Topic 6

Functions and Graphs

1. (a) Sketch the graph of $y = -(x-3)(x+1)$. [2]

(b) Write down the equation of the line of symmetry of the graph. [1]
(c) Find the coordinates of the maximum point. [1]

(N2011/P1/Q21)

2. *Answer the whole of this question on a sheet of graph paper.*
The variables x and y are connected by the equation

$$y = \frac{1}{5}x^2(x-4).$$

Some corresponding values of x and y are given in the following table.

x	-2	-1	0	1	2	3	4	5
y	p	-1	0	-0.6	-1.6	-1.8	0	5

(a) Find the value of p. [1]
(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for
$-2 \leqslant x \leqslant 5$.
Using a scale of 1 cm to 1 unit, draw a vertical y-axis for
$-6 \leqslant y \leqslant 6$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]
(c) Use your graph to find three solutions of $\frac{1}{5}x^2(x-4) = -1$. [2]
(d) By drawing a tangent, find the gradient of the curve at the point (4, 0). [2]
(e) (i)   On the same axes, draw the line $y = 4 - 2x$ for $0 \leqslant x \leqslant 5$. [2]
(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]
(iii) This value of x is a solution of the equation
$x^3 - 4x^2 + Ax + B = 0$.
Find the value of A and the value of B. [2]

(N2011/P2/Q9)

3. The sketch shows the graph of $y = ka^x$.
    The points $(0, 5)$ and $(6, 320)$ lie on the graph.
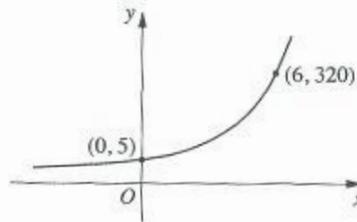
    Find the values of k and a. [2]

    (N2012/P1/Q5)

4. *Answer the whole of this question on a sheet of graph paper.*
    The variables x and y are connected by the equation

    $$y = \frac{x^3}{20} + \frac{10}{x} - 5.$$

    Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	1	2	3	4	5	6
y	5.05	0.40	−0.32	0.70	3.25	p

    (a) Calculate the value of p. [1]
    (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $1 \leqslant x \leqslant 6$.
        Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for
        $-1 \leqslant y \leqslant 8$.
        On your axes, plot the points given in the table and join them with a smooth curve. [3]
    (c) Use your graph to find the solutions to the equation $\frac{x^3}{20} + \frac{10}{x} = 5$,
        in the range $1 \leqslant x \leqslant 6$. [2]
    (d) By drawing a tangent, find the gradient of the curve at $(4, 0.70)$. [2]
    (e) Use your graph to find the coordinates of the minimum point of
        $y = \frac{x^3}{20} + \frac{10}{x} - 5$, in the range $1 \leqslant x \leqslant 6$. [2]

        (N2012/P2/Q7)

5. (a) Express $x^2 - 8x + 14$ in the form $(x - p)^2 - q$. [1]
    (b) Hence write down the minimum value of $x^2 - 8x + 14$. [1]
    (c) Write down the equation of the line of symmetry of the graph of $y = x^2 - 8x + 14$. [1]
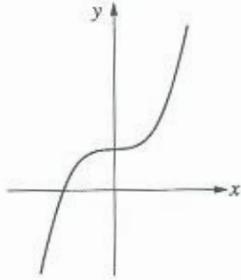
        (N2013/P1/Q10)

6.

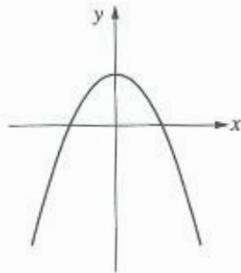|		
$y = x^3 - 5$	$y = 5 - x^3$	$y = 5^x$
$y = 5 - x^2$	$y = x^2 + 5$	$y = x^3 + 5$

Write down a possible equation for each of the sketch graphs below.
In each case, select one of the equations from the box above.
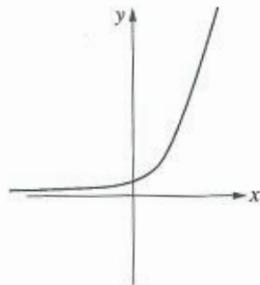
(a)

[1]

(b)

[1]

(c)

[1]
(N2013/P1/Q12)

7. *Answer the whole of this question on a sheet of graph paper.*
    A stone is thrown from the top of a cliff next to the sea. The height, h metres, of the stone above sea level t seconds after it is released can be modeled by the equation
    $h = 80 + 16t - 5t^2$.
    Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6
h	80	91	92	83	64	35	p

    (a) Calculate the value of p. [1]
    (b) Using a scale of 2 cm to represent 1 seconds, draw a horizontal t-axis for $0 \le t \le 6$.
         Using a scale of 1 cm to represent 10 metres, draw a vertical h-axis for $-20 \le h \le 100$.
         On your graph, plot the points given in the table and join them with a smooth curve. [3]
    (c) Use your graph to estimate
         (i)   the maximum height of the stone above sea level, [1]
         (ii) the length of time that the stone was more than 85 m above sea level, [2]
         (iii) the time taken for the stone to hit the water. [1]
    (d) By drawing a tangent, find the gradient of the curve at (4, 64).
         State the units of your answer. [3]
    (N2013/P2/Q5)

8. *Answer the whole of this question on a sheet of graph paper.*
    The variables x and y are connected by the equation
    $$y = \frac{x}{5}(2 + 4x - x^2).$$
    Some corresponding values of x and y are given in the table below.

x	-3	-2	-1	0	1	2	3	4	5
y	p	4	0.6	0	1	2.4	3	1.6	-3

    (a) Find the value of p. [1]
    (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-3 \le x \le 5$.
         Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-4 \le y \le 12$.
         On your axes, plot the points given in the table and join them with a smooth curve. [3]
    (c) Use your graph to find the solutions of $\frac{x}{5}(2 + 4x - x^2) = 2$. [2]
    (d) By drawing a tangent, find the gradient of the curve at the point (-2, 4). [2]
    (e) (i)   On the same axes, draw the line $3y = 2x - 6$ for $-3 \le x \le 5$. [2]
         (ii) Write down the coordinates of the point where the line intersects the curve. [1]
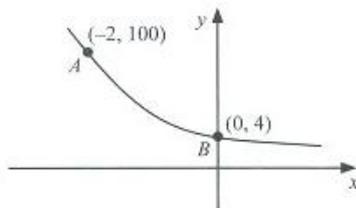   (N2014/P2/Q3)

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9. *Answer the whole of this question on a sheet of graph paper.*
    The variables x and y are connected by the equation

    $$y = \frac{x^2}{5} + \frac{4}{x} - 3.$$

    Some corresponding values of x and y, correct to two decimal places, are given in the table below.

x	0.5	1	1.5	2.0	2.5	3	4	5	6
y	p	1.20	0.12	−0.20	−0.15	0.13	1.20	2.80	4.87

    (a) Find the value of p. [1]
    (b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$ and a vertical y-axis for $-2 \leqslant y \leqslant 6$.
       On your axes, plot the points given in the table and join them with a smooth curve. [3]
    (c) Use your graph to find the solutions to the equation $\frac{x^2}{5} + \frac{4}{x} = 3$ in the range $0 \leqslant x \leqslant 6$. [2]
    (d) By drawing a tangent, find the gradient of the curve at (4, 1.20). [2]
    (e) (i) On the same axes, draw the line with gradient −1.5 that passes through the point with coordinates (2, 1). [1]
       (ii) Write down the equation of this line. [1]
       (iii) Write down the coordinates of the points where the line intersects the curve. [2]
       (N2015/P2/Q4)

10. (a) Express $9 - 8x + x^2$ in the form $p + (x + q)^2$. [2]
    (b) Write down the coordinates of the minimum point of the graph of $y = 9 - 8x + x^2$. [1]
       (N2016/P1/Q11)

11. The sketch shows the graph of $y = ka^{-x}$.
    The points $A(-2, 100)$ and $B(0, 4)$ lie on the graph.

    (a) Find the values of k and a. [2]
    (b) A straight line is drawn from A to B.
       Find the equation of the line AB. [2]
   (N2016/P1/Q17)

12. **Answer the whole of this question on a sheet of graph paper.**
    The variables x and y are connected by the equation

    $$y = \frac{x^3}{2} - 5x - 2.$$

    Some corresponding values of x and y are given in the table below.

 | x | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
    |---|---|---|---|---|---|---|---|---|
    | y | p | 4 | 2.5 | -2 | -6.5 | -8 | -3.5 | 10 |

    (a) Find the value of p. [1]
    (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-3 \leqslant x \leqslant 4$.
        Using a scale of 1 cm to represent 2 units, draw a vertical y-axis for $-10 \leqslant y \leqslant 12$.
        On your axes, plot the points given in the table and join them with a smooth curve. [3]
    (c) The equation $\frac{x^3}{2} - 5x = 8$ has only one solution.
        Explain how this can be seen from your graph. [2]
    (d) By drawing a tangent, find the gradient of the curve at $(1, -6.5)$. [2]
    (e) (i) On the same axes, draw the line $y = 4 - 3x$ for $-1 \leqslant x \leqslant 4$. [2]
        (ii) Write down the x-coordinate of the point where this line intersects the curve. [1]
        (iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.
            Find the value of A and the value of B. [2]

    (N2016/P2/Q5)

13. **Sketch** the graph of $y = -(x - 8)(x + 3)$ on the axes below.
    Indicate clearly the values where the graph crosses the x- and y- axes.
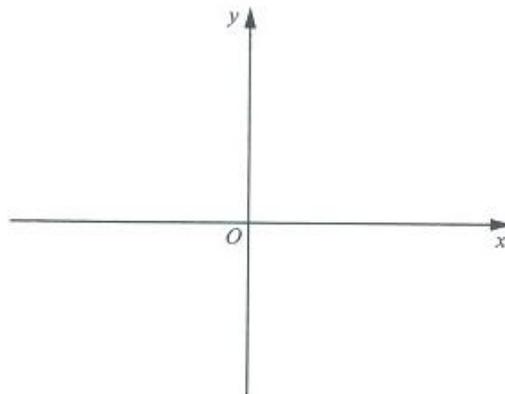
    [2]
    (N2017/P1/Q4)

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14. The graphs of $y = 2x^2 - 3x - 7$ and $y = 5 - 2x$ are drawn on the grid.
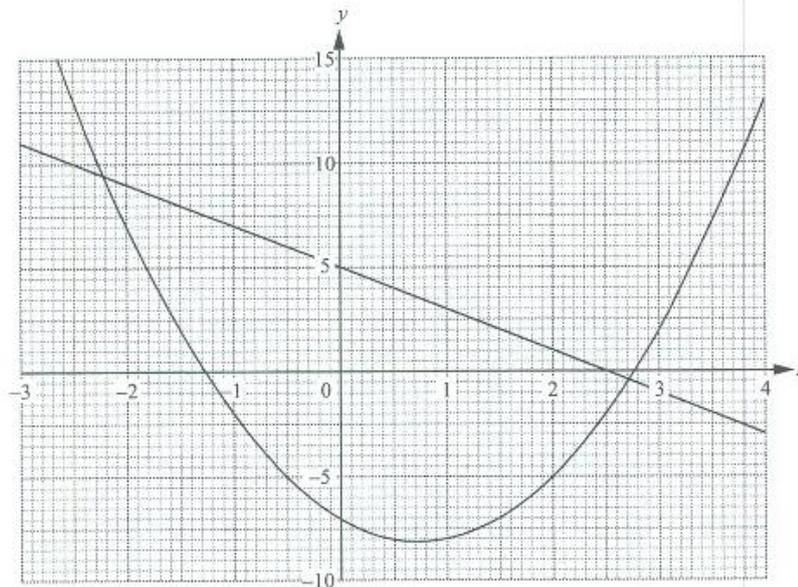
(a) Explain why the equation $2x^2 - 3x - 7 = k$ does not have solutions for some values of k. [1]

(b) The points of intersection of the curve and the straight line give the solutions of a quadratic equation.
Find the quadratic equation, giving your answer in the form $ax^2 + bx + c = 0$. [1]

(c) The equation $2x^2 - 6x - 3 = 0$ can be solved by drawing a suitable straight line on the grid.
   (i) Find the equation of the straight line. [1]
   (ii) By drawing this straight line, solve the equation $2x^2 - 6x - 3 = 0$. [3]

(N2017/P1/Q21)

15. **Answer the whole of this question on a sheet of graph paper.**
A number of bacteria are introduced to a culture.
The number of bacteria, y, in the culture t hours after they are first introduced is given by the formula

$$y = 50 \times 2^t.$$

The table shows some corresponding values of t and y, correct to 3 significant figures.

t	0.5	1	1.5	2	2.5	3	3.5	4
y	p	100	141	200	283	400	566	800

(a) How many bacteria are introduced to the culture at the start? [1]

(b) Calculate the value of p. [1]

(c) Using a scale of 4 cm to represent 1 hour, draw a horizontal t-axis for $0 \leqslant t \leqslant 4$.
Using a scale of 2 cm to represent 100 bacteria, draw a vertical y-axis for $0 \leqslant y \leqslant 800$.
On your axes, draw a graph to show the number of bacteria in the culture for $0 \leqslant t \leqslant 4$. [3]

(d) Use your graph to find how many hours it takes for the number of bacteria to reach 500. [1]

(e) (i) By drawing a tangent, find the gradient of the curve at (2, 200). [2]
(ii) What does this gradient represent? [1]

(N2017/P2/Q3)

16. The graph of $y = \frac{1}{2}(x - x^2)$ is drawn on the grid.
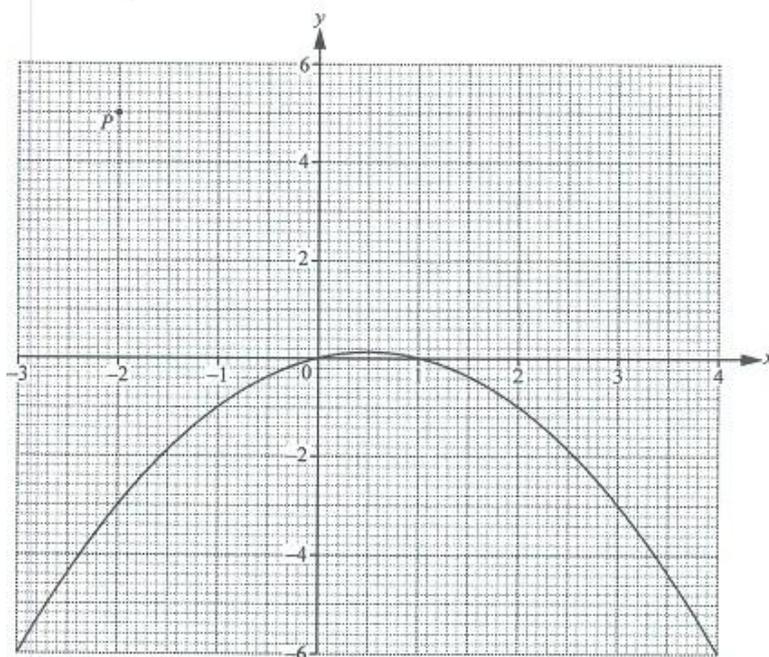
(a) Write down the equation of the line of symmetry of the curve. [1]
(b) Use the graph to solve the equation $x - x^2 = -4$. [2]
(c) The point P has coordinates $(-2, 5)$.
A tangent to the curve can be drawn so that the tangent passes through P.
(i) Draw this tangent on the grid above. [1]
(ii) Find the equation of this tangent. [2]

(N2018/P1/Q23)

17. **Sketch** the graph of $y = -(x-2)^2 + 9$ on the axes below.
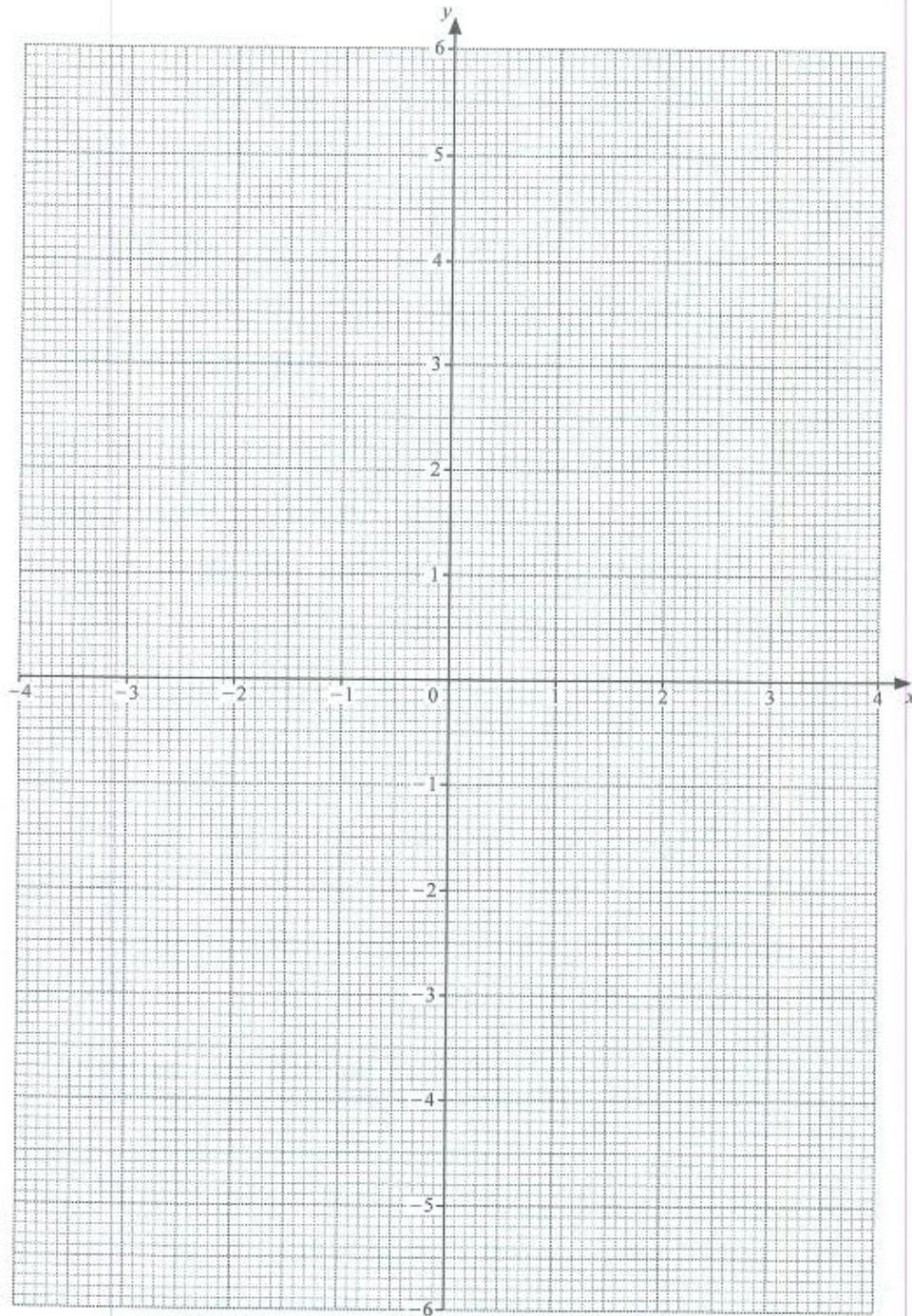Indicate clearly the coordinates of the points where the graph crosses the axes and the maximum point on the curve.
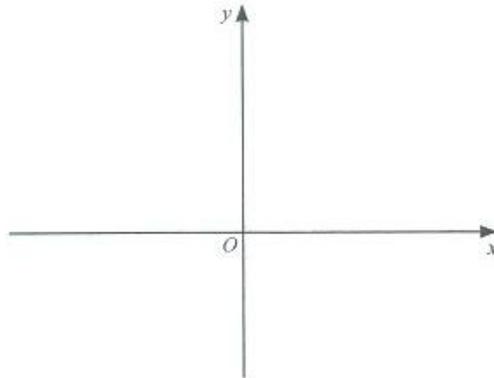
[3]
(N2019/P1/Q15)

18. The variables x and y are connected by the equation $y = \frac{x^3}{5} - 2x + 1$.
Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
y	p	1.6	3.4	2.8	1	-0.8	-1.4	0.4	5.8

(a) Find the value of p. [1]
(b) On the grid opposite, draw the graph of $y = \frac{x^3}{5} - 2x + 1$ for $-4 \leqslant x \leqslant 4$. [3]
(c) Use your graph to write down an inequality in x to describe the range of values where $y > 4$. [1]
(d) (i) On the same grid, draw the graph of $5y + x = 10$ for $-4 \leqslant x \leqslant 4$. [2]
    (ii) Show that the points of intersection of the line and the curve give the solutions of the equation $x^3 - 9x - 5 = 0$. [2]
    (iii) Use your graphs to solve the equation $x^3 - 9x - 5 = 0$. [2]

(N2019/P2/Q4)