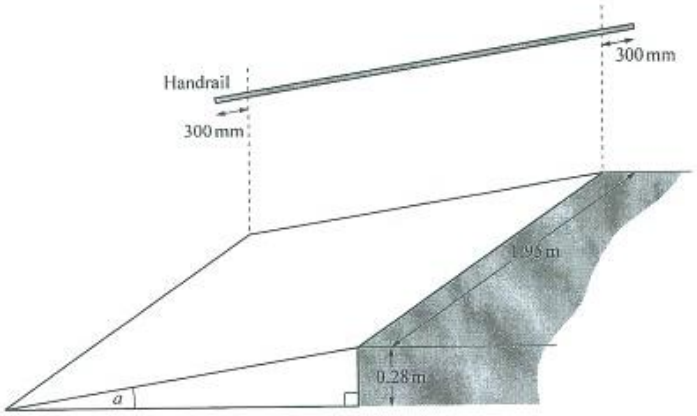
10. The cross-section of a gold pendant is a quadrilateral with two right angles and a circular hole, as shown. All measurements are in centimetres.



The diameter of the circular hole is 0.8 cm.
The uniform thickness of the pendant is 0.3 cm.
The mass of 1 cubic centimetre of gold is 19.3 grams.
The price of 1 gram of the gold is $69.95.
Calculate the value of the gold in the pendant.
Give your answer to the nearest cent. [5]
(N2014/P1/Q22)

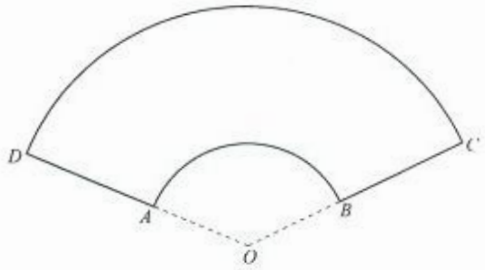
11. Ravi is playing with 385 one-centimetre cubes.
(a) He uses some of the cubes to make a cuboid measuring 9 cm by 8 cm by 5 cm.
Calculate the total surface area of the cuboid. [2]
(b) Ravi uses all 385 cubes to make a cuboid.
All the sides of the cuboid are longer than 1 cm.
Find the dimensions of the cuboid. [2]
(c) Ravi makes the largest cube possible using some of the 385 cubes.
How many cubes does he have left over? [2]
(N2014/P1/Q23)

12.



There is a vertical step 0.28 m high on horizontal ground at the entrance to a building.
The width of the step is 1.95 m.
A ramp in the shape of a prism is to be installed so that wheelchairs can enter the building.
The gradient of the ramp is such that the ratio vertical distance : horizontal distance is 1 : 12.
The ramp is to be made of concrete.
When it is set, the mass of 1 m^3 of concrete is 2300 kg.
Calculate the mass of the completed ramp. [4]
(N2014/P2/Q6b)

13.



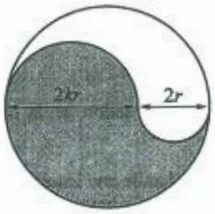
The diagram shows a mirror $ABCD$.
AB and DC are arcs of circles centre O with radii 20 cm and 50 cm respectively.
The perimeter of the mirror is 235 cm.

(a) Calculate the angle AOB in radians. [2]

(b) Calculate the area of the mirror. [2]

(N2014/P2/Q7b)

14. This design is drawn using a large circle and semicircles.
The diameters, in centimetres, of two of the semicircles are shown.



(a) Show that the total area, A, of the large circle is given by the formula $A = \pi r^2(k+1)^2$. [2]

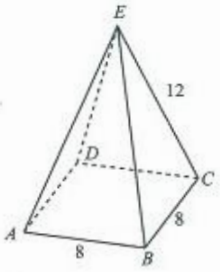
(b) Find, in terms of π and r, the difference in area between the shaded section and the unshaded section when $k = 2$. [4]

(N2015/P1/Q23)

15. The perimeter of a sector of another circle is 14.8 cm.
The angle of the sector is 1.7 radians.
Calculate the area of the sector. [3]

(N2015/P2/Q5b)

16.



The diagram shows a candle in the shape of a pyramid $ABCDE$.
$ABCD$ is a square of side 8 cm and $AE = BE = CE = DE = 12$ cm.

(a) Calculate the volume of the candle. [4]

Another candle is made in the shape of a sphere.
The volume of this candle is the same as the volume of candle *ABCDE*.

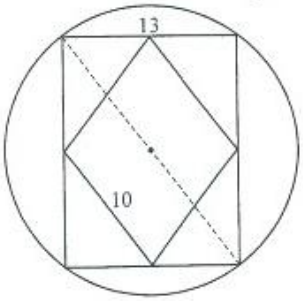
(b) Show that the radius of the spherical candle is 3.78 cm, correct to 3 significant figures. [2]

The diagram shows the plan view of a box holding six of the spherical candles.
The box is in the shape of a cuboid and the candles just fit into the box.

(c) Calculate the volume of empty space in the box. [3]

(N2015/P2/Q7)

17. The diagram shows a rhombus drawn inside a rectangle inside a circle.



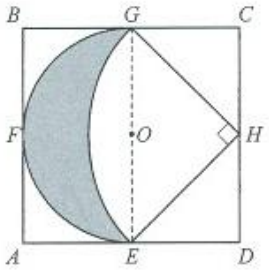
The rhombus has sides of length 10 cm.
The length of the shorter side of the rectangle is 13 cm.

(a) Calculate the circumference of the circle. [2]

(b) Calculate the area of the rectangle. [2]

(N2016/P1/Q16)

18.



ABCD is a square, centre *O*.
$BC = 2r$.
E, *F*, *G* and *H* are the midpoints of the sides of the square.
EFG is a semi-circular arc, centre *O*.
The other arc *EG* has centre *H*.
$HG = \sqrt{2}r$.
What fraction of the square *ABCD* is **not** shaded? [5]

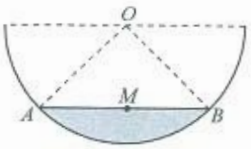
(N2016/P1/Q24)

19.

A solid cylinder has radius r cm and height h cm.
A solid hemisphere has radius r cm.
The volumes of the cylinder and hemisphere are equal.
Work out, in terms of r, the total surface area of the cylinder. [3]

(N2017/P1/Q9)

20.



The diagram shows a semicircle, centre O, radius 30 cm.
M is the midpoint of the chord AB.
$OM = 20$ cm.

(a) Show that angle $AOB = 96.4°$, correct to 3 significant figures. [2]

(b) Calculate the shaded area. [4]

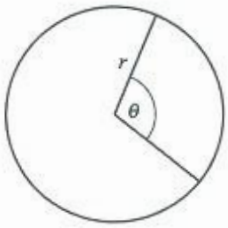
(c) The semicircle is the cross-section of a water trough of length 1.5 m, standing on level ground.
The shaded area represents the water in the trough.

(i) Calculate the volume of water, in cm^3, in the trough. [2]

(ii) Calculate the number of litres of water that must be added to fill the trough.
Give your answer correct to the nearest 10 litres. [3]

(N2017/P2/Q8)

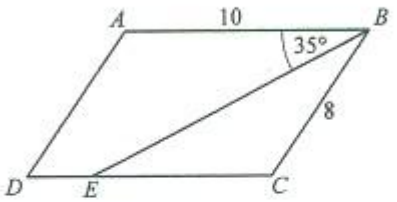
21.



The diagram shows a circle with radius r cm.
The circle is divided into two sectors.
The angle of the minor sector is θ radians.
The perimeter of the major sector is twice the perimeter of the minor sector.
Find the value of θ.
Give your answer correct to three decimal places. [4]

(N2018/P1/Q16)

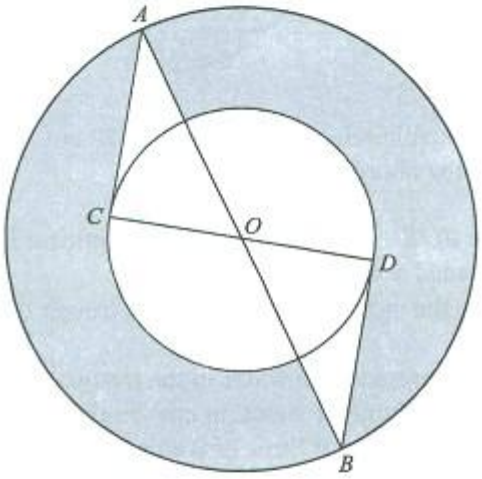
22.



$ABCD$ is a parallelogram.
BE bisects angle ABC.
$AB = 10$ cm, $BC = 8$ cm and angle $ABE = 35°$.
Calculate the area of trapezium $ABED$.

[5]
(N2018/P1/Q22)

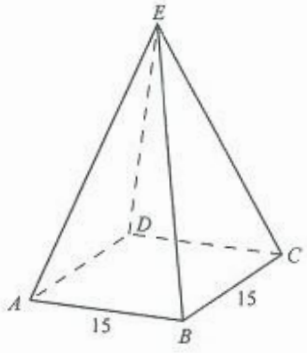
23.



AB is a diameter of the large circle, centre O.
CD is a diameter of the small circle, centre O.
AC and BD are tangents to the small circle.
The radius of the large circle is 7 cm and $O\hat{A}C = 30°$.
Calculate the shaded area.

[3]
(N2018/P2/Q7b(ii))

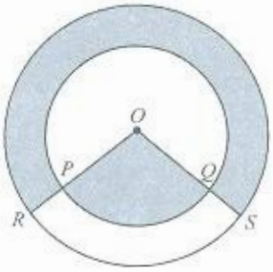
24.



The diagram shows a pyramid $ABCDE$.
The base of the pyramid is a square of side 15 cm.
E is vertically above the centre of the square base.
The vertical height of the pyramid is 20 cm.
Calculate the total surface area of the pyramid. [3]

(N2018/P2/Q8c)

25.



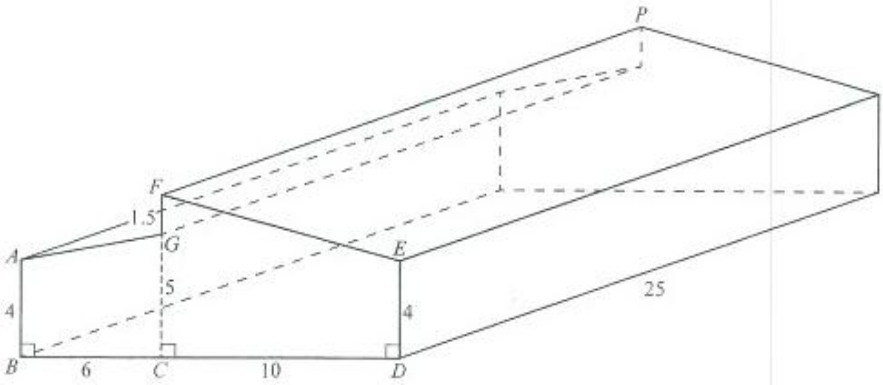
P and Q are points on the circle centre O with radius 4 cm.
R and S are points on the circle centre O with radius 6 cm.
OPR and OQS are straight lines.
The perimeter of the minor sector OPQ is 15.2 cm.

(a) Calculate angle POQ in radians. [2]

(b) Calculate the total shaded area. [3]

(N2019/P2/Q6b)

26.



The diagram shows a barn in the shape of a prism of length 25 m with a rectangular base.
The barn has two sloping rectangular roofs.
$AB = DE = 4$ m, $BC = 6$ m, $CD = 10$ m, $CG = 5$ m and $FG = 1.5$ m.
The barn is positioned on horizontal ground and the walls are vertical.

(a) Calculate the volume of the barn. [3]

(b) Calculate the total area of the two sloping roofs of the barn. [4]

(c) Calculate the angle of elevation of P from D. [4]

(N2019/P2/Q8)