Topic 17

Vectors in Two Dimensions

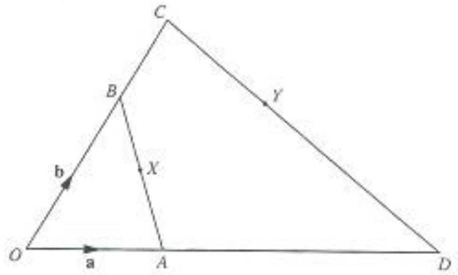
1. (a) P is the point (3, 4). Q is the point (–1, 2).

   (i) Write down the column vector $\overrightarrow{PQ}$. [1]

   (ii) Find $|\overrightarrow{PQ}|$. [2]

   (iii) Given that $\overrightarrow{PL} = \frac{1}{2}\overrightarrow{PQ}$, find $\overrightarrow{PL}$. [1]

   (b)



In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OB} = 2\overrightarrow{BC}$, $\overrightarrow{AD} = 2\overrightarrow{OA}$ and $\overrightarrow{AB} = 2\overrightarrow{AX}$.

(i) Express, as simply as possible, in terms of **a** and **b**,

   (a) $\overrightarrow{AB}$, [1]

   (b) $\overrightarrow{OX}$, [1]

   (c) $\overrightarrow{CD}$. [1]

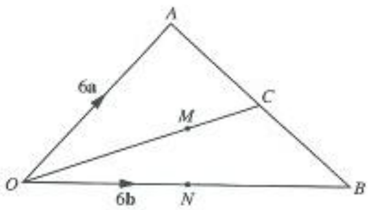
Y is the point on CD such that $CY : YD = 1 : 2$.

(ii) Express $\overrightarrow{OY}$ in terms of **a** and **b**. [2]

(iii) Hence write down two facts about O, X and Y. [1]

(N2011/P2/Q8)

2.



The position vectors of A, B and C, relative to O, are 6**a**, 6**b** and (3**a** + 3**b**) respectively.

$ON = NB$ and $MC = \frac{1}{3}OC$.

(a) Express in terms of **a** and **b**, as simply as possible,

   (i) $\overrightarrow{OM}$, [1]

   (ii) $\overrightarrow{AM}$, [1]

   (iii) $\overrightarrow{AN}$. [1]

(b) Use your answers to parts (a)(ii) and (a)(iii) to explain why A, M and N lie in a straight line. [1]

(c) Write down the ratio $AM : AN$. [1]

(N2012/P1/Q25)

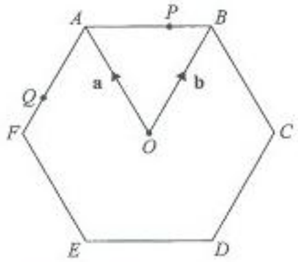
3. (a) L is the point (3, 2).
The point M is the result of the translation of point L by $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$.

(i) Find the coordinates of point M. [1]

(ii) Find the equation of line LM. [2]

(iii) Given that $\overrightarrow{MN} = 2\overrightarrow{ML}$, find the coordinates of point N. [1]

(b)



$ABCDEF$ is a regular hexagon, centre O.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
P is the point on AB such that $AP : PB = 2 : 1$ and Q is the point on AF such that $AQ : QF = 2 : 1$.

(i) Express, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{AB}$, [1]

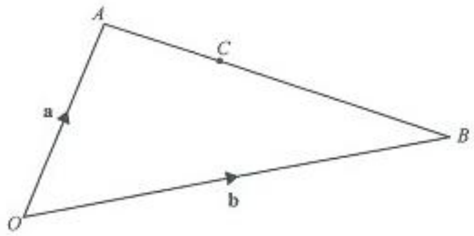
(b) $\overrightarrow{OP}$, [1]

(c) $\overrightarrow{AQ}$. [1]

(ii) QP intersects OA at the point X. Find $\overrightarrow{QX}$. [2]

(N2013/P2/Q6)

4. OAB is a triangle.
C is the point on AB such that $AC = \frac{1}{3}AB$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(a) Find $\overrightarrow{AC}$ in terms of **a** and **b**. [1]

(b) D is the point on OB such that $OD : DB = 1 : 3$.
Use vectors to show that OA and DC are **not** parallel. [2]

(N2014/P1/Q14)

5. (a) The position vector of point A is $\begin{pmatrix}1\\5\end{pmatrix}$ and the position vector of point B is $\begin{pmatrix}-5\\-2\end{pmatrix}$.
   (i) Find the column vector $\overrightarrow{AB}$. [1]
   (ii) Find $|\overrightarrow{AB}|$. [2]
   (iii) Given that $\overrightarrow{BA} = 2\overrightarrow{AC}$, find the coordinates of point C. [2]

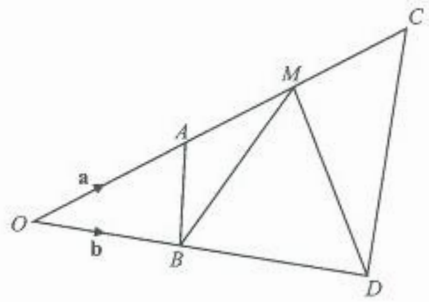
   (b) The point P has coordinates $(8, -2)$ and $\overrightarrow{PQ} = \begin{pmatrix}4\\-3\end{pmatrix}$.
   (i) Find the equation of the line PQ. [2]
   (ii) The equation of the line RS is $2y - 4x = 19$.
   Find the coordinates of the point of intersection of PQ and RS. [3]

(N2014/P2/Q5)

6. A is the point $(-1, 4)$ and $\overrightarrow{BA} = \begin{pmatrix}-3\\6\end{pmatrix}$.
   (a) Find the coordinates of point B. [1]
   (b) Calculate $|\overrightarrow{AB}|$. [1]

(N2015/P1/Q8)

7.



OCD is a triangle where A and M are points on OC and B is a point on OD.
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{MB} = \mathbf{b} - 2\mathbf{a}$, and $\overrightarrow{MD} = 3\mathbf{b} - 2\mathbf{a}$.
$AM = MC$.

(a) Express, as simply as possible, in terms of **a** and/or **b**,
   (i) $\overrightarrow{AB}$ [1]
   (ii) $\overrightarrow{BD}$ [1]

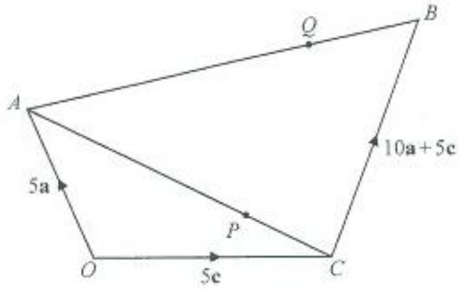
(b) Show that $\overrightarrow{CD} = 3\mathbf{b} - 3\mathbf{a}$. [1]

(c) Show that triangles OAB and OCD are similar.
Give a reason for each statement you make. [3]

(d) Find the ratio of the area of triangle OAB to the area of quadrilateral $ABDC$. [2]

(N2015/P2/Q2)

8.



$OABC$ is a quadrilateral.
$\overrightarrow{OA} = 5\mathbf{a}$, $\overrightarrow{OC} = 5\mathbf{c}$, $\overrightarrow{CB} = 10\mathbf{a} + 5\mathbf{c}$ and $\overrightarrow{AQ} = 4\mathbf{a} + 8\mathbf{c}$.
$AP : PC = 4 : 1$.

(a) Write each of the following in terms of **a** and **c**.
Give your answers in their simplest form.

(i) $\overrightarrow{OP}$ [2]

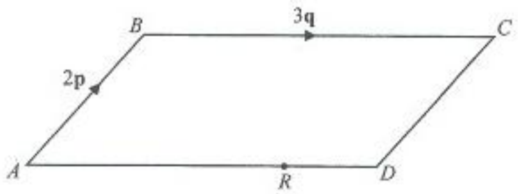
(ii) $\overrightarrow{PQ}$ [2]

(b) (i) Explain why PQ is parallel to CB. [1]

(ii) Find the ratio $CB : PQ$. [1]

(N2016/P1/Q25)

9.



$ABCD$ is a parallelogram.
$\overrightarrow{AB} = 2\mathbf{p}$ and $\overrightarrow{BC} = 3\mathbf{q}$.
R is the point on AD such that $AR : RD = 2 : 1$.

(a) Express $\overrightarrow{RC}$, as simply as possible, in terms of **p** and **q**. [2]

(b) S is a point on AD and T is a point on CD such that $\overrightarrow{RC} = 2\overrightarrow{ST}$.

(i) Show that triangles RCD and STD are similar.
Give a reason for each statement you make. [3]

(ii) Find the ratio area triangle STD : area parallelogram $ABCD$. [2]

(iii) Q is a point on BC.
Triangle QAB is congruent to triangle RCD.
From $\overrightarrow{QR}$, as simply as possible, in terms of **p** and **q**. [2]

(N2017/P2/Q6)

10. (a) $\overrightarrow{PQ} = \begin{pmatrix} a \\ 3a \end{pmatrix}$.

$|\overrightarrow{PQ}| = \frac{5\sqrt{10}}{2}$

Find the two possible values of a. [2]

(b) A line joins the two points A (–5, 1) and B (43, 38).

(i) Find $\overrightarrow{AB}$. [1]

(ii) Use vectors to show whether or not the point C (11, 13) lies on this line. [2]

(N2018/P1/Q18)

11. The point A is translated to B (5, –2) by the vector $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$.

(a) Find the coordinates of point A. [1]

(b) Find the magnitude of $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$. [2]

(N2019/P1/Q13)

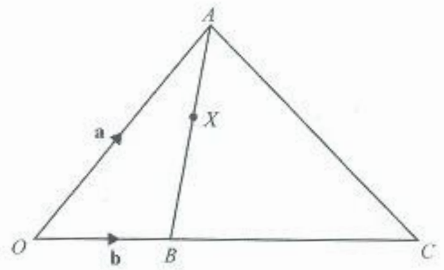
12. (a) P is the point (4, –3), Q is the point (–5, 1).

(i) Write down the column vector $\overrightarrow{PQ}$. [1]

(ii) R has coordinates (h, 3) and $\overrightarrow{PR} = k\overrightarrow{PQ}$.

Find the value of h and the value of k. [2]

(b)



OAC is a triangle and B is a point on OC.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $OB : BC = 2 : 3$.

X is the point on AB such that $AX : XB = 1 : 2$.

(i) Express $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible. [2]

(ii) Express $\overrightarrow{XB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible. [2]

(iii) Y is the point on OC such that $AXYC$ is a trapezium.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, $\overrightarrow{XY}$. [2]

(N2019/P2/Q7)