Topic 5 Algebraic Expressions and Formulae

1. Petrol costs x cents per litre.
John buys some petrol and it costs him y dollars.
Find an expression, in terms of x and y, for the number of litres that John buys. [2]
(N2011/P1/Q5)

2. (a) Simplify $\frac{5c}{2} \div \frac{20c^2}{d}$. [1]
(b) Factorise fully $6x^2 + 14x - 12$. [2]
(N2011/P1/Q8)

3. (a) Given that $xy = 2(x + 3)$, express x in terms of y. [2]
(b) Express as a single fraction in its simplest form $\frac{4}{x-2} + \frac{2}{2x+1}$. [2]
(c) Simplify $\frac{4x^2 - y^2}{2x^2 + xy}$. [2]
(N2011/P2/Q1b, c, d)

4. (a) The nth term of a sequence is given by $T_n = \frac{n(n+1)}{4}$.
(i) Use the formula to find T_{20}. [1]
(ii) Which term of the sequence has value 33? [2]
(b) p is a positive integer.
(i) Explain why $(2p + 1)$ is an odd number. [1]
(ii) Write down an expression for the next odd number which is greater than $(2p + 1)$. [1]
(iii) Find and simplify expressions for the squares of these two odd numbers. [2]
(iv) Hence explain why the difference between the squares of two consecutive odd numbers is always a multiple of 8. [1]
(N2011/P2/Q5)

5. The first four terms of a sequence are 5, 9, 13 and 17.
(a) Write down the 8th term of the sequence. [1]
(b) Find an expression, in terms of n, for the nth term of the sequence. [1]
(c) One term in the sequence is 205.
Find the value of n for this term. [1]
(N2012/P1/Q7)

6. (a) Simplify $2(3x + 2y) - 5(x - 2y)$. [1]
(b) Write as a single fraction in its simplest form $\frac{5}{(x+2)^2} - \frac{1}{(x+2)}$. [2]
(N2012/P1/Q11)

7. (a) Factorise fully $xy - 3x + 2y - 6$. [2]
(b) Factorise fully $6x^2 - 15x - 9$. [2]
(N2012/P1/Q18)

8. (a) Factorise completely $3x^2 - 48y^2$. [2]
(b) Express as a single fraction in its simplest form $\frac{2x}{5} - \frac{x-1}{15}$. [1]
(c) Simplify $\frac{x}{2y} \div \frac{3x^2y}{4}$. [1]
(d) It is given that $V = \frac{4\pi}{3}(a^3 - b^3)$.
(i) Evaluate V when $a = 2.5$ and $b = 1.9$. [1]
(ii) Express a in terms of V, π and b. [2]
(N2012/P2/Q1)

9. Simplify $5p - 3(p - 2)$. [1]
(N2013/P1/Q1a)

10. (a) Simplify $12x^2y \div 3xy^{-5}$. [1]
(b) Write as a single fraction in its simplest form $\frac{3x}{(2x-1)^2} - \frac{2}{2x-1}$. [2]
(N2013/P1/Q17)

11. (a) Factorise completely $3xy - 6ay - 4x + 8a$. [2]
(b) Factorise $3x^2 + 10x - 8$. [2]
(N2013/P1/Q18)

12. (a) It is given that $s = ut + \frac{1}{2}at^2$.
(i) Find s when $u = 0$, $a = 0.6$ and $t = 15$. [1]
(ii) Express a in terms of s, u and t. [2]
(b) (i) Factorise completely $18p^2 - 8$. [2]
(ii) Simplify $\frac{18p^2 - 8}{6p^2 - 14p - 12}$. [2]
(c) Express as a single fraction in its simplest form $\frac{6}{3-2x} - \frac{4}{2-x}$. [2]
(N2013/P2/Q1)

13. The first three terms in a sequence of numbers, T_1, T_2, T_3, ... are given below.
$T_1 = 1^2 + 2 = 3$
$T_2 = 2^2 + 4 = 8$
$T_3 = 3^2 + 6 = 15$

(a) (i) Find T_4. [1]
(ii) Find an expression, in terms of n, for T_n. [2]
(iii) Evaluate T_{50}. [1]
(b) The first four terms in a different sequence are -5, -1, 3, 7.
Find an expression, in terms of n, for the nth term, P_n, of this sequence. [2]
(c) By forming an equation in n, find the values of n for which
$\frac{P_n}{T_n} = \frac{1}{5}$. [3]
(N2013/P2/Q4)

14. Some bacteria were introduced into a culture.
The number, B, of bacteria t hours after being introduced is given by

$$B = 1000 \times 3^t.$$

(a) How many bacteria were introduced into the culture? [1]
(b) Find the percentage increase in the number of bacteria at the end of the first hour. [1]
(N2014/P1/Q9)

15. Simplify $\frac{1}{x-5} + \frac{7x}{(x-5)^2}$. [2]
(N2014/P1/Q16b)

16. Each term in this sequence is found by adding the same number to the previous term.

$$a, \quad 12, \quad b, \quad c, \quad 33, \quad \ldots$$

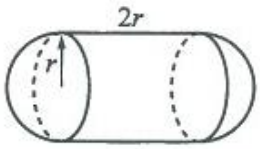
(a) Find the values of a, b and c. [2]
(b) Write down an expression, in terms of n, for the nth term. [1]
(c) Explain why 109 is not a term of this sequence. [1]
(N2014/P1/Q17)

17. (a) It is given that $W = \frac{1}{2}m(v^2 - u^2)$.
(i) Find W when $m = 3$, $u = 4$ and $v = 10$. [1]
(ii) Express u in terms of W, m and v. [2]
(b) Simplify $\frac{8xy + 2x^2}{x^2 - 16y^2}$. [2]
(N2014/P2/Q1b, c)

18. Simplify $\frac{4x}{3} - \frac{3(2-5x)}{4}$. [2]
(N2015/P1/Q5)

19. Factorise fully $4ax - 3ay - 8bx + 6by$. [2]
(N2015/P1/Q7)

20. (a) The surface area of a solid is given by $A = \pi p(2p + q)$.
Make q the subject of the formula. [2]
(b) Another solid is made from a cylinder and two hemispheres.
The cylinder has radius r and length $2r$ and the hemispheres have radius r.



The total surface area of the solid is twice the total surface area of a cone with radius r and slant height l.
Find l in terms of r. [3]
(N2015/P1/Q21)

21. (a) Factorise $9x^2 - 16y^2$. [1]

(b) Express as a single fraction in its simplest form

(i) $\frac{15xy}{12} \div \frac{9x^2}{4y}$, [1]

(ii) $\frac{6}{2x-3} - \frac{1}{x+2}$. [2]

(N2015/P2/Q1a, b)

22. (a) The first four terms in a sequence are 55, 51, 47 and 43.

(i) Find an expression, in terms of n, for the nth term, T_n, of this sequence. [2]

(ii) Evaluate T_{25}. [1]

(b) The diagram shows part of a number grid.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35					

A square outlining four numbers, as shown, can be placed anywhere on the grid.

(i) If n represents the number in the top left corner of the square, write down an expression, in terms of n, for the number in the bottom right corner of the square. [1]

(ii) Show that the difference between the products of the numbers in the opposite corners of the square is always 8. [2]

(iii) Show that the sum of the four numbers in the square cannot be 260. [3]

(N2015/P2/Q8)

23. (a) Simplify $3(2x - 1) + 1$. [1]

(b) Factorise $6x + 18xy$. [1]

(N2016/P1/Q1)

24. Factorise completely $4ax + 12by - 16ay - 3bx$. [2]

(N2016/P1/Q2)

25. n is a positive integer.

Show that, for all n, $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 20. [2]

(N2016/P1/Q4)

26. Write as a single fraction in its simplest from $\frac{3}{(x-4)^2} - \frac{1}{(4-x)}$. [2]

(N2016/P1/Q9)

27. (a) It is given that $a = \frac{4b - 5c}{b + c}$.

(i) Find a when $b = 5$ and $c = -2$. [1]

(ii) Express b in terms of a and c. [2]

(b) Simplify $\frac{9x^2 - 4}{3x^2 - 10x - 8}$. [3]

(N2016/P2/Q1a, d)

28. The first four terms in a sequence of numbers are given below.
$T_1 = 2^2 + 5 = 9$
$T_2 = 3^2 + 8 = 17$
$T_3 = 4^2 + 11 = 27$
$T_4 = 5^2 + 14 = 39$
(a) Find T_5. [1]
(b) Explain why the value of T_n must be odd for all values of n. [1]
(c) Show that the nth term of the sequence, T_n, is given by $n^2 + 5n + 3$. [3]
(d) T_p and T_{p+1} are consecutive terms in the sequence.
Find and simplify an expression, in terms of p, for $T_{p+1} - T_p$. [3]
(e) Explain why two consecutive terms of the sequence cannot have a difference of 4. [1]
(N2016/P2/Q4)

29. (a) Factorise completely $2x^2 - 5x - 12$. [2]
(b) Hence factorise completely $2(2y - 3)^2 - 5(2y - 3) - 12$.
Write your answer as simply as possible. [2]
(N2017/P1/Q14)

30. In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are
$$39 \quad p \quad q \quad r \quad 11 .$$
(a) Find the values of p, q and r. [2]
(b) Write down an expression for the nth term of this sequence. [2]
(c) Explain why –246 is not a term of this sequence. [1]
(N2017/P1/Q20)

31. $A = \dfrac{b(c+2)}{5-c}$
(a) Calculate the value of A when $b = 12.17$ and $c = 1.615$.
Write your answer correct to two decimal places. [2]
(b) Rearrange the formula to make c the subject. [4]
(N2017/P1/Q22)

32. (a) Express as a single fraction in its simplest form $\dfrac{2y}{5-2y} - \dfrac{3y}{(5-2y)^2}$. [2]
(b) Simplify $\dfrac{18h^3j^3}{5k^3} \div \dfrac{3h^5k}{10j^2}$. [2]
(c) Simplify $\left(\dfrac{16t^8}{v^{12}}\right)^{-\frac{1}{4}}$. [2]
(N2017/P2/Q1b, c, d)

33. Show that $(3n - 1)^2 + 2$ is a multiple of 3 for all integer values of n. [2]
(N2018/P1/Q5)

34. Factorise completely
(a) $8p^2q - 6pq^3$, [2]
(b) $6x^2y - 2xy + 3x - 1$. [2]
(N2018/P1/Q13)

35. Rearrange the formula $y = \frac{x^2+3}{x^2-a}$ to make x the subject. [4]
(N2018/P1/Q17)

36. (a) Write as a single fraction in its simplest form
(i) $\frac{5t^2}{v} \div \frac{25t}{v^3}$, [1]
(ii) $\frac{4}{3-2y} - \frac{5}{y+3}$. [2]
(b) Simplify $\frac{16x^2-9}{4x^2-9x-9}$. [3]
(N2018/P2/Q1a, b)

37. (a) These are the first four terms in a sequence.

11 17 23 29

(i) Find an expression, in terms of n, for the nth term of the sequence. [2]
(ii) Explain why it is not possible for a term in the sequence to be a multiple of 3. [1]
(b) The nth term of a difference sequence is given by $T_n = \frac{4n-1}{205-5n}$.
(i) Use the formula to find T_5.
Give your answer as a fraction. [1]
(ii) The value of T_k can be simplified to $\frac{7}{32}$.
Find the value of k. [3]
(iii) Find the least value of n for which $T_n > 1$. [3]
(N2018/P2/Q4)

38. Write as a single fraction in its simplest form $\frac{1}{2x-3} - \frac{3}{3x-1}$. [2]
(N2019/P1/Q6)

39. (a) Factorise $x^2 - y^4$. [1]
(b) Factorise completely $6ab + 1 - 3a - 2b$. [2]
(N2019/P1/Q11)

40. (a) The first four terms of a sequence are 8, 11, 14 and 17.
The sum of the first n terms of this sequence is given by $pn^2 + qn$.
(i) When $n = 1$, $p + q = 8$.
Show that $4p + 2q = 19$. [1]
(ii) Solve $p + q = 8$
$4p + 2q = 19$. [3]
(b) The sum of the first n terms of a different sequence is given by $3n^2 + 7n$.
Find the 10th term of this sequence. [2]
(N2019/P1/Q25)

41. (a) Simplify $\frac{4p^2r}{3} \div \frac{2r^3}{p}$. [1]

(b) $a = \frac{3b + 4c}{5 - b}$

(i) Evaluate a when $b = 6$ and $c = -2$. [1]

(ii) Express b in terms of a and c. [2]

(c) (i) Express $9 - 7x + x^2$ in the form $p + (q + x)^2$. [2]

(ii) Write down the coordinates of the minimum point of the graph of $9 - 7x + x^2$. [1]

(N2019/P2/Q1a, b, c)

42. In a sequence, S, the difference between consecutive terms is constant.
The third term of this sequence is 36.
The sixth term of this sequence is 60.

(a) Find an expression, in terms of n, for the nth term of this sequence. [2]

(b) Explain why the terms of the sequence are all multiples of 4. [1]

(N2019/P2/Q5b)