EPH O LEVEL YEARLY MATHEMATICS
WORKED SOLUTIONS (2011–2020)

Must-Know Concepts INCLUDED Questions Analysis

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2020 EXAMINATION PAPER

Paper 1

1. −1.46 (3 s.f.)

2. Diagram 3

3. Angle BOA = 66° (alt. ∠s, OA // CB)
   Angle OBA = 90° (tangent ⊥ radius)
   Angle OAB = 180° − 90° − 66° (∠ sum in △)
   = **24°**

4. (a) The vertical axis does not start from 0.

   (b) The decrease in newspaper circulation is exaggerated, making percentage fall seem more than it actually is.

5. (a) $4^5 = (2^2)^5$
   $= \mathbf{2^{10}}$

   (b) $\frac{4a^2}{3b} \div \frac{10ab}{21} = \frac{{}^{2}\cancel{4}a^{\cancel{2}}}{\cancel{3}b} \times \frac{\cancel{21}^{7}}{{}_{5}\cancel{10}ab}$
   $= \mathbf{\frac{14a}{5b^2}}$

6. Let the length of one side of the hexagon and the triangle be l_h and l_t respectively.

   $\frac{6l_h}{3l_t} = \frac{1}{2}$
   $12l_h = 3l_t$
   $l_t = 4l_h$
   Ratio of area of hexagon : triangle
   $= 6 \times \frac{1}{2}(l_h)^2 \sin 60° : \frac{1}{2}(4l_h)^2 \sin 60°$
   $= \frac{3\sqrt{3}}{2}l_h^2 : \frac{8\sqrt{3}}{2}l_h^2$
   = **3 : 8**

   Must-Know Concept:
   Area of triangle = $\frac{1}{2}ab \sin c$
   Use the exact value of $\sin 60° = \frac{\sqrt{3}}{2}$

7. (a) 12th man → **83** minutes

   (b) 75 + 33 = **108** minutes, or 102 − 33 = **69** minutes

8. (a) P(a yellow counter) = 1 − 0.45 − 0.3
   = **0.25**

   (b) Let the total number of counters in the bag be x.

   $0.45x - 0.3x = 9$
   $0.15x = 9$
   $x = \mathbf{60}$

9. $\frac{x}{5} - \frac{2x-3}{4} = -3$
   $\frac{4x - 5(2x-3)}{20} = -3$
   $4x - 10x + 15 = -60$
   $-6x = -75$
   $x = \mathbf{12.5}$

10. (a) $x^2 + 9x - 4 = \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 4 = \left(x + \frac{9}{2}\right)^2 - \frac{97}{4}$

    (b) $\left(-\frac{9}{2}, -\frac{97}{4}\right)$

    Must-Know Concept:
    Use completing the square to solve for question.

11. (a) nth term = $11 + 9(n-1)$
    $= 11 + 9n - 9$
    $= \mathbf{2 + 9n}$

    (b) $2 + 9n = 335$
    $9n = 333$
    $n = \mathbf{37}$

    Must-Know Concept:
    The pattern in the sequence is by adding 9 to obtain the next number.

12. Amount that Kim receives = $\frac{3}{3+2}$ × \$285
    = **\$171**

    Amount that Pat and Xin receive = \$285 − \$171
    = \$114

    Amount that Pat receives = (\$114 + \$24) ÷ 2
    = \$69

    Amount that Xin receives = \$69 − \$24
    = **\$45**

13. (a) $y = 16 - 4(-2)^2$
$= \mathbf{0}$

(b) $y = 16 - 4x^2$
$4x^2 = 16 - y$
$x^2 = \frac{16-y}{4}$
$x = \pm\frac{\sqrt{16-y}}{2}$

Must-Know Concept:
Make x the subject by shifting terms that have x to the left-hand side and terms that do not have x to the right-hand side.

14. Percentage of tank that is filled with water
$= \frac{84}{100} \times 60\%$
$= 50.4\%$

Capacity of the tank $= 378 \div \frac{50.4}{100}$
$= \mathbf{750}$ litres

15. $850\left(1 + \frac{r}{100}\right)^{12} = 1120$
$1 + \frac{r}{100} = \sqrt[12]{\frac{1120}{850}}$
$r = \mathbf{2.33}$ (3 s.f.)

16. (a) (i) $A \cap B = \{\mathbf{s, a, r, e}\}$

(ii) $(A \cup B)' = \varnothing$

(b) $(P \cap Q)'$

17. (a) 1 cm represents 20 km.

Distance on map $= \frac{950}{20}$
$= \mathbf{47.5}$ cm

(b) 1 cm : 20 km
1 cm^2 : 400 km^2

Area on map $= \frac{330\ 803}{400}$
$= \mathbf{827.0075}$ cm^2

Must-Know Concept:
Convert the scale to km^2 to solve for part (b).

18. $\tan 36° = \frac{45}{BC}$
$BC = \frac{45}{\tan 36°}$
$= 61.937$ m

Angle of elevation $= \tan^{-1}\frac{45}{61.937 + 22}$
$= \mathbf{28.2°}$ (1 d.p.)

19. (a) $3(3x + 2y) - 5(x - 3y) = 9x + 6y - 5x + 15y$
$= \mathbf{4x + 21y}$

(b) $12ab - 9ax - 8by + 6xy$
$= 3a(4b - 3x) - 2y(4b - 3x)$
$= \mathbf{(3a - 2y)(4b - 3x)}$

Must-Know Concept:
Expand the brackets before simplifying the expression. Take note of the change in sign when putting into brackets.

20. (a) $1188 = \mathbf{2^2 \times 3^3 \times 11}$

(b) (i) LCM $= p^2 \times q^{r+2} \times 11$

$p = \mathbf{2}, q = \mathbf{3}, r = \mathbf{1}$

(ii) HCF $= 2 \times 3 \times 11$
$= \mathbf{66}$

21. (a) (i) Angle EDC = **102°** because **angle EDC + angle AED = 180° (int. ∠s, AE // CD).**

(ii) Angle BCD = **96°** because **sum of interior angles of a pentagon = (5 – 2) × 180° = 540°.**

(b) Angle BAC = angle BCA
$= (180° - 144°) \div 2$
$= 18°$

Angle $EAC = 120° - 18°$
$= 102°$

Since angle EAC + angle AED = 180°, AC is parallel to ED (interior angles).
$ACDE$ is a parallelogram as AC is parallel to ED and AE is parallel to CD.

22. (a) Angle DBG = angle GBA
= 90°
Angle BGD = angle BAG (given)

By AA similarity test, triangle DBG is similar to triangle GBA.

$\frac{36}{24} = \frac{BD}{36}$
$BD = 36 \times 36 \div 24$
$= 54$ m **(shown)**

(b) $\frac{1}{2} \times (30 + CF) \times (37 + 24) = 1586$
$61(30 + CF) = 3172$
$30 + CF = 52$
$CF = 22$ m

Total area of the field $AEFDG$
$= \frac{1}{2}(22)(54 - 37) + \frac{1}{2}(54 + 24)(36) + 1586$
$= 187 + 1404 + 1586$
$= \mathbf{3177}$ m^2

23. (a) Mean = [(6 × 2.5) + (13 × 7.5) + (32 × 11.5) + (62 × 14.5) + (22 × 18) + (5 × 22.5)] ÷ 140
= 1888 ÷ 140
= **13.5** hours (3 s.f.)

(b) (i) 40% × 140 = 56
140 − 56 = 84
n = **14.75**

(ii) P(the adult watches between 8 and 13.5 hours)
$= \frac{60-12}{140}$
$= \frac{48}{140}$
$= \mathbf{\frac{12}{35}}$

Must-Know Concept:
Use the formula $\frac{\Sigma ft}{\Sigma f}$ to find the mean since we are given the frequency and values of t.

24. (a) $\vec{OP} = \vec{OC} + \vec{CP}$
$= \mathbf{c} + m(\mathbf{a} - \mathbf{c})$
$= \mathbf{ma + (1 - m)c}$

(b) (i) $\vec{OB} = 4[m\mathbf{a} + (1 - m)\mathbf{c}] = 4\,\vec{OP}$
$\vec{OB}$ is parallel to $\vec{OP}$

Since O is a common point, O, B and P lie on a straight line. **(shown)**

(ii) $\vec{CB} = \vec{OB} - \vec{OC}$
$k\mathbf{a} = 4m\mathbf{a} + 4(1 - m)\mathbf{c} - \mathbf{c}$
$k\mathbf{a} = 4m\mathbf{a} + (3 - 4m)\mathbf{c}$

$3 - 4m = 0$.............. (1)
$k = 4m$..................... (2)

From (1), $m = \mathbf{\frac{3}{4}}$
Sub into (2), $k = \mathbf{3}$

(iii) $\vec{CP} = \frac{3}{4}(\mathbf{a} - \mathbf{c})$
$= \frac{3}{4}\vec{CA}$
$CP : CA = \mathbf{3 : 4}$

Paper 2

1. (a) $\frac{2x+1}{2} \geqslant \frac{5-4x}{3}$
$3(2x + 1) \geqslant 2(5 - 4x)$
$6x + 3 \geqslant 10 - 8x$
$14x \geqslant 7$
$\mathbf{x \geqslant \frac{1}{2}}$

(b) $6x - 3y = 16$ .... (1)
$9x + 2y = 11$ .... (2)

(1) × 2 + (2) × 3: $12x + 27x = 32 + 33$
$39x = 65$
$x = \mathbf{\frac{5}{3}}$

Sub $x = \frac{5}{3}$ into (1): $10 - 3y = 16$
$3y = 10 - 16$
$y = \mathbf{-2}$

(c) $\frac{x}{(3-2x)^2} - \frac{5}{3-2x} = \frac{x - 5(3-2x)}{(3-2x)^2}$
$= \frac{x - 15 + 10x}{(3-2x)^2}$
$= \mathbf{\frac{11x - 15}{(3-2x)^2}}$

(d) $\left(\frac{a^9}{27b^{15}}\right)^{-\frac{1}{3}} = \left(\frac{27b^{15}}{a^9}\right)^{\frac{1}{3}}$
$= \mathbf{\frac{3b^5}{a^3}}$

(e) $\frac{4x^2 - 16}{3x^2 + x - 10} = \frac{4(x+2)(x-2)}{(3x-5)(x+2)}$
$= \mathbf{\frac{4(x-2)}{3x-5}}$

Must-Know Concept:
Perform cross multiplication across the inequality. Do take note of the signs of the terms. Solve for x.
Use the Law of Indices: $\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}$.

2. (a) Number of girls = 16 − 9
= **7**

(b) $F = \begin{pmatrix} \mathbf{30} \\ \mathbf{26} \\ \mathbf{24} \end{pmatrix}$

(c) $M = \begin{pmatrix} 10 & 12 & 16 \\ 14 & 16 & 20 \end{pmatrix}\begin{pmatrix} 30 \\ 26 \\ 24 \end{pmatrix}$
$= \begin{pmatrix} 300 + 312 + 384 \\ 420 + 416 + 480 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{996} \\ \mathbf{1316} \end{pmatrix}$

(d) Elements of M represents the amount of fees collected for the morning and afternoon session respectively.

(e) Total amount in one week = 5 × ($996 + $1316)
= **$11 560**

(f) Total amount taken in week 2

$= 5 \times \left(\frac{75}{100} \times 24 \times 30 + \frac{150}{100} \times 28 \times 26 + \frac{75}{100} \times 36 \times 24\right)$

$= \$14\ 100$

Percentage change $= \frac{14\ 100 - 11\ 560}{11\ 560} \times 100\%$

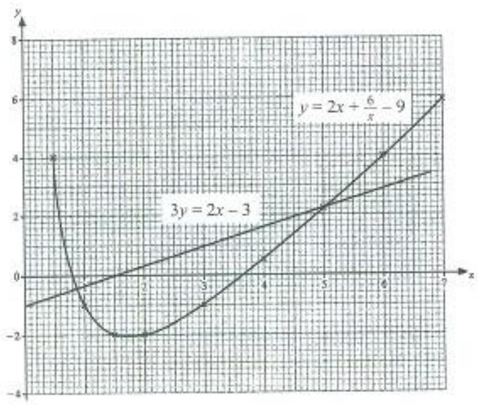
$=$ **22.0%** (3 s.f.)

Must-Know Concept:
The product of a 2 × 3 matrix and a 3 × 1 matrix gives a resulting 2 × 1 matrix.

3. (a)

x	0.5	1	1.5	2	3	4	5	6	7
y	4	–1	–2	–2	–1	0.5	2.2	4	**5.9**

(b)



(c) $2x + \frac{6}{x} = 10$

$2x + \frac{6}{x} - 9 = 10 - 9$

When $y = 1$,

$x =$ **0.7** or **4.3**

(d) (i) $3y = 2x - 3$

$y = \frac{2}{3}x - 1$

Graph as shown in part (b).

(ii) $x =$ **0.9** or **5.1**

(iii) $2x + \frac{6}{x} - 9 = \frac{2}{3}x - 1$

$6x^2 + 18 - 27x = 2x^2 - 3x$

$4x^2 - 24x + 18 = 0$

$2x^2 - 12x + 9 = 0$

$A =$ **–12**, $B =$ **9**

Must-Know Concept:
Equate both $y = 2x + \frac{6}{x} - 9$ and $3y = 2x - 3$ to get the values for A and B.

4. (a) $AC^2 = 660^2 + 950^2 - 2(660)(950)\cos 80°$

$AC = \sqrt{660^2 + 950^2 - 2(660)(950)\cos 80°}$

$= 1058.46$

$= 1060$ m (3 s.f.) **(shown)**

(b) $\frac{\sin \angle ADC}{1058.46} = \frac{\sin 24°}{480}$

$\sin \angle ADC = 0.896\ 91$

$\angle ADC = 180 - \sin^{-1} 0.896\ 91$ (angle ADC is obtuse)

$= 116.245°$

$\angle DAC = 180° - 116.245° - 24°$ ($\angle$ sum in $\triangle$)

$= 39.755°$

Bearing of D from C $= 360° - 24° - 39.755°$

$=$ **296.2°** (1 d.p.)

(c) Time taken $= \frac{660 + 950 + 1058.46}{9500}$

$= 0.280\ 89$ hour

$=$ **16** min **50** seconds (to the nearest ten seconds)

Must-Know Concept:
Use the Cosine rule to solve for AC.

5. (a) Slant height $l = \sqrt{8^2 + 5.5^2}$

$= 9.7082$ cm

Curved surface area $= \pi \times 5.5 \times 9.7082$

$=$ **168** cm^2 (3 s.f.)

(b) (i) $\frac{\text{Height of water}}{\text{Height of glass}} = \frac{6}{8}$,

ratio of volume is $\left(\frac{3}{4}\right)^3 \neq 75\%$.

(ii) Percentage of total capacity that is filled

$= \left(\frac{3}{4}\right)^3 \times 100\%$

$=$ **42.2%** (3 s.f.)

(iii) Volume of water $= \frac{1}{3} \times \pi \times \left(\frac{11}{2}\right)^2 \times 8 \times \left(\frac{3}{4}\right)^3$

$= 106.91$ cm^3

Let the radius of the cylindrical glass be r cm.

$\pi \times r^2 \times 2.5 = 106.91$

$r =$ **3.69** cm (3.s.f.)

Must-Know Concept:
We can apply Pythagoras' Theorem to find the slant height.

6\. (a) (i) $BF = CF$ (tangents from external point)
$DF = EF$ (tangents from external point)
Angle BFD = angle CFE (vert. opp. $\angle$s)

By SAS test, triangle BDF is congruent to triangle CEF. (shown)

(ii) (a) Angle OBC = $\mathbf{(90 - x)^\circ}$ ($\angle$ in semicircle)

(b) Angle DPE
$= 360^\circ - (2 \times 90^\circ)$ – angle DFE (tangent $\perp$ radius)
$= 360^\circ - (2 \times 90^\circ)$ – angle BFC (vert. opp. $\angle$s)
$= 180^\circ - (180 - 2x)^\circ$
$= \mathbf{2x^\circ}$ (exterior angle of triangle)

(b) (i) Major arc $LM = 12 \times (2\pi - 1.8)$
$= \mathbf{53.8}$ **cm** (3 s.f.)

(ii) Area of shaded region
$= \left(\frac{1}{2} \times 12^2 \times 1.8\right) - \left(\frac{1}{2} \times 12^2 \times \sin 1.8\right)$
$= 59.483 \text{ cm}^2$

Percentage of circle that is shaded
$= \frac{59.483}{\pi \times 12^2} \times 100\%$
$= \mathbf{13.1\%}$ (3 s.f.)

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians). Subtract the area of the triangle LKM from the area of the sector LKM to find the shaded part.

7\. (a) $p + 10 + 13 + 9 + 6 + q + 2 = 50$
$p + q = 10$ ....(1)

$0 \times p + 10 + 26 + 27 + 24 + 5q + 12$
$= 2.68 \times 50$ ....(2)
From (2): $q = \mathbf{7}$
Sub $q = 7$ into (1): $p = \mathbf{3}$

(b) S.D. $= \sqrt{\frac{486}{50} - 2.68^2}$
$= \mathbf{1.59}$ (3 s.f.)

(c) 1. Students watch more movies than adults on average as the mean for students is higher.
2. The number of movies watched by students is more consistent than the number of movies watched by adults as the S.D. for students is lower.

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$ to find the standard deviation since we have the required values.

8\. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= \begin{pmatrix} -4 \\ 5 \end{pmatrix} - \begin{pmatrix} -7 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

Length of $AB = \sqrt{3^2 + 6^2}$
$= \mathbf{6.71}$ **units** (3 s.f.)

(b) $\overrightarrow{CD} = \overrightarrow{BA} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Gradient of $CD = \frac{-6}{-3} = 2$

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$
$= \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$C = (4, 3)$

Sub $x = 4$, $y = 3$ into $y = 2x + c$

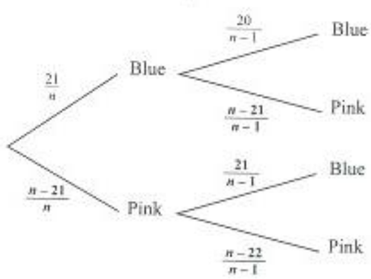
$3 = 8 + c$
$c = -5$

Equation of CD is $\mathbf{y = 2x - 5}$.

(c) (i) $\overrightarrow{XC} = \frac{1}{2}\overrightarrow{AC}$
$= \frac{1}{2}\left[\begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -7 \\ -1 \end{pmatrix}\right]$
$= \begin{pmatrix} \mathbf{5.5} \\ \mathbf{2} \end{pmatrix}$

(ii) $\overrightarrow{OX} = \overrightarrow{OC} - \overrightarrow{XC}$
$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 5.5 \\ 2 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{-1.5} \\ \mathbf{1} \end{pmatrix}$

9\. (a)



(b) $\frac{n-21}{n}\left(\frac{n-22}{n-1}\right) = \frac{1}{8}$

$8(n-21)(n-22) = n(n-1)$

$8(n^2 - 43n + 462) = n^2 - n$

$7n^2 - 343n + 3696 = 0$

$n^2 - 49n + 528 = 0$ **(shown)**

(c) $n^2 - 49n + 528 = 0$

$(n-33)(n-16) = 0$

$n =$ **33** or **16**

(d) 16 must be rejected as $n \geqslant 21$.

(e) P(Leon takes one blue ball and one pink ball)

$= \left(\frac{21}{33} \times \frac{12}{32}\right) + \left(\frac{12}{33} \times \frac{21}{32}\right)$

$= \mathbf{\frac{21}{44}}$

10. (a) (i) $1.824

(ii) Difference between greatest and least possible value

= 217 × 1.86 − 217 × 1.796

= **$13.89** (to nearest cent)

(b) Total cost per year

= (217 + 60 + 57 + 25 + 15) × 52 + (12 × 56.90)

= £20 130.80

Average exchange rate in past 2 years = 0.553 75

Estimated cost $= \frac{20\,130.8}{0.553\,75} \times 115\%$

= $41 806.63

A suitable amount to give Ying for the year is **$41 807**.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2019 EXAMINATION PAPER

Paper 1

1. 350

2. The widths of the pictures are not constant, leading to an exaggerated increase in card spending if the area of the cards are used for comparison instead of height.

3. (a) x^4
(b) $3^a = 3^7 + 3^7 + 3^7$
$3^a = 3^7 \times 3$
$3^a = 3^8$
$a = \mathbf{8}$

4. (a) $360 = \mathbf{2^3 \times 3^2 \times 5}$
(b) $p = \mathbf{3}, q = \mathbf{5}$

Must-Know Concept:
For a number to be a perfect cube, powers of all its prime factors have to be multiples of 3.

5. $3200(1 + r\%)^6 = 3890$
$(1 + r\%)^6 = \frac{3890}{3200}$
$1 + r\% \approx 1.033\ 078$
$r = \mathbf{3.31}$ (3 s.f.)

6. $\frac{1}{2x-3} - \frac{3}{3x-1}$
$= \frac{(3x-1)-3(2x-3)}{(3x-1)(2x-3)}$
$= \frac{3x-1-6x+9}{(3x-1)(2x-3)}$
$= \mathbf{\frac{-3x+8}{(3x-1)(2x-3)}}$

7. (a) Probability that it is not a yellow counter
$= \frac{8+9}{8+9+3}$
$= \mathbf{\frac{17}{20}}$

(b) $\frac{8}{20-x} = \frac{2}{3}$
$24 = 40 - 2x$
$2x = 16$
$x = \mathbf{8}$

8. $3p^2 + p - 10 = 0$
$(3p-5)(p+2) = 0$
$p = \mathbf{\frac{5}{3}}$ or $\mathbf{-2}$

9. (a) $33\% < \frac{1}{3}$

(b) $6 \leqslant 2x + 11$ and $2x + 11 < 19$
$2x \geqslant -5$ and $2x < 8$
$x \geqslant -\frac{5}{2}$ $\quad x < 4$
$\mathbf{-\frac{5}{2} \leqslant x < 4}$

10. (a) $P = kQ^3$
$2 = k\left(\frac{1}{3}\right)^3$
$\therefore k = 54$

When $Q = \frac{1}{6}$, $P = 54\left(\frac{1}{6}\right)^3$
$= \mathbf{\frac{1}{4}}$

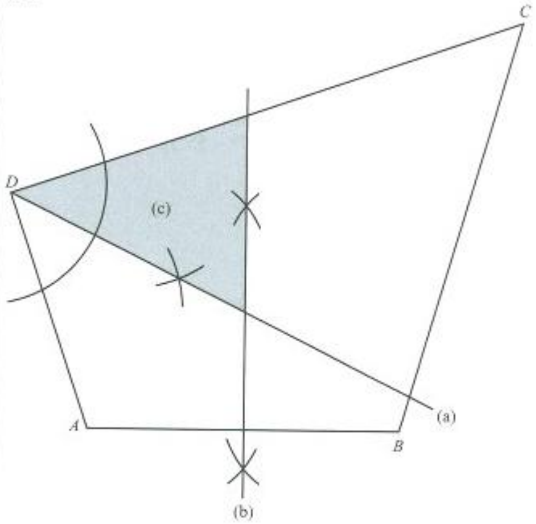
(b) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$
$m = \mathbf{\frac{1}{8}}$

Must-Know Concept:
The factor here refers to scale factor.

11. (a) $x^2 - y^4 = x^2 - (y^2)^2$
$= \mathbf{(x + y^2)(x - y^2)}$

(b) $6ab + 1 - 3a - 2b$
$= (6ab - 3a) + (1 - 2b)$
$= 3a(2b-1) - (2b-1)$
$= \mathbf{(3a-1)(2b-1)}$

12.



13. (a) $\vec{OA} = \vec{OB} - \vec{AB}$

$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -9 \end{pmatrix}$

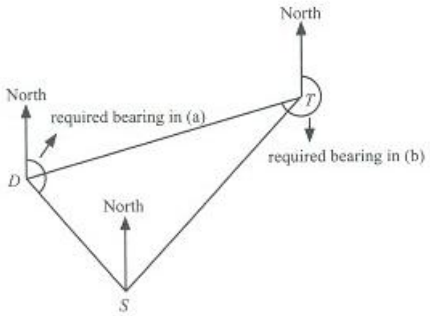
A = **(8, –9)**

Must-Know Concept:
Translation from A to B means direction vector.

(b) $\left|\begin{pmatrix} -3 \\ 7 \end{pmatrix}\right| = \sqrt{(-3)^2 + 7^2}$

= **7.62 units** (3 s.f.)

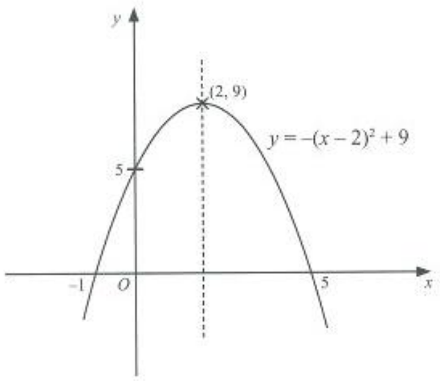
14. From the diagram,



(a) Bearing of Singapore from Delhi
= 180° – (360° – 317°)
= **137°**

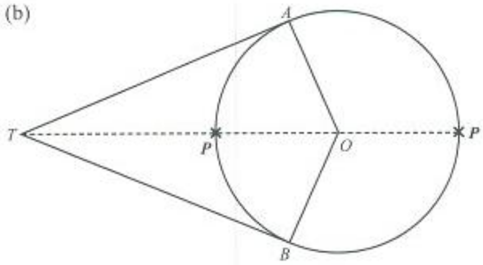
(b) Bearing of Delhi from Tokyo
= 360° – [180° – (137° – 55°)]
= **262°**

15.



16. (a) $\angle TAO = \angle TBO = 90°$ (tangent ⊥ radius)
$AT = BT$ (tangents from external point)
OT is a common side.
$\triangle OAT$ is congruent to $\triangle OBT$ (RHS). (shown)

(b)



17. (a) Nur's speed = **3.6** km/h

(b) (i) 12.24 p.m.
(ii) Distance from village B
= 14.8 – 6.8
= **8** km

(c) 8 km/h = $\frac{8000 \text{ m}}{3600 \text{ s}}$

$= \mathbf{2\frac{2}{9}}$ m/s

18. $\mathscr{E}$ = {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
P = {5, 7, 11, 13}
Q = {8, 12}

(a) (i) R = **{5, 6, 10, 15}**
(ii) $P \cap R$ = **{5}**

(b) $R \cap Q = \varnothing$ and $16 \notin P$

Must-Know Concept:
The universal set includes all possible elements in this question.

19. Let Ann's savings be \$7x and Ben's savings be \$5x.

$\frac{7x - 60}{5x - 60} = \frac{3}{2}$

$14x - 120 = 15x - 180$

$x = 60$

Ann's current savings = \$(7 × 60) – \$60
= **\$360**

Must-Know Concept:
Ratios can be written as $a : b$ or $\frac{a}{b}$.

20. (a) $\frac{\text{Height of the smaller bottle}}{30} = \frac{4}{10}$

Height of the smaller bottle = **12** cm

(b) $\frac{125}{500} = \left(\frac{4}{\text{diameter of the base}}\right)^3$

Diameter of the base = **6.35 cm** (3 s.f.)

Must-Know Concept:
For two solids that are geometrically similar, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$, where V_1 and V_2 are their volumes respectively and l_1 and l_2 are the lengths of their corresponding sides respectively.

21\. (a) $Q = \begin{pmatrix} \mathbf{24} & \mathbf{14} \\ \mathbf{29} & \boldsymbol{x} \\ \mathbf{20} & \mathbf{15} \end{pmatrix}$

(b) $T = QP$

$= \begin{pmatrix} 24 \times 10 + 14 \times 40 & 24 \times 8 + 14 \times 45 \\ 29 \times 10 + x \times 40 & 29 \times 8 + x \times 45 \\ 20 \times 10 + 15 \times 40 & 20 \times 8 + 15 \times 45 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{800} & \mathbf{822} \\ \mathbf{40x + 290} & \mathbf{45x + 232} \\ \mathbf{800} & \mathbf{835} \end{pmatrix}$

(c) $40x + 290 + 70 = 800$
$40x = 800 - 70 - 290$
$40x = 440$
$x = \mathbf{11}$

22\. Gradient of $AB = \frac{1-(-5)}{-2-7}$
$= -\frac{2}{3}$

Gradient of $PQ = -1 \div \left(-\frac{2}{3}\right)$
$= 1.5$

Sub (5, 4) into $y = 1.5x + c$

$4 = 7.5 + c$
$c = -3.5$

Equation of PQ is $\mathbf{y = 1.5x - 3.5}$

23\. (a) Length = 6 cm
Breadth = 5 cm
Area of rectangle on the floor plan $= 6 \times 5$
$= 30 \text{ cm}^2$

30 cm² represents 19.2 m²
1 cm² represents 0.64 m²
1 cm represents 0.8 m
$n = \mathbf{0.8}$

Must-Know Concept:
If 1 : n is the scale for length, then 1 : n^2 is the scale for area.

(b) Amount of varnish needed $= \frac{19.2}{16}$
$= 1.2\ l$

Least amount of money Cheryl needs to pay
(2 × 500 ml tin and 1 × 250 ml tin)
= 2 × \$42.50 + \$24
= **\$109**

24\. (a) (i) 180 cm to 200 cm

(ii) Mean $= \frac{4 \times 110 + 5 \times 130 + 8 \times 150 + 11 \times 170 + 14 \times 190 + 12 \times 210 + 6 \times 230}{60}$
$= \frac{10\,720}{60}$
$= \mathbf{178\frac{2}{3}}$ cm

(b) (i) Number of trees with a height of at least 1.92 m
= 60 – 36
= **24**

(ii) Interquartile range = 204 – 156
= **48** cm

(c) The trees in the second set have heights that are more consistent since the interquartile range is smaller.

Must-Know Concept:
The smaller the interquartile range, the more consistent the data is.

25\. (a) (i) When $n = 2$, $4p + 2q = 8 + 11$
$\mathbf{4p + 2q = 19}$ (shown)

(ii) $p + q = 8$..................(1)
$4p + 2q = 19$.................(2)

(1) × 2 – (2):

$-2p = -3$
$p = \mathbf{1.5}$

$q = 8 - 1.5$
$= \mathbf{6.5}$

(b) The 10th term of this sequence
= sum of first 10 terms – sum of first 9 terms
$= [3(10)^2 + 7(10)] - [3(9)^2 + 7(9)]$
= 370 – 306
= **64**

Paper 2

1\. (a) $\frac{4p^2r}{3} \div \frac{2r^3}{p} = \frac{4p^2r}{3} \times \frac{p}{2r^3}$
$= \mathbf{\frac{2p^3}{3r^2}}$

(b) (i) $a = \frac{3(6) + 4(-2)}{5 - 6}$
$= \frac{18 - 8}{-1}$
$= \mathbf{-10}$

(ii) $a = \frac{3b + 4c}{5 - b}$
$5a - ab = 3b + 4c$
$5a - 4c = 3b + ab$
$(3 + a)b = 5a - 4c$
$b = \mathbf{\frac{5a - 4c}{3 + a}}$

(c) (i) $9 - 7x + x^2 = \left(x - \frac{7}{2}\right)^2 - \left(-\frac{7}{2}\right)^2 + 9$
$= \mathbf{-3\frac{1}{4} + \left(-\frac{7}{2} + x\right)^2}$

(ii) $\left(\frac{7}{2}, -3\frac{1}{4}\right)$

(d) $\frac{1}{x-3} + \frac{6}{x-1} = 2$
$x - 1 + 6(x - 3) = 2(x - 3)(x - 1)$
$x - 1 + 6x - 18 = 2(x^2 - 4x + 3)$
$2x^2 - 15x + 25 = 0$
$(2x - 5)(x - 5) = 0$
$x = \mathbf{2.5}$ or $\mathbf{5}$

2. (a) $CX^2 = 8.3^2 + 6.4^2 - 2(8.3)(6.4)\cos 27°$
$CX = \sqrt{109.85 - 106.24\cos 27°}$ $(CX > 0)$
$= \mathbf{3.90}$ cm (3 s.f.)

(b) $\frac{\sin \angle XAB}{6.4} = \frac{\sin 112°}{7.5}$
$\angle XAB = \sin^{-1}\left(\frac{6.4 \sin 112°}{7.5}\right)$
$= \mathbf{52.3°}$ (1 d.p.)

(c) $\angle XBA = 180° - 112° - 52.297°$
$= \mathbf{15.703°}$ (3 d.p.)

Area of triangle ABC
$= \frac{1}{2}(7.5)(8.3)\sin(15.703 + 27)°$
$= \mathbf{21.1}$ cm² (3 s.f.)

3. (a) Price of the smartphone in the UK
$= 389 \div 0.58$
$= \$670.6897$ (4 d.p.)

Difference in price $= \$670.6897 - \$620$
$= \mathbf{\$50.69}$ (2 d.p.)

(b) Marked price $= \$785 \div 94\%$
$= \mathbf{\$835}$ (to nearest dollar)

Must-Know Concept:
Percentage discount $= \frac{\text{Marked price} - \text{Sale price}}{\text{Marked price}} \times 100\%$

(c) (i) Difference in mobile phone subscriptions
$= 8.21 \times 10^6 - 7.29 \times 10^6$
$= 0.92 \times 10^6$
$= \mathbf{9.2 \times 10^5}$

(ii) Percentage increase
$= \frac{1.20 \times 10^7 - 7.85 \times 10^6}{7.85 \times 10^6} \times 100\%$
$= \mathbf{52.9\%}$ (3 s.f.)

(iii) Mean number of SMS messages per person per day
$= \frac{1.14 \times 10^{10}}{5.54 \times 10^6 \times 365}$
$= \mathbf{5.64}$ (3 s.f.)

4. (a) $p = \mathbf{-3.8}$

(b) Refer to Appendix 1.

(c) $3.75 < x \leqslant 4$

(d) (ii) Sub $y = \frac{x^3}{5} - 2x + 1$ into $5y + x = 10$
$5\left(\frac{x^3}{5} - 2x + 1\right) + x = 10$
$x^3 - 10x + 5 + x = 10$
$\mathbf{x^3 - 9x - 5 = 0}$ (shown)

(iii) $x = \mathbf{-2.65}$ or $\mathbf{-0.6}$ or $\mathbf{3.25}$

5. (a) (i) Product of the top number and the bottom number $= t(t + 18)$
$= \mathbf{t^2 + 18t}$

(ii) Product of the middle two numbers
$= (t + 6)(t + 12)$
$= \mathbf{t^2 + 18t + 72}$

∴ The difference between the two products
$= (t^2 + 18t + 72) - (t^2 + 18t)$
$= \mathbf{72}$

Must-Know Concept:
Difference between 2 numbers = Larger number – Smaller number

(iii) $t + (t + 6) + (t + 12) + (t + 18) = 360$
$4t + 36 = 360$
$4t = 324$
$t = 81$

The largest number in the column
$= 81 + 18$
$= \mathbf{99}$

(b) (i) Common difference $= (60 - 36) \div 3$
$= 8$

1st term in the sequence $= 36 - 2(8)$
$= 20$

nth term $= 20 + (n - 1) \times 8$
$= \mathbf{12 + 8n}$

(ii) $12 + 8n = 4(3 + 2n)$

Since n is a positive integer, $3 + 2n$ is also a positive integer.
Hence $12 + 8n$ is always a multiple of 4.

6. (a) (i) $\angle ABC = 90°$ (∠ in a semicircle)

$\angle DBC = \angle DAC$ (∠s in same segment)
$= 23°$

$\angle ABD = 90° - 23°$
$= \mathbf{67°}$

(ii) $\angle AED = 180° - \angle ABD$ (∠s in opp. segments)
$= 180° - 67°$
$= 113°$

$\angle EAD = 180° - 113° - 42°$ (∠ sum of △)
$= \mathbf{25°}$

(b) (i) $8 + 4(\angle POQ) = 15.2$
$\angle POQ = \mathbf{1.8}$ radians

(ii) Total shaded area
$= \frac{1}{2}(6)^2(2\pi - 1.8) - \frac{1}{2}(4)^2(2\pi - 1.8) + \frac{1}{2}(4)^2(1.8)$
$= \mathbf{59.2}$ cm^2 (3 s.f.)

7. (a) (i) $\vec{PQ} = \vec{OQ} - \vec{OP}$
$= \begin{pmatrix} -5 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$
$= \begin{pmatrix} \mathbf{-9} \\ \mathbf{4} \end{pmatrix}$

(ii) $\vec{OR} = \vec{OP} + \vec{PR}$
$\begin{pmatrix} h \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + k\begin{pmatrix} -9 \\ 4 \end{pmatrix}$
$\begin{pmatrix} h \\ 3 \end{pmatrix} = \begin{pmatrix} 4 - 9k \\ -3 + 4k \end{pmatrix}$
$h = 4 - 9k$ .... (1)
$3 = -3 + 4k$ .... (2)

From (2): $k = \mathbf{1.5}$

Sub $k = 1.5$ into (1),

$h = 4 - 9(1.5)$
$= \mathbf{-9.5}$

Must-Know Concept:
If $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$, then $a = c$ and $b = d$.

(b) (i) $\vec{AC} = \vec{OC} - \vec{OA}$
$= -\mathbf{a} + \frac{5}{2}\mathbf{b}$

(ii) $\vec{XB} = \frac{2}{3}\vec{AB}$
$= \frac{2}{3}(\mathbf{b} - \mathbf{a})$
$= -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(iii) $\vec{XY} = \frac{2}{3}\vec{AC}$
$= \frac{2}{3}\left(-\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$
$= -\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}$

8. (a) Volume of the barn
$= \left[\frac{1}{2}(4 + 5) \times 6 + \frac{1}{2}(4 + 6.5) \times 10\right] \times 25$
$= (27 + 52.5) \times 25$
$= \mathbf{1987.5}$ m^3

Must-Know Concept:
Volume of prism = cross-sectional area × length

(b) $AG = \sqrt{6^2 + (5 - 4)^2}$
$= \sqrt{37}$ m

$EF = \sqrt{10^2 + (6.5 - 4)^2}$
$= \sqrt{106.25}$ m

Total area of the sloping roofs
$= \sqrt{37} \times 25 + \sqrt{106.25} \times 25$
$= \mathbf{410}$ m^2 (3 s.f.)

(c) Let the foot of perpendicular from P to the ground be X.

$PX = FC$
$= 6.5$ m

$DX = \sqrt{10^2 + 25^2}$
$= \sqrt{725}$ m

Angle of elevation of P from $D = \tan^{-1} \frac{6.5}{\sqrt{725}}$
$= \mathbf{13.6°}$ (1 d.p.)

9. (a) (i) Median score for group A $= \frac{71 + 72}{2}$
$= \mathbf{71.5}$

(ii) Range of scores for group B $= 93 - 50$
$= \mathbf{43}$

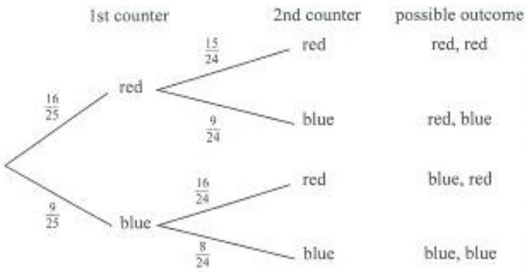
(iii) Percentage of students who scored more than 70 in group A
$= \frac{14}{24} \times 100\%$
$= \mathbf{58\frac{1}{3}}\%$

(iv) 1. There is a higher percentage of students who were awarded a merit in group B (45%) than group A (37.5%).
2. There is a higher percentage of students who were awarded a distinction in group A (20.8%) than group B (20%).

Note that students who scored between 71 and 85 marks will be awarded a merit.

(b) (i) When the first blue counter is taken and not replaced, the probability of taking another blue counter should be $\frac{8}{24}$ instead of $\frac{9}{25}$. As a result, the probability that both counters are blue is not $\frac{9}{25} \times \frac{9}{25} = \frac{81}{625}$.

(ii)



(iii) Probability that only one of the counters is red $= \frac{16}{25} \times \frac{9}{24} + \frac{9}{25} \times \frac{16}{24}$
$= \mathbf{\frac{12}{25}}$

10. (a) Amount of fuel needed
$= 92 \div 100 \times 6.3$
$= 5.796$
$=$ **5.8 litres** (1 d.p.) (shown)

(b) Distance travelled $= 85 \times \frac{45}{60}$
$= 63.75$ km

Amount of fuel needed $= 63.75 \div 100 \times 4.2$
$=$ **2.6775** litres

(c) Let the distance of the first stage of the journey be x km.

$$\frac{x}{60} + \frac{x-25}{75} = 3\frac{1}{4}$$
$$5x + 4(x-25) = 975$$
$$9x = 1075$$
$$x = \frac{1075}{9}$$

Total distance travelled during the journey
$= 2x - 25$
$= \frac{1925}{9}$ km

Amount of fuels needed $= \frac{1925}{9} \div 100 \times 5.0$
$= \frac{385}{36}$ litres

Estimated total cost $= \frac{385}{36} \times \$2.07 \times 95\%$
$= \$21.03$ (2 d.p.)

$\$21.03 \div 2 \approx \$10.50$
A suitable amount would be **\$10.50**, which is about half of the total cost.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2018 EXAMINATION PAPER

Paper 1

1. $\frac{3}{4} = 0.75$

$\left(\frac{3}{4}\right)^2 = 0.5625$

$\frac{3}{5} = 0.6$

Arranging the answers from the smallest to largest

$= \left(\frac{3}{4}\right)^2, \frac{3}{5}, 0.65, \frac{3}{4}$

Must-Know Concept:
Express all the numbers in decimal form to compare them.

2. $\left(\frac{x^8}{y^6}\right)^{-\frac{3}{2}} = \left(\frac{y^6}{x^8}\right)^{\frac{3}{2}}$

$= \frac{y^{\left(6 \times \frac{3}{2}\right)}}{x^{\left(8 \times \frac{3}{2}\right)}}$

$= \frac{y^9}{x^{12}}$

Must-Know Concept:
Use the Law of Indices: $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$.

3. (a) $kx^2 + (k+1)x - 4 = 0$

When $x = -2$,

$k(-2)^2 + (k+1)(-2) - 4 = 0$

$4k - 2k - 2 - 4 = 0$

$2k - 6 = 0$

$2k = 6$

$k = \frac{6}{2}$

$= \mathbf{3}$

Must-Know Concept:
Substitute $x = -2$ into the given equation to find the value of k.

(b) When $k = 3$,

$3x^2 + (3+1)x - 4 = 0$

$3x^2 + 4x - 4 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-4)}}{2(3)}$

$= \frac{-4 \pm \sqrt{64}}{6}$

$= \frac{-4 \pm 8}{6}$

$= \frac{2}{3}$ or -2

$\therefore x = \frac{2}{3}$

Must-Know Concept:
Substitute the value of k found in (a) into the given equation. Then, solve the equation.

4. $\angle DBC = 180° - 129°$ (adj. ∠s on a str. line)

$= 51°$

$\angle CDB = 180° - 38° - 51°$ (∠ sum of △)

$= 91°$

Since $\angle CDB$ is not a right angle (90°), by the property of angle in a semicircle, BC is not a diameter of the circle.

Must-Know Concept:
Note that there is a right angle in a circle if the hypotenuse is the diameter of the circle.

5. $(3n-1)^2 + 2$

$= 9n^2 - 6n + 1 + 2$

$= 9n^2 - 6n + 3$

$= \mathbf{3(3n^2 - 2n + 1)}$, which is a multiple of 3 for all integer values of n. (shown)

Must-Know Concept:
'Multiple of 3' implies that the expression is divisible by 3.

6. (a) Males in 2014 generally married at a later age compared to males in 2004.

(b) Mode (males) = 30 – 34 years
Mode (females) = 30 – 34 years
No, because **the modal groups for both males and females in 2014 are 30 – 34 years.** Therefore, we cannot conclude that males married at a higher age than females in 2014.

Must-Know Concept:
The modal group is the age group which has the greatest number of marriages.

7. (a) $A \cup B'$

(b) $(A' \cap B) \cup (A \cap B')$

8. Total amount of monthly payments = 12 × \$113

= \$1356

Amount of deposit = \$1651 – \$1356

= \$295

Required percentage = $\frac{\$295}{\$1475} \times 100\%$

= **20%**

Must-Know Concept:
An item bought on hire-purchase always costs more than its cash price.

9. (a) $OC = OB = OA$

$\angle OCB = \angle OBC$ (base ∠s of isos. △)

$= 41°$

$\angle OAB = \angle OBA$ (base ∠s of isos. △)

$= 23°$

$\angle ABC = 41° + 23°$

$= \mathbf{64°}$

Must-Know Concept:
When solving questions on circles, keep a lookout for isosceles triangles.

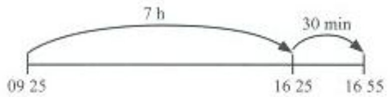
(b) $\angle AOC = 2 \times \angle ABC$ ($\angle$ at centre = $2\angle$ at circumference)
$= 2 \times 64°$
$= \mathbf{128°}$

Must-Know Concept:
Note that an angle at centre is twice the angle at circumference.

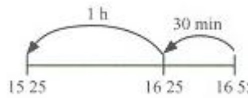
(c) $\angle CDA = 180° - 64°$ ($\angle$s in opp. segments)
$= 116°$
$\angle OAD = 180° - 116°$ ($\angle$s between two // lines, $OA \parallel CD$)
$= \mathbf{64°}$

Must-Know Concept:
Note that angles in opposite segments are supplementary.

10. Time taken = 5408 km ÷ 721 km/h
= 7.5006 h
≈ 7 h 30 min



Time of arrival in Adelaide time = 16 55



Time difference between Adelaide and Singapore
= 1 h 30 min
= 1.5 h
Singapore is **behind** by **1.5** hours.

Must-Know Concept:
Time taken = $\frac{\text{Distance}}{\text{Speed}}$

11.
$1 : 20$
$1 \text{ cm} : 20 \text{ cm}$
$0.01 \text{ m} : 0.2 \text{ m}$
$0.01^3 \text{ m}^3 : 0.2^3 \text{ m}^3$
$1 \times 10^{-6} \text{ m}^3 : 0.008 \text{ m}^3$
$\frac{1 \times 10^{-6}}{0.008} \text{ m}^3 : 1 \text{ m}^3$
$184 \times \frac{1 \times 10^{-6}}{0.008} \text{ m}^3 : 184 \text{ m}^3$
$2.3 \times 10^{-2} \text{ m}^3 : 184 \text{ m}^3$
∴ cargo volume of the model = $\mathbf{2.3 \times 10^{-2}}$ m^3

Must-Know Concept:
Convert the scale to m^3.

12. (a) 4 workers ⟶ 17.5 h
1 worker ⟶ 4 × 17.5 h
= 70 h
5 workers ⟶ 70 h ÷ 5
= 14 h
Time it would take = **14** hours

(b) $F \propto \frac{1}{d^2}$
$F = \frac{k}{d^2}$
When d increased by 20%,
$\frac{120}{100} \times d = 1.2d$
$F_1 = \frac{k}{(1.2d)^2}$
$= \frac{k}{1.44d^2}$
$= \frac{1}{1.44}F$
$= \frac{25}{36}F$
Change in the force between the objects
$= F - \frac{25}{36}F$
$= \frac{11}{36}F$
Percentage reduction in the force between the objects $= \frac{\frac{11}{36}F}{F} \times 100\%$
$= \frac{11}{36} \times 100\%$
$= \mathbf{30\frac{5}{9}}\%$

Must-Know Concept:
Percentage reduction = $\frac{\text{Reduction}}{\text{Original value}} \times 100\%$

13. (a) $8p^2q - 6pq^3 = \mathbf{2pq(4p - 3q^2)}$

Must-Know Concept:
Factorise the common terms.

(b) $6x^2y - 2xy + 3x - 1 = 2xy(3x - 1) + (3x - 1)$
$= \mathbf{(2xy + 1)(3x - 1)}$

Must-Know Concept:
Factorise terms with common factors.

14. Interior angle of $C = \frac{(6-2) \times 180°}{6}$
$= 120°$
Interior angle of $B = 360° - 90° - 120°$
$= 150°$
Let n be the number of sides in polygon B.
$\frac{(n-2) \times 180°}{n} = 150°$
$(n-2) \times 180° = 150°n$
$180°n - 360° = 150°n$
$180°n - 150°n = 360°$
$30°n = 360°$
$n = \mathbf{12}$

Must-Know Concept:
Each interior angle of an n-sided regular polygon $= \frac{(n-2) \times 180°}{n}$

15. Let x be the number of males in the group.

Number of males $= x$

Number of females $= x + 4$

Total height of males $= x \times 183$ cm $= 183x$ cm

Total height of females $= (x + 4) \times 162$ cm $= (162x + 648)$ cm

Total number of students $= x + x + 4 = 2x + 4$

Total height of the group of students $= (2x + 4) \times 171$ cm $= (342x + 684)$ cm

$$183x + 162x + 648 = 342x + 684$$
$$345x + 648 = 342x + 684$$
$$345x - 342x = 684 - 648$$
$$3x = 36$$
$$x = 12$$

Total number of students in the group $= 2x + 4 = 2(12) + 4 = \mathbf{28}$

Must-Know Concept:
Find the total height of males and females respectively before equating them to the total height of the group of students. This will form an equation in x. Solve for x.

16. Perimeter of the minor sector $= r\theta + 2r$

Angle of major sector $= 2\pi - \theta$

Perimeter of the major sector $= r(2\pi - \theta) + 2r$

$$r(2\pi - \theta) + 2r = 2(r\theta + 2r)$$
$$2\pi r - r\theta + 2r = 2r\theta + 4r$$
$$2\pi r + 2r - 4r = 2r\theta + r\theta$$
$$2\pi r - 2r = 3r\theta$$
$$\theta = \frac{2\pi r - 2r}{3r} = \frac{2\pi - 2}{3} = 1.4277$$
$= \mathbf{1.428}$ radians (3 d.p.)

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians)
Perimeter of sector = Arc length + Radius + Radius

17.
$$y = \frac{x^2 + 3}{x^2 - a}$$
$$y(x^2 - a) = x^2 + 3$$
$$x^2y - ay = x^2 + 3$$
$$x^2y - x^2 = ay + 3$$
$$x^2(y - 1) = ay + 3$$
$$x^2 = \frac{ay + 3}{y - 1}$$
$$x = \pm\sqrt{\frac{ay + 3}{y - 1}}$$

Must-Know Concept:
Make x the subject by shifting terms that have x to the left-hand side and terms that do not have x to the right-hand side, and simplify the equation.

18. (a)
$$|\overrightarrow{PQ}| = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2}$$
$$\frac{5\sqrt{10}}{2} = \sqrt{10a^2}$$
$$5\sqrt{10} = 2\sqrt{10a^2}$$
$$\sqrt{250} = \sqrt{40a^2}$$
$$40a^2 = 250$$
$$a^2 = \frac{250}{40} = \frac{25}{4}$$
$$a = \pm\sqrt{\frac{25}{4}} = \frac{5}{2} \text{ or } -\frac{5}{2}$$

Must-Know Concept:
The formula for the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2 + y^2}$.

(b) (i)
$$\overrightarrow{AB} = \begin{pmatrix} 43 \\ 38 \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{48} \\ \mathbf{37} \end{pmatrix}$$

(ii)
$$\overrightarrow{AC} = \begin{pmatrix} 11 \\ 13 \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 16 \\ 12 \end{pmatrix}$$

Since $\overrightarrow{AC}$ cannot be expressed as $k\overrightarrow{AB}$ where k is a scalar, the point C does not lie on line AB.

Must-Know Concept:
When two vectors are parallel and have a common point, they lie on the same line.

19. (a) Median position $= \frac{30 + 1}{2} = 15.5$

Median mass of the apples $= \frac{127 \text{ g} + 129 \text{ g}}{2} = \mathbf{128}$ g

Must-Know Concept:
As there is an even number of values, the median is the average of the two middle values when arranged in ascending order.

(b) Minimum mass of the bananas = 103 g

Maximum mass of the bananas = 103 g + 44 g = 147 g

Value of $x = \mathbf{7}$

Must-Know Concept:
The range of a data set is the spread of the data.
Range = Largest value − Smallest value

(c) Median position of the bananas $= \frac{25+1}{2}$
$= 13$

Median mass of the bananas = 125 g
Range of the masses for apples = 159 g – 92 g
= 67 g

The apples have a larger mass than the bananas, as the median mass of the apples is heavier than the median mass of the bananas.
The masses of the bananas are more consistent than the masses of the apples as the range for the masses of the bananas is smaller.

Must-Know Concept:
Compare both distributions using their median values and ranges as these values are found in the previous parts.

20. (a) $126 = \mathbf{2 \times 3^2 \times 7}$

(b) For $126k$ to be a perfect square, the index of each prime number must be 2.
$126k = 2^2 \times 3^2 \times 7^2$
$k = 2 \times 7$
$= \mathbf{14}$

(c) $126 = 2 \times 3^2 \times 7$
HCF = 21
$= 3 \times 7$
Smallest possible value of $x = 3 \times 7 \times 11$
$= \mathbf{231}$

Must-Know Concept:
x cannot have factors 2 and 3 since it is given that the HCF is 21. Therefore, to find the smallest possible value of x, we multiply 21 by prime factors larger than 3 and so on, until the number is within the range given in the question.

21. $y = ax^2 + bx + 3$
When $x = -2$ and $y = 11$,
$11 = a(-2)^2 + b(-2) + 3$
$11 = 4a - 2b + 3$
$4a - 2b = 8$
$2a - b = 4$ .... (1)
When $x = 4$ and $y = -1$,
$-1 = a(4)^2 + b(4) + 3$
$-1 = 16a + 4b + 3$
$16a + 4b = -4$
$4a + b = -1$
$b = -1 - 4a$ .... (2)
Substitute (2) into (1):
$2a - (-1 - 4a) = 4$
$2a + 1 + 4a = 4$
$6a = 4 - 1$
$a = \frac{1}{2}$
Substitute $a = \frac{1}{2}$ into (2):
$b = -1 - 4\left(\frac{1}{2}\right)$
$= -1 - 2$
$= -3$
$\therefore a = \mathbf{\frac{1}{2}}, b = \mathbf{-3}$

Must-Know Concept:
Substitute the two pairs of coordinates and solve the two equations simultaneously to find the values of a and b.

22. $\angle ABC = 2 \times 35°$
$= 70°$
Area of parallelogram $ABCD$
$= (10)(8)(\sin 70°)$
$= 75.1754 \text{ cm}^2$
$\angle BEC = 35°$ (alt. ∠s, $AB // EC$)
$\therefore \triangle BEC$ is an isosceles triangle, $EC = 8$ cm
$\angle BCE = 180° - 35° - 35°$ (∠ sum of △)
$= 110°$
Area of $\triangle BEC$
$= \frac{1}{2}(8)(8)(\sin 110°)$
$= 30.0701 \text{ cm}^2$
Area of trapezium $ABED$
$= 75.1754 \text{ cm}^2 - 30.0701 \text{ cm}^2$
$= 45.1053 \text{ cm}^2$
$= \mathbf{45.1} \text{ cm}^2$ (3 s.f.)

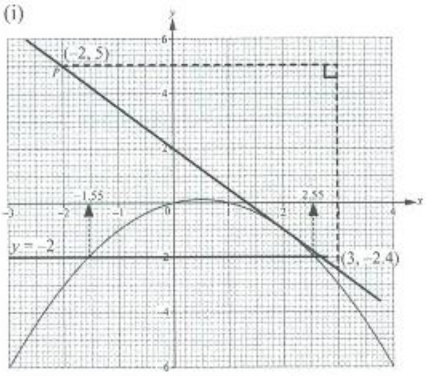
Must-Know Concept:
Area of a parallelogram $= ab \sin C$
Area of a triangle $= \frac{1}{2}ab \sin C$

23. (a) Equation of the line of symmetry:
$x = \frac{0+1}{2}$
$\mathbf{x = 0.5}$

(b) $x - x^2 = -4$
$\frac{1}{2}(x - x^2) = -2$
Draw the line $y = -2$.
From the graph,
$x = \mathbf{-1.55}$ or $\mathbf{2.55}$

Must-Know Concept:
The points of intersection of the line and the graph is the solution of the equation $x - x^2 = -4$.

(c) (i)



(ii) Gradient of the tangent $= \frac{-2.4 - 5}{3 - (-2)}$
$= -1.48$

Substitute $y = 5$, $x = -2$ and $m = -1.48$ into $y = mx + c$:
$5 = -1.48(-2) + c$
$c = 2.04$
$\therefore$ Equation of the tangent:
$\mathbf{y = -1.48x + 2.04}$

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient using the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

24. (a) Using Sine Rule,

$$\frac{2392}{\sin 29^\circ} = \frac{4140}{\sin \angle DMS}$$

$$\sin \angle DMS = \frac{4140 \sin 29^\circ}{2392}$$

$$= 0.839\ 093$$

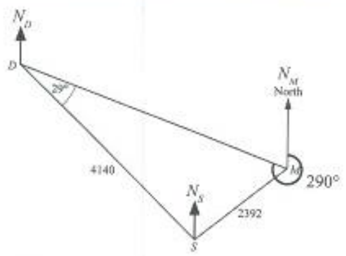
$$\angle DMS = \sin^{-1} 0.839\ 093$$

$$= 57.044^\circ$$

$$= \mathbf{57.0^\circ} \text{ (1 d.p.)}$$

Must-Know Concept:
Apply the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

(b)



$\angle N_M MD = 360^\circ - 290^\circ$ ($\angle$s at a pt.)
$= 70^\circ$
$\angle N_D DM = 180^\circ - 70^\circ$ (int. $\angle$s, DN_D // MN_M)
$= 110^\circ$
$\angle DSN_S = 180^\circ - 110^\circ - 29^\circ$ (int. $\angle$s, DN_D // SN_S)
$= 41^\circ$
Bearing of New Delhi from Singapore
$= 360^\circ - 41^\circ$ ($\angle$s at a pt.)
$= \mathbf{319^\circ}$

Must-Know Concept:
When calculating bearings, always move in a clockwise direction to identify the required angle.

Paper 2

1. (a) (i)

$$\frac{5t^2}{v} \div \frac{25t}{v^3} = \frac{5t^2}{v} \times \frac{v^3}{25t}$$

$$= \mathbf{\frac{tv^2}{5}}$$

(ii)

$$\frac{4}{3-2y} - \frac{5}{y+3} = \frac{4(y+3) - 5(3-2y)}{(3-2y)(y+3)}$$

$$= \frac{4y + 12 - 15 + 10y}{(3-2y)(y+3)}$$

$$= \mathbf{\frac{14y - 3}{(3-2y)(y+3)}}$$

Must-Know Concept:
Convert the two denominators into a common denominator to become a single fraction.

(b)

$$\frac{16x^2 - 9}{4x^2 - 9x - 9} = \frac{(4x+3)(4x-3)}{(4x+3)(x-3)}$$

$$= \mathbf{\frac{4x-3}{x-3}}$$

Must-Know Concept:
Factorise the numerator and the denominator. Cancel out the common term.

(c)

$$\frac{20}{x+1} = 2x + 5$$

$$(2x+5)(x+1) = 20$$

$$2x^2 + 2x + 5x + 5 = 20$$

$$2x^2 + 7x - 15 = 0$$

$$(2x-3)(x+5) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = \mathbf{1.5} \quad \text{or} \quad x = \mathbf{-5}$$

Must-Know Concept:
Perform cross multiplication and solve for x.

2. (a) Amount of earnings in 2017 $= 12 \times \$3900$
$= \$46\ 800$
Increase in his earnings $= \$46\ 800 - \$44\ 000$
$= \$2800$
Percentage increase in his earnings
$= \frac{\$2800}{\$44\ 000} \times 100\%$
$= \mathbf{6.36\%}$ (3 s.f.)

Must-Know Concept:
Percentage increase $= \frac{\text{Increase}}{\text{Original value}} \times 100\%$

(b) Total amount of money after 6 years
$= \$6500\left(1 + \frac{2.35}{100}\right)^6$
$= \$7472.06$ (nearest cent)
Total amount of interest earned
$= \$7472.06 - \$6500$
$= \mathbf{\$972.06}$

Must-Know Concept:
The formula for compound interest is $P\left(1 + \frac{r}{100}\right)^n$.
This formula can be found in the Mathematical Formulae page.

(c) (i) €0.66 = \$1
€1 $= \$\left(\frac{1}{0.66}\right)$
$= \$1.52$ (nearest cent)
£1 = \$1.97
Since it costs more Singapore dollars to purchase 1 pound than 1 euro, for the same amount of pounds and euros, the hotel in London costs more per night than the hotel in Paris.

Must-Know Concept:
Convert both currencies to a common currency, i.e. Singapore dollars.

(ii) €1 $= \$\left(\frac{1}{0.66}\right)$
€145 $= 145 \times \$\left(\frac{1}{0.66}\right)$
$= \$219.70$ (nearest cent)
£1 = \$1.97
£145 $= 145 \times \$1.97$
$= \$285.65$
Cost of hotel in London per night in Singapore dollars = \$285.65
Cost of hotel in Paris per night in Singapore dollars = \$219.70
Total cost of both hotels
$= 3 \times \$285.65 + 4 \times \$219.70$
$= \$1735.75$
Total amount Ken pays for the two hotels
$= 101.8\% \times \$1735.75$
$= \mathbf{\$1767}$ (nearest dollar)

3. (a) $\mathbf{P} = 7\mathbf{M}$

$$= 7\begin{pmatrix} 70 & 40 & 30 \\ 50 & 30 & 30 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{490} & \mathbf{280} & \mathbf{210} \\ \mathbf{350} & \mathbf{210} & \mathbf{210} \end{pmatrix}$$

Must-Know Concept:
When a matrix is multiplied by a scalar, every element in the matrix is multiplied by the scalar.

(b) $\mathbf{N} = \begin{pmatrix} \mathbf{0.75} \\ \mathbf{1.5} \\ \mathbf{2.25} \end{pmatrix}$

(c) $\mathbf{T} = \mathbf{PN}$

$$= \begin{pmatrix} 490 & 280 & 210 \\ 350 & 210 & 210 \end{pmatrix}\begin{pmatrix} 0.75 \\ 1.5 \\ 2.25 \end{pmatrix}$$

$$= \begin{pmatrix} 490(0.75) + 280(1.5) + 210(2.25) \\ 350(0.75) + 210(1.5) + 210(2.25) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1260} \\ \mathbf{1050} \end{pmatrix}$$

Must-Know Concept:
The product of a 2 × 3 matrix and a 3 × 1 matrix gives a resulting 2 × 1 matrix.

(d) The elements in T represent **the total cost of making apple pies and cherry pies respectively in one week**.

(e) Total cost of making the pies $= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1260 \\ 1050 \end{pmatrix}$

$$= (2310)$$

Selling price of the pies $= 1.4\begin{pmatrix} 0.75 \\ 1.5 \\ 2.25 \end{pmatrix}$

$$= \begin{pmatrix} 1.05 \\ 2.1 \\ 3.15 \end{pmatrix}$$

Number of pies of each size sold

$$= \begin{pmatrix} \frac{6}{7} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} 490 & 280 & 210 \\ 350 & 210 & 210 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{7}(490) + \frac{4}{5}(350) & \frac{6}{7}(280) + \frac{4}{5}(210) & \frac{6}{7}(210) + \frac{4}{5}(210) \end{pmatrix}$$

$$= \begin{pmatrix} 700 & 408 & 348 \end{pmatrix}$$

Total amount of money collected

$$= \begin{pmatrix} 700 & 408 & 348 \end{pmatrix}\begin{pmatrix} 1.05 \\ 2.1 \\ 3.15 \end{pmatrix}$$

$$= (700(1.05) + 408(2.1) + 348(3.15))$$

$$= (2688)$$

Total amount of profit made that week

$= \$2688 - \$2310$

$= \mathbf{\$378}$

4. (a) (i) $T_1 = 11 = 5 + 6(1)$

$T_2 = 17 = 5 + 6(2)$

$T_3 = 23 = 5 + 6(3)$

$T_4 = 29 = 5 + 6(4)$

$\therefore$ nth term $= \mathbf{5 + 6n}$

Must-Know Concept:
Observe that the pattern in the sequence is 6 more than the previous term.

(ii) $5 + 6n = 5 + 3(2n)$

Since 5 is not a multiple of 3, the expression $5 + 6n$ cannot be expressed as a multiple of 3. Therefore, it is not possible for a term in the sequence to be a multiple of 3.

Must-Know Concept:
'Multiple of 3' implies that the expression is a multiple of 3.

(b) (i) When $n = 5$,

$$T_5 = \frac{4(5) - 1}{205 - 5(5)} = \frac{20 - 1}{205 - 25} = \mathbf{\frac{19}{180}}$$

(ii) When $n = k$,

$$\frac{4k - 1}{205 - 5k} = \frac{7}{32}$$

$$32(4k - 1) = 7(205 - 5k)$$

$$128k - 32 = 1435 - 35k$$

$$128k + 35k = 1435 + 32$$

$$163k = 1467$$

$$k = \mathbf{9}$$

(iii)

$$T_n > 1$$

$$\frac{4n - 1}{205 - 5n} > 1$$

$$4n - 1 > 205 - 5n$$

$$4n + 5n > 205 + 1$$

$$9n > 206$$

$$n > 22\tfrac{8}{9}$$

Least value of $n = \mathbf{23}$

Must-Know Concept:
The least value of n should be the smallest integer value that satisfies the inequality.

5. (a) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OC} - \overrightarrow{BC}$

$$= \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-1} \\ \mathbf{2} \end{pmatrix}$$

(b) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$

$= \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 0 \end{pmatrix}$

Since $\overrightarrow{DC} \parallel \overrightarrow{AB}$,

$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$

$\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{DC}$

$= \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

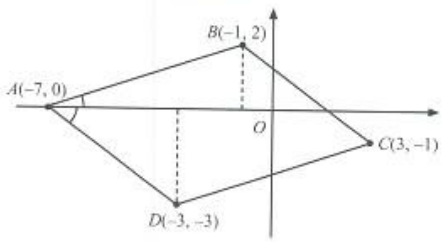
$= \begin{pmatrix} -3 \\ -3 \end{pmatrix}$

Coordinates of A = **(–7, 0)**

Coordinates of D = **(–3, –3)**

Must-Know Concept:
Since $ABCD$ is a parallelogram, $AB \parallel DC$.

(c)



$\tan \angle BAO = \frac{2}{6}$

$\angle BAO = \tan^{-1} \frac{1}{3}$

$= 18.4349°$

$\tan \angle DAO = \frac{3}{4}$

$\angle DAO = \tan^{-1} \frac{3}{4}$

$= 36.8698°$

$\angle BAD = 18.4349° + 36.8698°$

$=$ **55.3°** (1 d.p.)

Must-Know Concept:
Identify right-angled triangles and apply the trigonometric property: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ to find the required angles.

(d) Length of $BD = \sqrt{[-3-(-1)]^2 + (-3-2)^2}$

$= \sqrt{29}$

$=$ **5.39 units** (3 s.f.)

Must-Know Concept:
The formula to find the length of a line segment is $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$.

6. (a) Number of minutes it would take to fill the tank using pump A = $\mathbf{\frac{3500}{x}}$

(b) Number of minutes it would take to fill the tank using pump B = $\mathbf{\frac{3500}{x-10}}$

(c) $\frac{3500}{x} + 21 = \frac{3500}{x-10}$

$\frac{3500 + 21x}{x} = \frac{3500}{x-10}$

$(3500 + 21x)(x - 10) = 3500x$

$3500x - 35\,000 + 21x^2 - 210x = 3500x$

$21x^2 - 210x - 35\,000 = 0$

$\mathbf{3x^2 - 30x - 5000 = 0}$ (shown)

Must-Know Concept:
Use the information given in the question and the expressions found in (a) and (b) to form the equation.

(d) $x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(-5000)}}{2(3)}$

$= 46.1298$ or -36.1298

$=$ **46.13** or **–36.13** (2 d.p.)

(e) Since the rate is positive, $x = 46.1298$.
Time taken to fill the tank from empty using pump A and pump B together

$= \frac{3500}{x + x - 10}$

$= \frac{3500}{46.1298 + 46.1298 - 10}$

$= 42.548$ min

$=$ **42 min 30 s** (nearest ten seconds)

7. (a) $\angle COA = \angle DOB$ (vert. opp. $\angle$s)

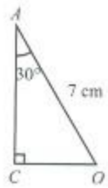
$\angle ACO = \angle BDO = 90°$ (tan $\perp$ rad)

$OC = OD$ (radius of small circle)

$\therefore$ **$\triangle OAC \equiv \triangle OBD$ (AAS)** (shown)

Must-Know Concept:
Two triangles are congruent if they have the same shape and the same size.
Conditions for two triangles to be congruent: SSS, SAS, ASA, AAS, RHS

(b) (i) $\sin \angle OAC = \frac{OC}{OA}$



$\sin 30° = \frac{OC}{7}$

$OC = 7 \sin 30°$

$= 3.5$ cm

$AC = \sqrt{7^2 - 3.5^2}$

$= \sqrt{36.75}$ cm

Area of $\triangle OAC = \frac{1}{2} \times \sqrt{36.75} \times 3.5$

$= 10.6088$ cm²

$=$ **10.6 cm²** (3 s.f.)

Must-Know Concept:
Since $\triangle OAC$ is a right-angled triangle, we can apply Pythagoras' Theorem to find the length of AC.

(ii) $\angle AOC = 180° - 30° - 90°$ ($\angle$ sum of $\triangle$)

$= 60°$

$\angle DOB = 60°$ (vert. opp. $\angle$s)

Area of unshaded parts

$= 2 \times 10.6088 + \frac{360° - 60° - 60°}{360°} \times \pi \times 3.5^2$

$= 46.873\,94$ cm²

Area of the large circle

$= \pi \times 7^2$

$= 153.938\,04$ cm²

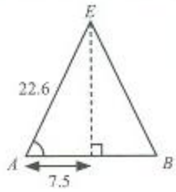
Shaded area $= 153.938\,04 - 46.873\,94$

$=$ **107 cm²** (3 s.f.)

8. (a) Slant height $= \sqrt{20^2 + 7.5^2}$
$= 21.36$ cm
$AE = \sqrt{21.36^2 + 7.5^2}$
$=$ **22.6 cm** (3 s.f.) (shown)

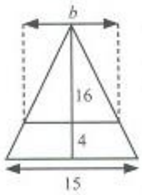
Must-Know Concept:
Apply Pythagoras' Theorem to find the unknown sides.

(b) $\cos \angle BAE = \frac{7.5}{22.6}$
$\angle BAE = \cos^{-1} \frac{7.5}{22.6}$
$= 70.6183°$
$=$ **70.6°** (1 d.p.)



(c) Total surface area of the pyramid
$= 4 \times \frac{1}{2} \times 15 \times 21.36 + 15 \times 15$
$=$ **866 cm²** (3 s.f.)

(d) Vertical height of the pyramid that was removed
$= 20 - 4$
$= 16$ cm
Let b be the length of the base of the pyramid that was removed.
$\frac{16}{20} = \frac{b}{15}$
$b = 12$
Area of $PQRS = 12 \times 12$
$=$ **144 cm²**



Must-Know Concept:
Since both pyramids are similar, we can use the property of similar triangles to find the base of the pyramid that was removed.

9. (a) (i)

Mass (m kg)	$2.0 \leqslant m < 2.5$	$2.5 \leqslant m < 3.0$	$3.0 \leqslant m < 3.5$	$3.5 \leqslant m < 4.0$	$4.0 \leqslant m < 4.5$
Frequency	4	11	(34 – 15 =) **19**	(46 – 34 =) **12**	(50 – 46 =) **4**

Must-Know Concept:
Find the difference between the values of the lower limit and the upper limit of the given range.

(ii) Mean mass
$= \frac{4 \times 2.25 + 11 \times 2.75 + 19 \times 3.25 + 12 \times 3.75 + 4 \times 4.25}{50}$
$= \frac{163}{50}$
$=$ **3.26 kg**

Must-Know Concept:
We use the middle value of the range as the value of x as this is a good estimate of the mean. Then, use the formula $\frac{\Sigma fx}{\Sigma f}$ to find the mean.

(iii) Standard deviation
$= \sqrt{\frac{4 \times 2.25^2 + 11 \times 2.75^2 + 19 \times 3.25^2 + 12 \times 3.75^2 + 4 \times 4.25^2}{50} - 3.26^2}$
$= \sqrt{\frac{545.125}{50} - 3.26^2}$
$=$ **0.524** (3 s.f.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ to find the standard deviation since we have the required values.

(iv) The mean and standard deviations are estimates as the exact masses of the 50 babies are not known.

(v) From the graph,
Number of babies of mass smaller than 2.8 kg = 9
Number of babies of mass greater than 2.8 kg = 50 – 9
= 41
$\frac{41}{50} \times 100\% = 82\%$ (< 90%)
∴ The data from the hospital does not support this claim.

(b) (i) P(under 30) $= \frac{3 + 7}{3 + 7 + 9 + 6}$
$= \frac{10}{25}$
$= \mathbf{\frac{2}{5}}$

(ii) (a) P(both have a baby girl) $= \frac{13}{25} \times \frac{12}{24}$
$= \mathbf{\frac{13}{50}}$

(b) P(both are over 30, only one has a baby boy)
$= \frac{9}{25} \times \frac{6}{24} + \frac{6}{25} \times \frac{9}{24}$
$= \mathbf{\frac{9}{50}}$

Must-Know Concept:
The selections are without replacements as the same person cannot be selected twice.

10. (a) Dimensions of the cardboard = 30 cm × 20 cm
Area of the original cardboard = 30 cm × 20 cm
= 600 cm²
Area of the remaining cardboard
= 600 cm² – 4 × 5 cm × 5 cm
= 500 cm²
= 0.05 m²
Mass of the box made = 0.05 × 210 g
= **10.5 g**

Must-Know Concept:
1 m² = 1 m × 1 m
= 100 cm × 100 cm
= 10 000 cm²

(b) (i) Height of the box $= x$

Breadth of the box $= 20 - x - x$

$= 20 - 2x$

Length of the box $= 30 - x - x$

$= 30 - 2x$

Volume of the box

$= x \times (20 - 2x) \times (30 - 2x)$

$= \mathbf{x(20 - 2x)(30 - 2x)}$ (shown)

(ii) Since the volume of the box made is $x(20 - 2x)(30 - 2x) > 0$,

$x > 0$ $\quad$ $20 - 2x > 0$ $\quad$ $30 - 2x > 0$

$20 > 2x$ $\quad$ $30 > 2x$

$x < 10$ $\quad$ $x < 15$

$\therefore\ 0 < x < 10$

Must-Know Concept:
The value of x is greater than 0 for the height of the box to exist and the value of x is smaller than 10 for the breadth of the box to exist. If the value of x is greater than 10, Huan will not be able to cut squares from the corners of the cardboard along its breadth.

(c) $V = x(20 - 2x)(30 - 2x)$

x	1	2	3	4	5	6	7	8	9
V	504	832	1008	1056	1000	864	672	448	216

Refer to Appendix 2.

When $V = 900$, $x = 2.3$ or 5.75.

Since the mass of the box is directly proportional to the surface area of the box, the smaller the surface area of the box, the smaller the mass of the box.

Surface area of the box

$= 600\text{ cm}^2 - 4 \times x\text{ cm} \times x\text{ cm}$

$= (600 - 4x^2)\text{ cm}^2$

When $x = 5.75$,

Smallest surface area of the box $= 600 - 4(5.75)^2$

$= 467.75\text{ cm}^2$

$= 0.046\ 775\text{ m}^2$

Smallest mass of the box she will make

$= 0.046\ 775 \times 210\text{ g}$

$= \mathbf{9.82\ g}$ (3 s.f.)

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2017 EXAMINATION PAPER

Paper 1

1. $\frac{1}{81} = \frac{1}{3^4}$
$= 3^{-4}$
$\therefore k = \mathbf{-4}$

Must-Know Concept:
Use the Law of Indices: $a^{-b} = \frac{1}{a^b}$

2. $\mathbf{-3 < x \leqslant 6.5}$

Must-Know Concept:
Closed circles are used for inequalities that are 'less / more than or equals to', whereas open circles are used for inequalities that are 'less / more than'.

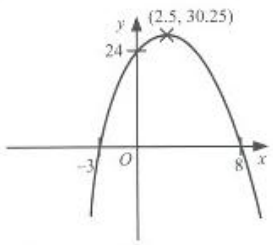
3. (a) Range $= 228 - 191$
$= \mathbf{37}$ grams

Must-Know Concept:
Range is the difference between the maximum value and the minimum value.

(b) Median position
$= \frac{20+1}{2}$
$= 10.5$
= average of 10th and 11th values
Median $= \frac{208 + 210}{2}$
$= \mathbf{209}$ grams

Must-Know Concept:
As there is an even number of values, the median is the average of the two middle values when arranged in ascending order.

4. Since the coefficient of x^2 is $-1 < 0$, the curve is '$\cap$' shaped.
When $x = 0$,
$y = -(0-8)(0+3)$
$= 24$
When $y = 0$,
$x = 8$ or $x = -3$
Turning point: $x = \frac{-3+8}{2}$
$= 2.5$
$y = -(2.5 - 8)(2.5 + 3)$
$= 30.25$



Must-Know Concept:
Find the important points such as turning point and intersection points with the axes. Label these points in the sketch.

5. (a) Distance travelled $= \frac{1}{2} \times 84 \times v$
$693 = 42v$
$v = 16.5$
Greatest speed, $v = \mathbf{16.5}$ m/s

Must-Know Concept:
In a speed-time graph, the area under the graph is the distance travelled.

(b) Acceleration
$= \frac{16.5}{34}$
$= \mathbf{0.485}$ m/s^2 (3 sig. fig.)

Must-Know Concept:
In a speed-time graph, the gradient of the graph is the acceleration.

6. (a) $\{2, 10\} \subset A$

Must-Know Concept:
Since the elements 2 and 10 are in set A, the set $\{2, 10\}$ is a subset of A.

(b) $3 \in B$

Must-Know Concept:
3 is an element in B.

(c) $B \cap C = \phi$

Must-Know Concept:
There is no element in the intersection of set B and set C.

7. Total number of members currently
$= 58 + 37$
$= 95$
Let x be the number of adults who join the club.
$\frac{58+x}{95+x} \times 100\% \geqslant 70\%$
$\frac{58+x}{95+x} \geqslant 0.7$
$58 + x \geqslant 0.7(95 + x)$
$58 + x \geqslant 66.5 + 0.7x$
$x - 0.7x \geqslant 66.5 - 58$
$0.3x \geqslant 8.5$
$x \geqslant 28\frac{1}{3}$
Smallest number of adults that would need to join the club $= \mathbf{29}$

Must-Know Concept:
The number of adults and total number of members will increase at the same time.

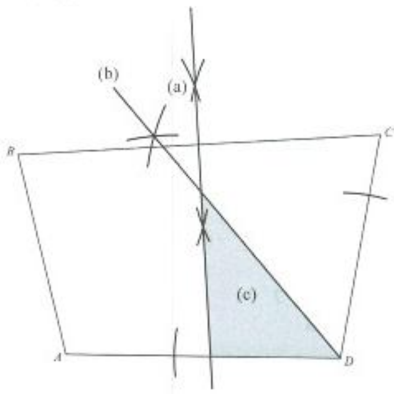
8. Interior angle of the second regular polygon
$= 360° - 96° - 108°$ ($\angle$s at a pt.)
$= 156°$
Exterior angle of the second regular polygon
$= 180° - 156°$
$= 24°$
Number of sides in the second regular polygon
$= 360° \div 24°$
$= \mathbf{15}$

Must-Know Concept:
The sum of all exterior angles of a polygon is 360°.

9. Volume of the cylinder = $\pi r^2 h$ cm^3
Volume of the hemisphere
$= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$
$= \frac{2}{3}\pi r^3$ cm^3
$\pi r^2 h = \frac{2}{3}\pi r^3$
$h = \frac{2}{3}r$
Total surface area of the cylinder
$= 2\pi r^2 + 2\pi rh$
$= 2\pi r^2 + 2\pi r\left(\frac{2}{3}r\right)$
$= 2\pi r^2 + \frac{4}{3}\pi r^2$
$= \mathbf{\frac{10}{3}\pi r^2}$ cm^2

Must-Know Concept:
Since the question only requires the surface area in terms of r, we need to eliminate h by substitution.

10. (a), (b), (c)



Must-Know Concept:
From (a), we can find the region nearer to C than B. From (b), we can find the region nearer to AD than to CD. The intersection of both regions is the answer to (c).

11. $\frac{3x-4}{2} - \frac{2x}{3} = 1$
$\frac{3(3x-4)-2(2x)}{(2)(3)} = 1$
$9x - 12 - 4x = 6$
$5x = 6 + 12$
$= 18$
$x = \frac{18}{5}$
$= \mathbf{3.6}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction.

12. $\vec{BC} = \vec{BA} + \vec{AC}$
$= -\vec{AB} + \vec{AC}$
$= -\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix}$
$= \begin{pmatrix} -6 \\ 9 \end{pmatrix}$
$|\vec{BC}| = \sqrt{(-6)^2 + 9^2}$
$= \sqrt{117}$
$=$ **10.8** units (3 sig. fig.)

Must-Know Concept:
The formula for the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2 + y^2}$.

13. Let \$$x$ be the sum of money Nur invested.
Amount of money in the account at the end of 4 years
$= \$x\left(1 + \frac{2.5}{100}\right)^4$
$= \$1.103\ 812x$
$\$1.103\ 812x = \$x + \$674.79$
$1.103\ 812x - x = 674.79$
$0.103\ 812x = 674.79$
$x = 6500.115\ 594$
Sum of money Nur invested in the account
= **\$6500.12** (nearest cent)

Must-Know Concept:
The formula for compound interest is $P(1 + \frac{r}{100})^n$.

14. (a) $2x^2 - 5x - 12 = \mathbf{(2x+3)(x-4)}$
(b) $2(2y-3)^2 - 5(2y-3) - 12$
$= [2(2y-3)+3][(2y-3)-4]$
$= (4y-6+3)(2y-7)$
$= \mathbf{(4y-3)(2y-7)}$

Must-Know Concept:
Substitute $x = 2y - 3$ into the expression in (a) to get the expression in (b).

15. $\angle ABC = 90°$ ($\angle$ in semicircle)
$\tan \angle BCA = \frac{8.1}{13.8}$
$\angle BCA = \tan^{-1} \frac{8.1}{13.8}$
$= 30.411°$
$\angle BDA = \angle BCA$ ($\angle$s in the same segment)
$= 30.4°$ (1 d.p.)
$\therefore x = \mathbf{30.4}$

Must-Know Concept:
Recall the two angle properties of circles:
An angle in a semicircle is a right angle.
The angles in the same segment of a circle are equal.

16. (a) Mean time

$= \frac{\Sigma ft}{\Sigma f}$

$= \frac{28(25) + 84(35) + 62(45) + 21(55) + 5(65)}{200}$

$= \frac{7910}{200}$

$=$ **39.55** minutes

Must-Know Concept:
Use the formula $\frac{\Sigma ft}{\Sigma f}$ to find the mean since we are given the frequency and values of t.

(b) $\frac{\Sigma ft^2}{\Sigma f} = \frac{28(25)^2 + 84(35)^2 + 62(45)^2 + 21(55)^2 + 5(65)^2}{200}$

$= \frac{330\,600}{200}$

Standard deviation

$= \sqrt{\frac{\Sigma ft^2}{\Sigma f} - (\bar{t})^2}$

$= \sqrt{\frac{330\,600}{200} - 39.55^2}$

$=$ **9.42** minutes (3 sig. fig.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma ft^2}{\Sigma f} - (\bar{t})^2}$ to find the standard deviation since we have the required values.

(c) Angle representing adults aged 21 to 30 years old = 99°
Angle representing adults aged 41 to 60 years old = 72°

$99° \longrightarrow 44$ adults

$72° \longrightarrow \frac{72°}{99°} \times 44$

$= 32$ adults

Number of adults aged 41 to 60 years old = **32**

17. $\angle ABO = 2x°$ (base angles of isos. △)
$\angle OBC = 5x°$ (base angles of isos. △)
$2x° + 5x° + 5x° = 180°$ (int. ∠s, AB // OC)
$12x° = 180°$
$x =$ **15**

Must-Know Concept:
When solving questions on circles, keep a lookout for isosceles triangles formed by radii of the circle.

18. (a) $28 = 2^2 \times 7$

$63 = 3^2 \times 7$

$28 \times 63 = 2^2 \times 7 \times 3^2 \times 7$

$= 2^2 \times 3^2 \times 7^2$

Since all the prime factors have powers of 2, 28 × 63 is a perfect square.

(b) $28k = 2^2 \times 7 \times k$
Smallest positive integer value of $k = 2 \times 7^2$
$=$ **98**

Must-Know Concept:
For $28k$ to be a perfect cube, the power of each of its prime factors must be a multiple of 3.

19. (a) $\mathbf{P} = \begin{pmatrix} \mathbf{28} & \mathbf{170} & \mathbf{95} \\ \boldsymbol{x} & \mathbf{176} & \mathbf{89} \end{pmatrix}$

(b) $\mathbf{R} = \begin{pmatrix} 28 & 170 & 95 \\ x & 176 & 89 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{1314} \\ \mathbf{1236 + 3x} \end{pmatrix}$

Must-Know Concept:
The product of a 2 × 3 matrix and a 3 × 1 matrix gives a resulting 2 × 1 matrix.

(c) The amount of money collected in week 1 is \$1314.
The amount of money collected in week 2 is \$(1236 + 3x).

(d) $1314 = 1236 + 3x$

$3x = 1314 - 1236$

$= 78$

$x = \frac{78}{3}$

$=$ **26**

20. (a) Difference between the first and the last number
$= 39 - 11$
$= 28$
Number subtracted each time $= 28 \div 4$
$= 7$

$p = 39 - 7$
$=$ **32**
$q = 32 - 7$
$=$ **25**
$r = 25 - 7$
$=$ **18**

Must-Know Concept:
The pattern in the sequence is subtracting 7 to obtain the next term.

(b) $T_1 = 39$
$= -7(1) + 46$
$T_2 = 32$
$= -7(2) + 46$
$T_3 = 25$
$= -7(3) + 46$
$T_4 = 18$
$= -7(4) + 46$
nth term: $\mathbf{-7n + 46}$

(c) For −246 to be a term in the sequence,
$-246 = -7n + 46$
$-7n = -292$
$n = 41\frac{5}{7}$
Since n **is not an integer**, −246 is not a term in the sequence.

Must-Know Concept:
Equate the expression to −246. If n is a whole number, then −246 is a term of this sequence.

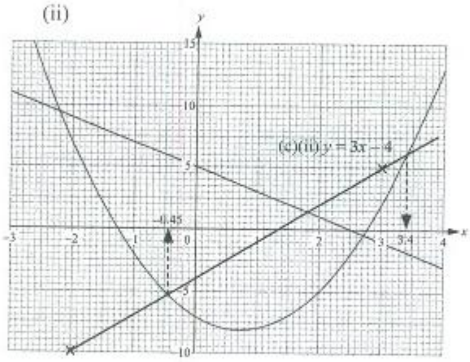
21. (a) Let $y = k$.
For $2x^2 - 3x - 7 = k$ to have no solutions, the horizontal line $y = k$ **does not have any intersection points** with the curve and **lies entirely below** the curve.

(b) $y = 2x^2 - 3x - 7$..............①
$y = 5 - 2x$.......................②
Substitute ① into ②:
$2x^2 - 3x - 7 = 5 - 2x$
$\mathbf{2x^2 - x - 12 = 0}$

Must-Know Concept:
The solutions to the quadratic equation are the two points of intersection of the graphs.

(c) (i)
$$2x^2 - 6x - 3 = 0$$
$$2x^2 - 3x - 7 - 3x + 4 = 0$$
$$2x^2 - 3x - 7 = 3x - 4$$
Equation of the straight line:
$\mathbf{y = 3x - 4}$

(ii)



From the graph,
$x = \mathbf{-0.45}$ or $x = \mathbf{3.4}$

Must-Know Concept:
The solution is the two points of intersection of the graphs.

22. (a) $A = \dfrac{12.17(1.615 + 2)}{5 - 1.615}$
$= \mathbf{13.00}$ (2 d.p.)

(b)
$$A = \frac{b(c + 2)}{5 - c}$$
$$5A - cA = bc + 2b$$
$$bc + cA = 5A - 2b$$
$$c(b + A) = 5A - 2b$$
$$\mathbf{c = \frac{5A - 2b}{b + A}}$$

Must-Know Concept:
Make c the subject by shifting terms that have c to the left-hand side and terms that do not have c to the right-hand side, and simplify the equation.

23. (a) Let the volume of the shampoo be x and the cost of the shampoo be y.
For the cost of the shampoo to be directly proportional to the quantity of shampoo,
$y = kx$, where k is a constant.
When $y = 30.80$ and $x = 400$,
$30.80 = k(400)$
$k = \frac{77}{1000}$
When $y = 19.25$ and $x = 250$,
$19.25 = k(250)$
$k = \frac{77}{1000}$
When $y = 5.95$ and $x = 75$,
$5.95 = k(75)$
$k = \frac{119}{1500}$
Since the **values of k are not the same**, the cost of the shampoo is not directly proportional to the quantity of shampoo. (shown)

(b) Let the height of the 75 ml bottle be h cm.
$$\left(\frac{h}{18.4}\right)^3 = \frac{75}{250}$$
$$\frac{h}{18.4} = \sqrt[3]{\frac{3}{10}}$$
$$h = 12.3 \text{ (3 s.f.)}$$
Height of the 75 ml bottle = **12.3** cm

Must-Know Concept:
For similar figures, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

Paper 2

1. (a)
$$\frac{x + 4}{5} \geqslant \frac{2 - x}{3}$$
$$3(x + 4) \geqslant 5(2 - x)$$
$$3x + 12 \geqslant 10 - 5x$$
$$3x + 5x \geqslant 10 - 12$$
$$8x \geqslant -2$$
$$x \geqslant \mathbf{-\frac{1}{4}}$$

Must-Know Concept:
Shift the x terms to one side of the inequality and the number terms to the other side.

(b)
$$\frac{2y}{5 - 2y} - \frac{3y}{(5 - 2y)^2} = \frac{2y(5 - 2y) - 3y}{(5 - 2y)^2}$$
$$= \frac{10y - 4y^2 - 3y}{(5 - 2y)^2}$$
$$= \frac{7y - 4y^2}{(5 - 2y)^2}$$
$$= \mathbf{\frac{y(7 - 4y)}{(5 - 2y)^2}}$$

(c)
$$\frac{18h^3j^3}{5k^3} \div \frac{3h^5k}{10j^2} = \frac{18h^3j^3}{5k^3} \times \frac{10j^2}{3h^5k}$$
$$= \frac{180h^3j^5}{15h^5k^4}$$
$$= \mathbf{\frac{12j^5}{h^2k^4}}$$

(d) $\left(\frac{16t^8}{v^{12}}\right)^{-\frac{1}{4}} = \left(\frac{v^{12}}{16t^8}\right)^{\frac{1}{4}}$

$= \frac{v^{\left(12 \times \frac{1}{4}\right)}}{16^{\frac{1}{4}} t^{\left(8 \times \frac{1}{4}\right)}}$

$= \mathbf{\frac{v^3}{2t^2}}$

Must-Know Concept:
Use the Law of Indices: $\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}$.

(e) $\frac{2}{x-2} + \frac{3}{x+4} = 1$

$\frac{2(x+4) + 3(x-2)}{(x+4)(x-2)} = 1$

$2(x+4) + 3(x-2) = (x+4)(x-2)$

$2x + 8 + 3x - 6 = x^2 - 2x + 4x - 8$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\therefore x = \mathbf{5}$ or $\mathbf{-2}$

Must-Know Concept:
Perform cross multiplication and solve for x.

2. (a) (i) Mean number of passengers each month

$= \frac{5.41 \times 10^7}{12}$

$= 4\ 508\ 333$ (nearest whole number)

$= 4.508\ 333 \times 10^6$

$=$ **4.51 million** (3 sig. fig.)

Must-Know Concept:
1 million $= 1 \times 10^6$

(ii) Percentage of the year's passengers that passed through in December

$= \frac{5.09 \times 10^6}{5.41 \times 10^7} \times 100\%$

$=$ **9.41%** (3 sig. fig.)

(b) Number of people from Europe that visited Singapore in 2014 $= \frac{101.6}{100} \times 1.591 \times 10^6$

$= 1\ 616\ 456$

$= \mathbf{1.616\ 456 \times 10^6}$

Must-Know Concept:
Since the answer obtained is exact, we leave the answer in the exact value in standard form.

(c) Number of international visitors in 2013

$= \frac{100}{96.9} \times 1.509 \times 10^7$

$= 15\ 572\ 755$ (nearest whole number)

$= \mathbf{1.557\ 275\ 5 \times 10^7}$

3. (a) When $t = 0$,

$y = 50 \times 2^0$

$= \mathbf{50}$

Must-Know Concept:
Infer that 'at the start' means $t = 0$.

(b) When $t = 0.5$,

$y = 50 \times 2^{0.5}$

$=$ **71** (nearest whole number)

Must-Know Concept:
Substitute $t = 0.5$ into the given equation to find the value of p.

(c) Refer to Appendix 3.

(d) From the graph,
when $y = 500$, $t = 3.32$
Number of hours it takes for the bacteria to reach $500 =$ **3.32**

(e) (i) Gradient $= \frac{265 - 130}{2.5 - 1.5}$

$=$ **135**

(ii) This gradient represents the rate of increase in the number of bacteria at the 2nd hour.

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient using the formula $\frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

4. (a) Length of the line $AB = \sqrt{(-4-4)^2 + [10-(-2)]^2}$

$= \sqrt{208}$

$=$ **14.4 units** (3 sig. fig.)

Must-Know Concept:
The formula to find the length of a line segment is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(b) Gradient of line $AB = \frac{10-(-2)}{-4-4}$

$= \frac{12}{-8}$

$= -1.5$

$y - 10 = -1.5[x - (-4)]$

$y - 10 = -1.5x - 6$

$y = -1.5x + 4$

Equation of line AB: $\mathbf{y = -1.5x + 4}$

Must-Know Concept:
Find the gradient of line AB using the given points. Then, substitute the values in the form $(y - y_1) = m(x - x_1)$ to find the equation of line AB.

(c) (i) $3x + 2y = 5$

$2y = -3x + 5$

$y = -1.5x + 2.5$

Since line p and line AB have the same gradient, they are parallel lines. However, they do not have the same y-intercept. Therefore, line p does not intersect line AB.

Must-Know Concept:
Both lines have the same gradient but do not have a common point, i.e. the y-intercept, therefore the two lines are parallel.

(ii) $3x + 2y = 5$

$2y = 5 - 3x$

$y = 2.5 - 1.5x$ .... ①

$6y = 4x - 37$.......................... ②

Substitute ① into ②:

$6(2.5 - 1.5x) = 4x - 37$

$15 - 9x = 4x - 37$

$4x + 9x = 15 + 37$

$13x = 52$

$x = 4$

Substitute $x = 4$ into ①:

$y = 2.5 - 1.5(4)$

$= -3.5$

Point of intersection: **(4, –3.5)**

Must-Know Concept:
Solve simultaneous equations of both lines to find the point of intersection.

5. (a) $\angle ACB = 122° - 64°$

$= 58°$

Using Cosine Rule,

$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB$

$= 104^2 + 135^2 - 2(104)(135)\cos 58°$

$= 14\ 160.867$

$AB = \sqrt{14\ 160.867}$

$= 118.999$

$=$ **119 m** (3 sig. fig.)

Must-Know Concept:
Given two sides and the included angle, use Cosine Rule to find the remaining side.

(b) Let N_B be the North direction of B.

$\angle CBN_B = 180° - 122°$ (int. ∠s, $CN_C // BN_B$)

$= 58°$

Using Sine Rule,

$\frac{\sin \angle ABC}{AC} = \frac{\sin \angle ACB}{AB}$

$\frac{\sin \angle ABC}{104} = \frac{\sin 58°}{118.999}$

$\sin \angle ABC = 0.741\ 157$

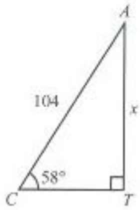
$\angle ABC = 47.830°$

Bearing of A from $B = 360° - 58° + 47.830°$

$=$ **349.8°** (1 d.p.)

Must-Know Concept:
Given two sides and a non-included angle, use Sine Rule to find the opposite angle.

(c) Let x be the shortest distance of T from A.



$\sin 58° = \frac{x}{104}$

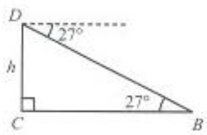
$x = 104 \sin 58°$

$= 88.197$ m

Shortest distance of T from A = **88.2 m** (3 sig. fig.)

Must-Know Concept:
The shortest distance is always perpendicular to the line of origin (line CB) to the point indicated (point A).

(d) Let h be the height of the cliff.



$\tan 27° = \frac{h}{135}$

$h = 135 \tan 27°$

$= 68.7859$ m

Height of the cliff = **68.8 m** (3 sig. fig.)

Must-Know Concept:
Since C is the foot of the cliff and D is the top of the cliff vertically above C, CD forms a right angle with CB.

6. (a) $\overrightarrow{RC} = \overrightarrow{RD} + \overrightarrow{DC}$

$= \frac{1}{3}\overrightarrow{AD} + \overrightarrow{DC}$

$= \frac{1}{3}\overrightarrow{BC} + \overrightarrow{AB}$

$= \frac{1}{3}(3\mathbf{q}) + 2\mathbf{p}$

$= \mathbf{q} + 2\mathbf{p}$

(b) (i) Since $\overrightarrow{RC} = 2\overrightarrow{ST}$, RC is parallel to ST.

$\angle CRD = \angle TSD$ (corr. ∠s, $CR // TS$)

$\angle RCD = \angle STD$ (corr. ∠s, $RC // ST$)

$\angle RDC = \angle SDT$ (common ∠)

By AAA, **ΔRCD and ΔSTD are similar**. (shown)

Must-Know Concept:
One method to show that two triangles are similar is to show that there are three pairs of equal corresponding angles.

(ii) Area of $\triangle STD$: Area of $\triangle RCD = 1^2 : 2^2$
$= 1 : 4$
Area of $\triangle RCD$: Area of $\triangle ACD$
$= \frac{1}{2} \times RD \times h : \frac{1}{2} \times AD \times h$
$= RD : AD$
$= 1 : 3$
$= 4 : 12$
Area of $\triangle ACD$: Area of parallelogram $ABCD$
$= 1 : 2$
$= 12 : 24$
Area of $\triangle STD$: Area of parallelogram $ABCD$
$= \mathbf{1 : 24}$

Must-Know Concept:
$\triangle RCD$ and $\triangle ACD$ have the same vertical height since their bases lie on the same line.

(iii) Given $\triangle QAB \equiv \triangle RCD$,
$\overrightarrow{QC} = \frac{2}{3}\overrightarrow{BC}$
$= 2\mathbf{q}$
$\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR}$
$= 2\mathbf{q} - (\mathbf{q} + 2\mathbf{p})$
$= \mathbf{q} - 2\mathbf{p}$

7. (a) Length of the box $= x$ cm
Height of the box $= \mathbf{(x - 5)}$ **cm**
Width of the box $= 2(x - 5)$ cm
$= \mathbf{(2x - 10)}$ **cm**

(b) $2(x)(x - 5) + 2(2x - 10)(x - 5) + (x)(2x - 10) = 48$
$2x(x - 5) + (4x - 20)(x - 5) + 2x^2 - 10x - 48 = 0$
$2x^2 - 10x + 4x^2 - 20x - 20x + 100 + 2x^2 - 10x - 48 = 0$
$8x^2 - 60x + 52 = 0$
$\mathbf{2x^2 - 15x + 13 = 0}$ (shown)

Must-Know Concept:
Since it is an open box, do not include the area of the top surface.

(c) $x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(13)}}{2(2)}$
$= \frac{15 \pm \sqrt{121}}{4}$
$= \mathbf{6.5}$ or $\mathbf{1}$

(d) When $x = 1$, the height of the box is $(x - 5) = (1 - 5) = -4$ cm. The height of the box cannot be negative. Therefore, $x = 1$ must be rejected.

Must-Know Concept:
Substitute both values of x into the expressions for length, height and width. Identify the value of x that gives dimensions that are not valid.

(e) Length of the box = 6.5 cm
Height of the box = 6.5 – 5
= 1.5 cm
Width of the box = 2(6.5) – 10
= 13 – 10
= 3 cm
Volume of the box = 6.5 cm × 3 cm × 1.5 cm
= **29.25 cm³**

8. (a) $OM \perp AB$ ($\perp$ bisector of chord)
$\therefore \angle OMA = 90°$
$\cos \angle AOM = \frac{20}{30}$
$\angle AOM = \cos^{-1} \frac{20}{30}$
$= 48.189°$
$\angle AOB = 2 \times 48.189°$
$= \mathbf{96.4°}$ (3 sig. fig.) (shown)

Must-Know Concept:
Recall the angle properties of circles.
The perpendicular bisector of a chord of a circle passes through the centre of the circle.

(b) Area of triangle AOB
$= \frac{1}{2} \times 30 \text{ cm} \times 30 \text{ cm} \times \sin 96.4°$
$= 447.195 \text{ cm}^2$
Area of sector $AOB = \frac{96.4°}{360°} \times \pi \times 30 \text{ cm} \times 30 \text{ cm}$
$= 757.123 \text{ cm}^2$
Area of the shaded area
$= 757.123 \text{ cm}^2 - 447.195 \text{ cm}^2$
$= 309.928 \text{ cm}^2$
$= \mathbf{310 \text{ cm}^2}$ (3 sig. fig.)

(c) (i) Volume of water in the trough
$= 309.928 \text{ cm}^2 \times 1.5 \text{ m}$
$= 309.928 \text{ cm}^2 \times 150 \text{ cm}$
$= 46\,489.2 \text{ cm}^3$
$= \mathbf{46\,500 \text{ cm}^3}$ (3 sig. fig.)

(ii) Volume of the trough
$= \frac{1}{2} \times \pi \times 30 \text{ cm} \times 30 \text{ cm} \times 150 \text{ cm}$
$= 212\,057.5041 \text{ cm}^3$
Volume of water that must be added to fill the trough
$= 212\,057.5041 \text{ cm}^3 - 46\,489.2 \text{ cm}^3$
$= 165\,568.3041 \text{ cm}^3$
$= 165.568\,304\ 1\ l$
$= \mathbf{170}\ \boldsymbol{l}$ (nearest 10 litres)

Must-Know Concept:
A cross-section refers to the shape of a uniform object when it is cut straight perpendicularly along the object.

9. (a) (i) (a) Median position = 80 ÷ 2
= 40
Median speed = **42 km/h**

Must-Know Concept:
The median is the value of the middle position of all the cars.

(b) Upper quartile position $= \frac{3}{4} \times 80$
$= 60$
Upper quartile = 46 km/h
Lower quartile position $= \frac{1}{4} \times 80$
$= 20$
Lower quartile = 37 km/h
Interquartile range
= 46 km/h − 37 km/h
= 9 km/h

Must-Know Concept:
The value of the lower quartile is at the 25% position and the value of the upper quartile is at the 75% position.

(ii) Number of cars that drove slower than 50 km/h = 73
Number of cars that exceeded the speed limit = 80 − 73
= 7
Percentage of the cars that exceeded the speed limit $= \frac{7}{80} \times 100\%$
= 8.75%

Must-Know Concept:
Find the number of cars that drove at a speed of 50 km/h and below. Then, minus this value from the total number of cars to find the number of cars that exceeded the speed limit.

(iii) The cars in the afternoon drove at a slower speed as the median speed of the cars in the afternoon was 40 km/h, as compared to the median speed of the cars in the morning at 42 km/h.
The speeds of the cars in the afternoon was more consistent that the speeds of the cars in the morning as the interquartile range for the speeds of the cars in the afternoon is smaller.

Must-Know Concept:
Compare both distributions using their median values and interquartile ranges as these values are found in previous parts.

(b) (i) Total number of cars
= 34 + 17 + 13 + 11 + 5
= 80

P(car has no passengers) $= \frac{34}{80}$
$= \mathbf{\frac{17}{40}}$

(ii) (a) P(both cars had four passengers)
$= \frac{5}{80} \times \frac{4}{79}$
$= \mathbf{\frac{1}{316}}$

(b) P(one car had more than two passengers and one had fewer than two passengers)
$= \left(\frac{16}{80} \times \frac{51}{79}\right) + \left(\frac{51}{80} \times \frac{16}{79}\right)$
$= \mathbf{\frac{102}{395}}$

Must-Know Concept:
As the selections are without replacement, remember to minus one from the total number after every selection.

10. (a) Length of Nurul's walk = 27.9 cm
Actual length of Nurul's route = 27.9 × 20 000
= 558 000 cm
= 5.58 km

(b) Total time she walks each time
= 5.58 km ÷ 6.5 km/h
$= \frac{279}{325}$ h
Total time she walks in a week $= 3 \times \frac{279}{325}$ h
$= 2\frac{187}{325}$ h
= 154.523 min
Since 155 min > 150 min, Nurul **meets** the weekly time target recommended in the health advice.

Must-Know Concept:
The question mentions weekly time target, therefore we multiply the time she walks by three as Nurul takes three brisk walks every week.

(c) Total time she jogs each time
= 5.58 km ÷ 9.5 km/h
$= \frac{279}{475}$ h
Total time she jogs in a week $= 3 \times \frac{279}{475}$ h
$= 1\frac{362}{475}$ h
= 105.7263 min
Amount of calories used for jogging
$= \frac{105.7263}{30} \times 345$
= 1215.852
Amount of calories used for walking
$= \frac{155.0769}{30} \times 150$
= 775.3845
2 × 775.3845 = 1550.769
Since 106 min > 75 min, Nurul **will meet** the weekly time target recommended in the health advice. However, she **will not** use more than double the amount of calories.

Must-Know Concept:
Compare the amount of calories used when she brisk walks and when she jogs.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2016 EXAMINATION PAPER

Paper 1

1. (a) $3(2x-1)+1 = 6x-3+1$
$= \mathbf{6x-2}$

Must-Know Concept:
Expand 3 into the term $(2x-1)$. Group and simplify the like terms together.

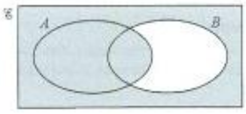
(b) $6x + 18xy = \mathbf{6x(1+3y)}$

Must-Know Concept:
Factorise the common term '6x'.

2. $4ax + 12by - 16ay - 3bx = 4ax - 16ay - 3bx + 12by$
$= 4a(x-4y) - 3b(x-4y)$
$= \mathbf{(4a-3b)(x-4y)}$

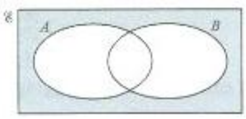
Must-Know Concept:
Group similar terms together and factorise the common factors out.

3. (a)



Must-Know Concept:
The complement of a set A, relative to the universal set ℰ, is the set that contains all the elements that are not in A but in ℰ. The union of two sets is the set of all elements which are either in one of the set or both.

(b)



Must-Know Concept:
The intersection of two sets is the set of all elements which are common in both sets.

4. $(5n+1)^2 - (5n-1)^2$
$= (5n+1+5n-1)(5n+1-5n+1)$
$= (10n)(2)$
$= \mathbf{20n}$, which is a multiple of 20. (shown)

Must-Know Concept:
'Multiple of 20' implies that the expression is divisible by 20.

5. Length + Breadth $= 22 \div 2$
$= 11$ cm
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 8 \times 3 \times 6$
Height of the cuboid = **6** cm

Must-Know Concept:
Express 144 as its prime factors first. Then, find the dimensions of the cuboid such that the length and the breadth is 11 cm.

6. Area of $\triangle ABC = \frac{1}{2} \times 18.7 \times 12.8 \times \sin \angle ABC$
$= 119.68 \sin \angle ABC$
$119.68 \sin \angle ABC = 58.6$
$\angle ABC = \sin^{-1}(0.489\ 639)$ or $180° - \sin^{-1}(0.489\ 639)$
$= \mathbf{29.3°}$ or $\mathbf{150.7°}$ (1 d.p.)

Must-Know Concept:
Area of a triangle $= \frac{1}{2}ab \sin C$
Note that $\sin(180° - \theta) = \sin \theta$ where $0 < \theta < 90°$.

7. Using similar figures,
$\left(\frac{h}{30}\right)^3 = \frac{1}{2}$
$h^3 = \frac{1}{2} \times 30^3$
$= 13\ 500$
$h = \mathbf{23.8}$ (3 s.f.)

Must-Know Concept:
Note that the smaller and larger cones in the figure are geometrically similar. Hence, we can apply the formula $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

8. There are no individual markings to show the cost of gas over the 4 years on the vertical axis. Therefore, we are not able to determine the magnitude of increase of the cost of gas, and this may lead to a misinterpretation of the graph.

Must-Know Concept:
Identify possible misleading information presented in data.

9. $\frac{3}{(x-4)^2} - \frac{1}{(4-x)} = \frac{3}{(x-4)^2} + \frac{1}{(x-4)}$
$= \frac{3}{(x-4)^2} + \frac{x-4}{(x-4)^2}$
$= \frac{3+x-4}{(x-4)^2}$
$= \mathbf{\frac{x-1}{(x-4)^2}}$

Must-Know Concept:
$(x-4) = -(4-x)$

10. (a) $\angle ADC = \frac{3}{2}x°$ (∠ at centre = 2∠s at circumference)

Alternatively,
$\angle ADC = 180° - (2x+5)°$ (∠s in opp. segments)
$= (180 - 2x - 5)°$
$= \mathbf{(175 - 2x)°}$

Must-Know Concept:
Note that an angle at centre is twice the angle at circumference.

(b) From part (a),
$\frac{3}{2}x = 175 - 2x$
$\frac{7}{2}x = 175$
$x = \mathbf{50}$

Must-Know Concept:
Note that angles in opposite segments are supplementary.

11. (a) $9 - 8x + x^2 = x^2 - 8x + 9$
$= x^2 - 8x + (-4)^2 - (-4)^2 + 9$
$= (x-4)^2 - 7$
$= \mathbf{-7 + (x-4)^2}$

Must-Know Concept:
Complete the square method.

(b) (4, −7)

12. 1 US dollar = 1.242 Singapore dollars
4.48 US dollars = 4.48 × 1.242
= 5.564 16 Singapore dollars
1 gallon ⟶ 5.564 16 Singapore dollars
3.785 litres ⟶ 5.564 16 Singapore dollars
1 litre ⟶ 5.564 16 ÷ 3.785
= 1.47 Singapore dollars (nearest cent)

Diesel is cheaper in **Hawaii**.

Must-Know Concept:
Find the cost of diesel in both countries in the same unit. For example, you can compare the price of 1 litre of diesel in Singapore and in Hawaii in Singapore dollars or 1 gallon of diesel in Singapore and in Hawaii in US dollars.

13. (a) When $T = 0$,
$0 = 21 - \frac{h}{120}$
$\frac{h}{120} = 21$
$h = 2520$

Height of the aircraft above sea level = **2520** m

Must-Know Concept:
Substitute $T = 0$ and solve for h.

(b) Let h_1 and h_2 be the heights of the two aircraft, where $h_2 > h_1$.
$21 - \frac{h_1}{120} = 21 - \frac{h_2}{120} + 25$
$\frac{h_2}{120} - \frac{h_1}{120} = 25$
$h_2 - h_1 = 25 \times 120$
$= 3000$

Difference between the heights of the two aircraft = **3000** m

Must-Know Concept:
As the difference in temperature is 25°, substitute $T = 25°$.

14. Length + Breadth = 63 − 15
= 48 units
Length = 46 − 10
= 36 units
Breadth = 48 − 36
= 12 units
x-coordinate of R = 15 + 12
= 27
y-coordinate of R = 10 + 12
= 22
Coordinates of R = **(27, 22)**

Must-Know Concept:
Since the rectangles are congruent, they have the same length and breadth.

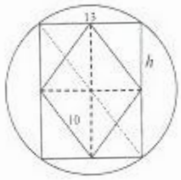
15. (a) $\angle ECD = 180° - 45° - 38°$ (∠ sum of △)
$= 97°$
$\angle CAD = 180° - 97° - 22° - 38°$ (∠ sum of △)
$= 23°$
$\angle ABC = 180° - 34° - 23°$ (∠s between two parallel lines, $BC \parallel AD$)
$= 123°$
Reflex $\angle ABC = 360° - 123°$ (∠s at a point)
$= \mathbf{237°}$

Must-Know Concept:
A reflex angle is greater than 180° but smaller than 360°.
When there are parallel lines, look out for alternate, corresponding or interior angles.

(b) Since $\angle ECD \neq 90°$, by the property of right angle in a semicircle, a semicircle with AD as diameter does not pass through C.

Must-Know Concept:
By geometrical properties of circles, there is a right angle in a semicircle.

16.



(a) Radius of the circle = 10 cm
Circumference of the circle = $2\pi(10)$
= **62.8** cm (3 s.f.)

Must-Know Concept:
Note that all sides of a rhombus are equal.
Circumference of a circle = $2\pi r$

(b) Let the length of the longer side of the rectangle be $2h$ cm.
By Pythagoras' Theorem,
$h^2 + 6.5^2 = 10^2$
$h = \sqrt{57.75}$ cm
Area of the rectangle = $2 \times \sqrt{57.75} \times 13$
= **198** cm² (3 s.f.)

Must-Know Concept:
Where there are right-angled triangles, consider using Pythagoras' Theorem: $c^2 = a^2 + b^2$

17. (a) $y = ka^x$
Substitute $x = -2$ and $y = 100$ into the equation:
$100 = ka^{(-2)}$
$100 = ka^2$ .... ①

Substitute $x = 0$ and $y = 4$ into the equation:
$4 = ka^0$
$k = \mathbf{4}$

Substitute $k = 4$ into ①:
$100 = 4a^2$
$a^2 = 25$
$a = \mathbf{5}$ or -5 (rejected)

Must-Know Concept:
Substitute the two pairs of coordinates in the equation to find k and a.

(b) Gradient of line $AB = \frac{100-4}{-2-0}$
$= \frac{96}{-2}$
$= -48$
Equation of the line AB: $\mathbf{y = -48x + 4}$

Must-Know Concept:
Equation of the line: $y = mx + c$
Substitute in $(-2, 100)$ and $(0, 4)$ to find the values of m and c.

18. (a) $784 = \mathbf{2^4 \times 7^2}$

(b) Since $784 = 2^4 \times 7^2 = (2^2 \times 7)^2$, it is a perfect square.

(c) $784 \times \frac{m}{n} = 2^4 \times 7^2 \times \frac{m}{n}$
$= 2^3 \times 7^3$, which is a perfect cube.
$\therefore m = \mathbf{7}, n = \mathbf{2}$

Must-Know Concept:
For $784 \times \frac{m}{n}$ to be a perfect cube, the index of each of its prime factors must be a multiple of 3.

19. (a) Total petrol consumption in 2015
$= \frac{19\,629}{100} \times 6.7$
$= 1315.143\ l$
Total amount Huma paid for petrol in 2015
$= 1315.143 \times \$2.42$
$= \mathbf{\$3182.65}$ (nearest cent)

(b) $135\% \longrightarrow 19\,629$ km
$100\% \longrightarrow \frac{19\,629}{135} \times 100$
$= 14\,540$ km

Total distance she drove in 2014 = **14 540** km

Must-Know Concept:
The distance she drove in 2014 is 100% and the distance she drove in 2015 is 35% more, so the distance she drove in 2015 is 135%.

20. (a) Actual length of the river Rhine
$= 49.3 \times 2\,500\,000$
$= 123\,250\,000$ cm $\div 100\,000$
$= \mathbf{1232.5}$ km

Must-Know Concept:
1 km = 100 000 cm

(b) 1 cm : 2 500 000 cm
1 cm : 25 km
1 cm^2 : 625 km^2

Area of Switzerland on the map $= 41\,285 \div 625$
$= \mathbf{66.056}$ cm^2

Must-Know Concept:
Convert the scale to cm^2 : km^2. The scale becomes 1 cm^2 : n^2 km^2.

21. (a) Number of students who did not overestimate the mass = 84
Number of students who overestimated the mass
$= 200 - 84$
$= 116$
Probability that a student, chosen at random, overestimated the mass $= \frac{116}{200}$
$= \mathbf{\frac{29}{50}}$

Must-Know Concept:
Find the number of students who estimated the mass to be 300 g and less. Then, minus this value from the total number of students to find the number of students who overestimated the mass.

(b) 10% of the actual mass = 10% × 300
= 30 g
Lower bound = 300 – 30
= 270 g
Upper bound = 300 + 30
= 330 g
Number of students who estimated the mass to be 270 g and below = 64
Number of students who estimated the mass to be 330 g and below = 100
Number of students who estimated the mass within 10% of the actual mass
= 100 – 64
= **36**

Must-Know Concept:
Find the number of students within the specific range.

22. (a) Bearing of B from $A = 360° - 47°$
$= \mathbf{313°}$

Must-Know Concept:
When calculating bearings, always move in a clockwise direction to identify the required angle.

(b) Length of the line within the shaded area = 5.1 cm
Distance the ship travels for which the light is visible = 5.1 × 10
= 51 km
Length of time for which the light is visible from the ship = 51 ÷ 35
$= 1\frac{16}{35}$ h
= **1** h **27** min (nearest minute)

Must-Know Concept:
$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

23. (a) $|\overrightarrow{AB}| = \sqrt{(-8)^2 + 15^2}$
$= \sqrt{289}$
= **17** units

Must-Know Concept:
The formula for the magnitude of a vector is $\sqrt{x^2 + y^2}$.

(b) $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$
$= \begin{pmatrix} 4 \\ 20 \end{pmatrix} + (-2\overrightarrow{AB})$
$= \begin{pmatrix} 4 \\ 20 \end{pmatrix} + (-2)\begin{pmatrix} -8 \\ 15 \end{pmatrix}$
$= \begin{pmatrix} 4 + 16 \\ 20 - 30 \end{pmatrix}$
$= \begin{pmatrix} 20 \\ -10 \end{pmatrix}$

Coordinates of D = **(20, –10)**

Must-Know Concept:
The opposite direction of a vector $\overrightarrow{AB}$ is $-\overrightarrow{BA}$.

(c) Since $\overrightarrow{DC} \,//\, \overrightarrow{AB}$ and $DC = 2AB$, $ABCD$ is a **trapezium**.

24. Area of the square = $(2r)^2$
$= 4r^2$

Area of quadrant $EGH = \frac{1}{4} \times \pi \times (\sqrt{2}r)^2$
$= \frac{1}{2}\pi r^2$

Area of semicircle $OGFE = \frac{1}{2} \times \pi \times r^2$
$= \frac{1}{2}\pi r^2$

Area of triangle $HGE = \frac{1}{2} \times \sqrt{2}r \times \sqrt{2}r$
$= r^2$

Area of region $GFEH = \frac{1}{2}\pi r^2 + r^2$

Area of the shaded region $= \frac{1}{2}\pi r^2 + r^2 - \frac{1}{2}\pi r^2$
$= r^2$

Area of the unshaded region = $4r^2 - r^2$
$= 3r^2$

Fraction of the square $ABCD$ that is not shaded $= \frac{3r^2}{4r^2}$
$= \mathbf{\frac{3}{4}}$

Must-Know Concept:
Area of a sector $= \frac{\theta}{360} \times \pi r^2$ (where θ is in degrees)
Area of a triangle $= \frac{1}{2} \times$ Base × Height

25. (a) (i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -5\mathbf{a} + 5\mathbf{c}$
$\overrightarrow{AP} = \frac{4}{5}\overrightarrow{AC}$
$= \frac{4}{5}(-5\mathbf{a} + 5\mathbf{c})$
$= -4\mathbf{a} + 4\mathbf{c}$
$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= 5\mathbf{a} + (-4\mathbf{a} + 4\mathbf{c})$
$= \mathbf{a + 4c}$

(ii) $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$
$= 4\mathbf{a} - 4\mathbf{c} + 4\mathbf{a} + 8\mathbf{c}$
$= \mathbf{8a + 4c}$

(b) (i) $\overrightarrow{PQ} = 8\mathbf{a} + 4\mathbf{c}$
$= 4(2\mathbf{a} + \mathbf{c})$
$\overrightarrow{CB} = 10\mathbf{a} + 5\mathbf{c}$
$= 5(2\mathbf{a} + \mathbf{c})$
Since they have the same direction vector, PQ is parallel to CB.

(ii) $CB : PQ =$ **5 : 4**

Must-Know Concept:
Two vectors are parallel if $\mathbf{a}$ can be written as the scalar multiple of $\mathbf{b}$, $\mathbf{a} = k\mathbf{b}$, where k is a scalar.

Paper 2

1. (a) (i) $a = \frac{4(5) - 5(-2)}{5 + (-2)}$
$= \frac{30}{3}$
= **10**

Must-Know Concept:
Substitute $b = 5$ and $c = -2$ into the given equation.

(ii) $a = \frac{4b - 5c}{b + c}$
$a(b + c) = 4b - 5c$
$ab + ac = 4b - 5c$
$ab - 4b = -5c - ac$
$b(a - 4) = -5c - ac$
$b = \frac{-5c - ac}{a - 4}$
$= \mathbf{\frac{5c + ac}{4 - a}}$

Must-Know Concept:
Make b the subject by shifting terms that have b to the left-hand side and terms that do not have b to the right-hand side, and simplify the equation.

(b) $\frac{2x-3}{4} + \frac{x}{3} = 3$

$\frac{3(2x-3)+4x}{12} = 3$

$6x - 9 + 4x = 36$

$10x = 45$

$x = \mathbf{4.5}$

Must-Know Concept:
Convert the denominators of the fractions on the left-hand side such that it becomes a single fraction. Perform cross multiplication and solve for x.

(c) $4x - 3y = 18$ .... ①

$6x + 2y = 1$ .... ②

① × 3:

$12x - 9y = 54$

$12x = 54 + 9y$ .... ③

② × 2:

$12x + 4y = 2$ .... ④

Substitute ③ into ④:

$54 + 9y + 4y = 2$

$13y = 2 - 54$

$= -52$

$y = -4$

Substitute $y = -4$ into ③:

$12x = 54 + 9(-4)$

$= 18$

$x = 1.5$

$\therefore x = \mathbf{1.5}, y = \mathbf{-4}$

(d) $\frac{9x^2 - 4}{3x^2 - 10x - 8} = \frac{(3x+2)(3x-2)}{(3x+2)(x-4)}$

$= \mathbf{\frac{3x-2}{x-4}}$

Must-Know Concept:
Factorise the numerator and denominator. Cancel out the common terms.

2. (a) $\mathbf{M} = 12\mathbf{L}$

$= 12\begin{pmatrix} 8 & 5 \\ 2 & 4 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{96} & \mathbf{60} \\ \mathbf{24} & \mathbf{48} \end{pmatrix}$

Must-Know Concept:
When a matrix is multiplied by a scalar, every element in the matrix is multiplied by the scalar.

(b) $\mathbf{N} = \begin{pmatrix} \mathbf{40} \\ \mathbf{65} \end{pmatrix}$

(c) $\mathbf{P} = \mathbf{MN}$

$= \begin{pmatrix} 96 & 60 \\ 24 & 48 \end{pmatrix}\begin{pmatrix} 40 \\ 65 \end{pmatrix}$

$= \begin{pmatrix} 96(40) + 60(65) \\ 24(40) + 48(65) \end{pmatrix}$

$= \begin{pmatrix} \mathbf{7740} \\ \mathbf{4080} \end{pmatrix}$

Must-Know Concept:
The product of a 2 × 2 matrix and a 2 × 1 matrix gives a resulting 2 × 1 matrix.

(d) The elements of **P** represent the total amount of money Yvette can earn from the 12-week block of sessions on weekdays and on weekends respectively.

(e) New price for each basic session = 95% × \$40

= \$38

New price of each advanced session

= 95% × \$65

= \$61.75

Total amount of money she earns

= (12 + 7)(12 × \$38) + (6 + 3)(12 × \$61.75)

= 19 × \$456 + 9 × \$741

= **\$15 333**

3. (a) $\angle ABC = \angle CDE$ (interior angles of a regular polygon)

$AB = CD$

$BC = DE$

$\therefore \triangle ABC \equiv \triangle CDE$ (SAS)

Must-Know Concept:
Two triangles are congruent if they have the same shape and the same size.

(b) (i) Interior angle of a regular polygon

$= \frac{(n-2) \times 180°}{n}$

$160° = \frac{(n-2) \times 180°}{n}$

$160°n = 180°n - 360°$

$180°n - 160°n = 360°$

$20°n = 360°$

$n = \mathbf{18}$

Must-Know Concept:
Each interior angle of an n-sided regular polygon $= \frac{(n-2) \times 180°}{n}$

(ii) $\angle BCA = \frac{180° - 160°}{2}$ (base ∠s of isos. △)

$= \mathbf{10°}$

Must-Know Concept:
Since $AB = BC$, $\angle BCA = \angle BAC$.

(iii) $\angle ACE = 160° - 10° - 10°$
$= 140°$
$\angle CEA = \frac{180° - 140°}{2}$ (base $\angle$s of isos. $\triangle$)
$= 20°$
$\angle DEA = 10° + 20°$
$= \mathbf{30°}$

Must-Know Concept:
Since triangles ABC and CDE are congruent, $AC = CE$

4. (a) $T_5 = 6^2 + 17 = \mathbf{53}$

(b) The value of T_n must be odd for all values of n since the sum of an odd number and an even number will always be odd.

Must-Know Concept:
Observe that each term is the addition of a squared number and an odd number.

(c) $T_n = (n+1)^2 + 5 + 3(n-1)$
$= n^2 + 2n + 1 + 5 + 3n - 3$
$= \mathbf{n^2 + 5n + 3}$ (shown)

(d) $T_p = p^2 + 5p + 3$
$T_{p+1} = (p+1)^2 + 5(p+1) + 3$
$= p^2 + 2p + 1 + 5p + 5 + 3$
$= p^2 + 7p + 9$
$T_{p+1} - T_p = p^2 + 7p + 9 - (p^2 + 5p + 3)$
$= \mathbf{2p + 6}$

(e) When the difference is 4,
$2p + 6 = 4$
$2p = 4 - 6$
$= -2$
$p = -1$
Since p cannot be -1, two consecutive terms of the sequence cannot have a difference of 4.

5. (a) When $x = -3$,
$y = \frac{(-3)^3}{2} - 5(-3) - 2$
$= -0.5$
$\therefore p = \mathbf{-0.5}$

Must-Know Concept:
Substitute $x = -3$ into the equation to find p.

(b) Refer to Appendix 4.

(c) $\frac{x^3}{2} - 5x - 2 = 8 - 2$
$= 6$
Draw the line $y = 6$.
The line only cuts the graph at 1 point, therefore the equation $\frac{x^3}{2} - 5x = 8$ has only one solution.

Must-Know Concept:
Solutions of the curve are the x-intercepts.

(d) Gradient $= \frac{0.8 - (-10)}{-1 - 2}$
$= \frac{10.8}{-3}$
$= \mathbf{-3.6}$

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient via the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

(e) (i) $y = 4 - 3x$

x	-1	2	4
y	7	-2	-8

(ii) x-coordinate of the point of intersection
$= \mathbf{2.85}$

(iii) $\frac{x^3}{2} - 5x - 2 = 4 - 3x$
$\frac{x^3}{2} - 2x - 6 = 0$
$x^3 - 4x - 12 = 0$
$\therefore A = \mathbf{-4}, B = \mathbf{-12}$

Must-Know Concept:
Solve the two equations simultaneously and compare the equation to $x^3 + Ax + B = 0$

6. (a) $\angle ADO = 90°$ (tan $\perp$ rad)
$\angle ABC = 90°$ ($\angle$ in a semicircle)
$\angle ADO = \angle ABC$
$\angle DAO = \angle BAC$ (common $\angle$)
$\therefore$ **$\triangle ABC$ and $\triangle ADO$ are similar. (AAA)** (shown)

Must-Know Concept:
Note that there is a right angle in a semicircle, and tangent AB is perpendicular to radius DO.

(b) Since $AC = 2AO$,
$\frac{\text{Area of } \triangle ADO}{\text{Area of } \triangle ABC} = \left(\frac{1}{2}\right)^2$
$= \frac{1}{4}$
Area of quadrilateral $ODBC = 4 - 1$
$= 3$ units^2
Area of $\triangle BOD$ = Area of $\triangle ADO$
$= 1$ unit^2
Area of $\triangle BOC = 3 - 1$
$= 2$ units^2
Required ratio = **2 : 1**

Must-Know Concept:
Since triangles ABC and ADO are similar, we can apply the formula $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

(c) $\angle DAO = 180° - 90° - 65°$ ($\angle$ sum of $\triangle$)
$= 25°$
$\sin 25° = \frac{3}{OA}$
$OA = \frac{3}{\sin 25°}$
$= 7.0986$ cm
Area of the shaded area $= \pi \times (7.0986)^2 - \pi \times (3)^2$
$= \mathbf{130\ cm^2}$ (3 s.f.)

7. (a) $(5 - 3 =)$ 2 units = 140 mugs
1 unit = 140 ÷ 2
= 70 mugs
$(3 + 7 + 5 =)$ 15 units = 15 × 70
= 1050 mugs

Total number of mugs made in the week = **1050**

Must-Know Concept:
Find the difference between export and local shop, and equate that to 140 mugs. Find the number of mugs representing 1 unit and multiply it by the total number of units.

(b) (i) Number of mugs David paints in an hour
$= \frac{3600}{x}$

Must-Know Concept:
1 h = 3600 s

(ii) Number of mugs Maryam paints in one hour
$= \frac{3600}{x - 50}$

(iii) $4 \times \left(\frac{3600}{x} + \frac{3600}{x-50}\right) = 68$
$\frac{3600}{x} + \frac{3600}{x-50} = 17$
$\frac{3600(x-50) + 3600x}{x(x-50)} = 17$
$3600x - 180\,000 + 3600x = 17x(x - 50)$
$7200x - 180\,000 = 17x^2 - 850x$
$\mathbf{17x^2 - 8050x + 180\,000 = 0}$ (shown)

Must-Know Concept:
68 mugs represent the total number of mugs both of them paint in 4 hours.

(iv) $x = \frac{-(-8050) \pm \sqrt{(-8050)^2 - 4(17)(180\,000)}}{2(17)}$
= **450** or **23.5** (3 s.f.)

(v) Since $x > 50$, $x = 450$.
Number of mugs Maryam paints in one hour
$= \frac{3600}{450 - 50}$
= **9**

8. (a) Using Cosine Rule,
$6^2 = 3^2 + 7^2 - 2(3)(7) \cos \angle BAC$
$\cos \angle BAC = 0.5238$
$\angle BAC = \cos^{-1} 0.5238$
= 58.4125°
= **58.4°** (1 d.p.) (shown)

Must-Know Concept:
Given all three lengths in $\triangle ABC$, we can apply the Cosine Rule to solve for the required angle.

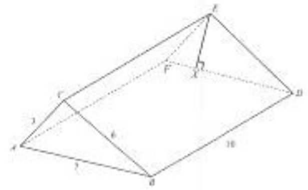
(b) Surface area of the prism
$= 2 \times (\frac{1}{2} \times 3 \times 7 \times \sin 58.4125) + (10 \times 6) + (10 \times 3) + (10 \times 7)$
= **178 cm²** (3 s.f.)

Must-Know Concept:
To find the surface area of a prism, find the area of all its faces.

(c) Let the vertical distance of C above AB be h.
$\sin 58.4° = \frac{h}{3}$
$h = 3 \sin 58.4$
= 2.5552 cm
= **2.56 cm** (3 s.f.)

Must-Know Concept:
Taking $\angle BAC$ as the reference angle, the vertical distance of C from AB is the opposite side while AC is the hypotenuse.

(d)



Let X be the point that is vertically below E on the line DF.
$EX = 2.5552$ cm
By Pythagoras' Theorem,
$2.5552^2 + FX^2 = 3^2$
$FX^2 = 2.4710$
By Pythagoras' Theorem,
$AX^2 = 10^2 + 2.4710$
= 102.4710
$AX = 10.1228$ cm
Angle of elevation of E from $A = \tan^{-1} \frac{2.5552}{10.1228}$
= **14.2°** (1 d.p.)

9. (a) (i) Number of students who took more than 30 minutes = 11
Percentage of students who took longer than expected $= \frac{11}{20} \times 100\%$
= **55%**

(ii) Position of median $= \frac{20 + 1}{2}$
= 10.5
Median time $= \frac{31 + 32}{2}$
= **31.5 min**

Must-Know Concept:
As there is an even number of values, the median is the average of the two middle values when arranged in ascending order.

(iii) The mean may not be useful in summarising the data as there is an extreme value of 69 min which may affect the mean of the data.

Must-Know Concept:
The presence of an extreme value will affect the mean value as it will increase the mean by a significant amount.

(iv) $\Sigma x = 24 + 25 + 25 + 27 + 27 + 27 + 28 + 28 + 29 + 31 + 32 + 32 + 32 + 33 + 37 + 44 + 46 + 47 + 48 + 69$
$= 691$

$\Sigma x^2 = 24^2 + 25^2 + 25^2 + 27^2 + 27^2 + 27^2 + 28^2 + 28^2 + 29^2 + 31^2 + 32^2 + 32^2 + 32^2 + 33^2 + 37^2 + 44^2 + 46^2 + 47^2 + 48^2 + 69^2$
$= 26\,239$

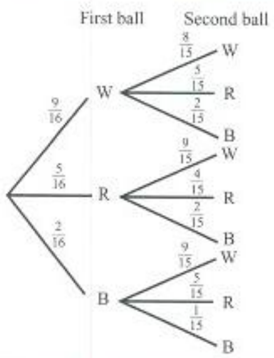
Standard deviation $= \sqrt{\frac{26\,239}{20} - \left(\frac{691}{20}\right)^2}$
$=$ **10.9 min** (3 s.f.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ to find the standard deviation.

(v) The times taken by the second group of students were more consistent as they had a lower standard deviation.

Must-Know Concept:
Compare the two distributions using their standard deviations.

(b) (i)



Must-Know Concept:
As the draws are without replacement, remember to minus one from the total number after every draw.

(ii) (a) P(two balls are the same colour)
$= P(W, W) + P(R, R) + P(B, B)$
$= \frac{9}{16} \times \frac{8}{15} + \frac{5}{16} \times \frac{4}{15} + \frac{2}{16} \times \frac{1}{15}$
$= \mathbf{\frac{47}{120}}$

(b) P(at least one of the balls is white)
$= 1 -$ P(none of the balls are white)
$= 1 - \left(\frac{5}{16} \times \frac{4}{15} + \frac{5}{16} \times \frac{2}{15} + \frac{2}{16} \times \frac{5}{15} + \frac{2}{16} \times \frac{1}{15}\right)$
$= 1 - \frac{7}{40}$
$= \mathbf{\frac{33}{40}}$

Must-Know Concept:
Use the tree diagram in (i) to find the required probabilities.

10. (a) Cost to post one copy of the newsletter = **\$0.37**

(b) Total number of copies of newsletter for a year
$= 150 \times 6$
$= 900$
Number of pages in 1 newsletter = 8
Total number of pages $= 900 \times 8$
$= 7200$
Number of toner cartridges he will need
$= \frac{7200}{1300}$
$=$ **6** (round up)

(c) Cost of 2 packs of 500 envelopes $= 2 \times \$34$
$= \$68$
Cost of 1 pack of 500 envelopes and 40 packs of 10 envelopes $= \$34 + 40 \times \$1.50$
$= \$94$
∴ He should buy 2 packs of 500 envelopes.
Number of A4 paper he needs $= 4 \times 900$
$= 3600$
Number of packs of 500 sheets of A4 paper he needs $= \frac{3600}{500}$
$= 7.2$
$= 8$ (round up)
Cost of 8 packs of A4 paper $= 8 \times \$3.90$
$= \$31.20$
Cost of 6 toner cartridges $= 6 \times \$147$
$= \$882$
Cost of stationery supplies including GST
$= 107\% \times (\$68 + \$31.20 + \$882)$
$= \$1049.88$ (nearest cent)
Cost of postage $= 900 \times \$0.37$
$= \$333$
Total cost of producing the newsletters for one year $= \$1049.88 + \$333$
$= \$1382.88$
Amount of money he should charge for a one-year subscription to the newsletter
$= \$1382.88 \div 150$
$=$ **\$9.22** (nearest cent)

Must-Know Concept:
There may be more than one answer for this question as the solution depends on the decision you make. For example, if you decide to save cost, you can choose to buy 2 packs of envelopes. However, if you decide to reduce wastage, you can choose to buy 1 pack of 500 envelopes and 40 packs of 10 envelopes. This will result in two different solutions based on your justification.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2015 EXAMINATION PAPER

Paper 1

1. $\frac{x}{5} + 14 = 8$
$x + 70 = 40$
$x = \mathbf{-30}$

Must-Know Concept:
Convert the denominators of the fractions on the left-hand side such that it becomes a single fraction. Perform cross multiplication and solve for x.

2. Profit earned $= \frac{180}{100} \times \$345$
$= \$621$
Selling price $= \$345 + \$621$
$= \mathbf{\$966}$

Must-Know Concept:
The original cost of the vase is 100% and his profit is 180%. So, the selling price of the vase is 280% of the original cost.

3. Arranged in increasing order,
middle position ⟶ 3rd number
3rd number ⟶ 5 (Given that the median is 5)
1st and 2nd number ⟶ 4 (Given that the mode is 4)
5th number ⟶ $3 \times 4 = 12$
Sum of all the numbers $= 5 \times 7$
$= 35$
4th number ⟶ $35 - 4 - 4 - 5 - 12 = 10$
∴ the five numbers are **4**, **4**, **5**, **10** and **12**.

Must-Know Concept:
Mean is the average of all the numbers, median is the middle value of the numbers when arranged in ascending order and mode is the value which has the highest frequency.

4. Average number of seeds produced by each flower
$= \frac{2 \times 10^8}{1400 \times 60 \times 20}$
$= \mathbf{119}$ (3 sig. fig.)

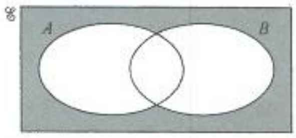
Must-Know Concept:
Average $= \frac{\text{Total number of seeds}}{\text{Total number of flowers}}$

5. $\frac{4x}{3} - \frac{3(2-5x)}{4} = \frac{16x - 18 + 45x}{12}$
$= \mathbf{\frac{61x - 18}{12}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Since there are no common term between both the denominators, the denominator would become $3 \times 4 = 12$.

6. $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 2, 3, 6\}$
$B = \{2, 3, 5, 7\}$

(a) $A' = \{4, 5, 7, 8\}$
$B' = \{1, 4, 6, 8\}$
$\therefore A' \cap B' = \mathbf{\{4, 8\}}$

(b)


Must-Know Concept:
The complement of a set A, relative to the universal set $\mathscr{E}$, is the set that contains all the elements that are not in A but in $\mathscr{E}$. The intersection of two sets is the set of all elements which are common in both sets.

7. $4ax - 3ay - 8bx + 6by = a(4x - 3y) - 2b(4x - 3y)$
$= \mathbf{(4x - 3y)(a - 2b)}$

Must-Know Concept:
Group similar terms together and factorise the common factors out.

8. (a) $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$
$\begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \overrightarrow{OB}$
$\overrightarrow{OB} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \end{pmatrix}$
$= \begin{pmatrix} 2 \\ -2 \end{pmatrix}$
Coordinates of B are **(2, −2)**.

(b) $|\overrightarrow{AB}| = |\overrightarrow{BA}|$
$= \sqrt{(-3)^2 + (6)^2}$
$= \sqrt{9 + 36}$
$= \mathbf{6.71}$ **units** (3 s.f.)

Must-Know Concept:
The formula for the magnitude of a vector is $\sqrt{x^2 + y^2}$.

9. Correct mean mass of each apple
$= 137 + 25$
$= \mathbf{162}$ **g**
Since the correct mass of each apple was 25 g more than what Simon recorded, the new standard deviation would be the same.
Correct standard deviation = **5.48 g**

Must-Know Concept:
Standard deviation measures the spread of the values, therefore the error in the readings will not affect the standard deviation.

10. (a) When $t = 4$, $\$A = \$(28\,700 \times 1.033^4)$
$\approx \$32\,680.0854$
$= \$\mathbf{32\,680}$ (nearest dollar)

(b) Percentage increase
$= \frac{\$32\,680.0854 - \$28\,700}{\$28\,700} \times 100\%$
$= \mathbf{13.9\%}$ (3 sig. fig.)

Must-Know Concept:
Percentage increase $= \frac{\text{Increase}}{\text{Original amount}} \times 100\%$

11. Let the second angle be x.
First angle ⟶ $x + 18°$
Third angle ⟶ $4x$
Sum of the three angles = 180° (∠ sum of △)
$x + x + 18° + 4x = 180°$
$6x = 162°$
$x = 27°$
First angle ⟶ 27° + 18° = 45°
Second angle ⟶ 27°
Third angle ⟶ 4 × 27° = 108°
∴ the angles of the triangle are **45°**, **27°** and **108°**.

Must-Know Concept:
Sum of angles in a triangle = 180°

12. (a) (i) Volume of apple juice he uses $= 5 \times \frac{2.4}{8}$
$= \mathbf{1.5\ litres}$

(ii) Volume of fruit drink he makes altogether
$= 1.5 + 2.4 + \left(2 \times \frac{2.4}{8}\right)$
$= \mathbf{4.5\ litres}$

Must-Know Concept:
Lemonade = 8 units = 2.4 l
Find the value each unit represents.

(b) Orange juice : lemonade = 2 : 5
= 6 : 15
Lemonade : pineapple juice = 3 : 4
= 15 : 20
∴ orange juice : lemonade : pineapple juice
= **6 : 15 : 20**

Must-Know Concept:
Observe that the common term between the two ratios is 'lemonade'. Convert the ratio such that the units representing lemonade is the same for both ratios.

13. Rate of Gina = 7 ÷ 5
= 1.4 panels/hour
Rate of Lim = 6 ÷ 4
= 1.5 panels/hour
Rate of both of them working together
= 1.4 + 1.5
= 2.9 panels/hour
Time taken to paint the 17 panels = 17 ÷ 2.9
$= 5\frac{25}{29}$ hours
= **5 hours 52 minutes**
(nearest minute)

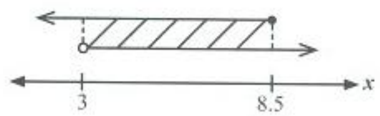
Must-Know Concept:
Find the rate of the number of fence each of them can paint in 1 hour. Add them together to find the rate of the total number of fence they can paint in 1 hour.
Time needed = Number of panels ÷ Rate

14. (a) $450 = \mathbf{2 \times 3^2 \times 5^2}$

(b) First number $= 2 \times 3^2 \times 5$
= **90**
Second number $= 3 \times 5^2$
= **75**

Must-Know Concept:
To find the lowest common multiple, select all the factors with the highest power. To find the highest common factor, select all the common factors with the lowest power. By observation, we can find the two numbers.

15. (a) $-10 \leqslant 7 - 2x < 1$
$-10 \leqslant 7 - 2x$ and $7 - 2x < 1$
$2x \leqslant 17$ $\quad$ $2x > 6$
$x \leqslant 8.5$ $\quad$ $x > 3$



∴ $\mathbf{3 < x \leqslant 8.5}$

Must-Know Concept:
Split the inequality into 2 inequalities, and solve each of them individually.

(b) **4, 5, 6, 7** and **8**

16. Sum of all the interior angles = (6 − 2) × 180°
= 720°
$\therefore x = 360° - \frac{720° - 3(96°)}{3}$ (∠s at a point)
= **216°**

Must-Know Concept:
Sum of interior angles of an n-sided regular polygon
$= (n - 2) \times 180°$
Angles at a point sum up to 360°.

17. (a) Let h_1 and h_2 be the height of the smaller and bigger bottles respectively.

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{1.25}{2}$$

$$\frac{h_1}{33.5} = \sqrt[3]{\frac{1.25}{2}}$$

$$h_1 = 33.5 \times \sqrt[3]{\frac{1.25}{2}}$$

$$\approx 28.642\ 097 \text{ cm}$$

$$= \mathbf{28.6 \text{ cm}} \text{ (3 sig. fig.)}$$

Must-Know Concept:
For similar figures, $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

(b) Surface area of larger bottle : Surface area of smaller bottle

$= 33.5^2 : 28.642\ 097^2$

$= \frac{33.5^2}{28.642\ 097^2} : \frac{28.642\ 097^2}{28.642\ 097^2}$

$\approx 1.36798 : 1$

$= 1.37 : 1$ (2 d.p.)

$\therefore k = \mathbf{1.37}$

Must-Know Concept:
For similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

18. Using Cosine Rule,

$$\cos \angle QPR = \frac{PQ^2 + PR^2 - QR^2}{2(PQ)(PR)}$$

$$= \frac{46.5^2 + 23.6^2 - 56.0^2}{2(46.5)(23.6)}$$

$$\angle QPR = \cos^{-1} \frac{46.5^2 + 23.6^2 - 56.0^2}{2(46.5)(23.6)}$$

$$\approx 100.947°$$

Bearing of R from P = 041° + 100.947°
= **141.9°** (1 d.p.)

Must-Know Concept:
Given all three lengths in ΔPQR, we can apply the Cosine Rule to solve for the required angle.

19. $OB = BD - OD$
$= 25 - 17$
$= 8$ cm

Using Pythagoras' Theorem,

$AB^2 = OA^2 - OB^2$
$= 17^2 - 8^2$
$= 225$

$AB = \sqrt{225}$
$= 15$ cm

$\sin \angle AOB = \frac{AB}{OA}$
$= \frac{15}{17}$

$\angle AOB = \sin^{-1} \angle AOB$
$\approx 61.927\ 513°$

$\angle AOC = 2 \times 61.927\ 513°$
$= 123.855\ 026°$

Reflex $\angle AOC = 360° - 123.855\ 026°$ ($\angle$s at a point)
$= 236.144\ 974°$

Area of the segment $= 2(\frac{1}{2})(15)(8) + \frac{236.144\ 974°}{360°}(\pi)(17)^2$
$= \mathbf{716\ cm^2}$ (3 sig. fig.)

Must-Know Concept:
Join A and C to O. After doing so, we can see that the segment is made up of the major sector $AOCD$ and ΔAOC.

20. (a) Gradient of PQ × Gradient of $QR = -1$

$$\frac{11-3}{4-0} \times \frac{3-11}{a-4} = -1$$

$$2 \times \frac{-8}{a-4} = -1$$

$$-\frac{16}{a-4} = -1$$

$$a - 4 = 16$$

$$a = \mathbf{20} \text{ (shown)}$$

Must-Know Concept:
Find the gradients of line PQ and line QR using the given points. The formula for the gradient of a line is $\frac{y_2 - y_1}{x_2 - x_1}$.

(b) $\vec{SR} = \vec{PQ}$

$\vec{OR} - \vec{OS} = \vec{OQ} - \vec{OP}$

$\vec{OS} = \vec{OR} + \vec{OP} - \vec{OQ}$

$= \begin{pmatrix} 20 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 11 \end{pmatrix}$

$= \begin{pmatrix} 16 \\ -5 \end{pmatrix}$

$\therefore$ the coordinates of the point S are **(16, −5)**.

Must-Know Concept:
Since $PQRS$ is a rectangle, its lengths are equal.

(c) $PQ = \sqrt{(4-0)^2 + (11-3)^2}$
$= \sqrt{80}$ units

$QR = \sqrt{(20-4)^2 + (3-11)^2}$
$= \sqrt{320}$ units

Area of the rectangle of PQRS $= \sqrt{80} \times \sqrt{320}$
$= \mathbf{160\ units^2}$

Must-Know Concept:
Find the length and breadth of the rectangle using the formula for the length of a line segment $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

21. (a) $A = \pi p(2p + q)$

$2p + q = \frac{A}{\pi p}$

$\therefore \mathbf{q = \frac{A}{\pi p} - 2p}$

Must-Know Concept:
Make q the subject by shifting terms that have q to the left-hand side and terms that do not have q to the right-hand side, and simplify the equation.

(b) Total surface area of the solid
$= 2 \times$ Total surface area of the cone
$2\pi r(2r) + 4\pi r^2 = 2 \times (\pi r^2 + \pi rl)$
$8\pi r^2 = 2\pi r(r + l)$
$r + l = 4r$
$\therefore l = \mathbf{3r}$

Must-Know Concept:
Curved surface area of a cylinder $= 2\pi rh$
Surface area of a hemisphere $= 2\pi r^2$
Make l the subject by shifting terms that have l to the left-hand side and terms that do not have l to the right-hand side, and simplify the equation.

22. (a) $\mathbf{P} = \begin{pmatrix} \mathbf{4} & \mathbf{2} & \mathbf{3} \\ \mathbf{3} & \mathbf{0} & \mathbf{4} \end{pmatrix}$

(b) $\mathbf{R} = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1.5 & 0.2 \\ 2.4 & -0.4 \\ 1.4 & -0.1 \end{pmatrix}$

$= \begin{pmatrix} 4(1.5) + 2(2.4) + 3(1.4) & 4(0.2) + 2(-0.4) + 3(-0.1) \\ 3(1.5) + 0(2.4) + 4(1.4) & 3(0.2) + 0(-0.4) + 4(-0.1) \end{pmatrix}$

$= \begin{pmatrix} \mathbf{15} & \mathbf{-0.3} \\ \mathbf{10.1} & \mathbf{0.2} \end{pmatrix}$

Must-Know Concept:
The product of a 2×3 matrix and a 3×2 matrix gives a resulting 2×2 matrix.

(c) \$0.20

(d) Amount of money she pays
$= \frac{100 - 10}{100} \times \$(15 - 0.30)$
$= \mathbf{\$13.23}$

23. (a) Total area, $A = \pi\left(\frac{2kr + 2r}{2}\right)^2$
$= \pi(kr + r)^2$
$= \boldsymbol{\pi r^2(k + 1)^2}$ (shown)

Must-Know Concept:
Area of a circle $= \pi r^2$

(b) Area of the larger semicircle $= \frac{1}{2}\pi\left(\frac{2 \times 2r}{2}\right)^2$
$= 2\pi r^2$ cm²

Area of the smaller semicircle $= \frac{1}{2}\pi\left(\frac{2r}{2}\right)^2$
$= 0.5\pi r^2$ cm²

Area of the large circle $= \pi r^2(2 + 1)^2$
$= 9\pi r^2$ cm²

Area of half of the large circle $= 9\pi r^2 \div 2$
$= 4.5\pi r^2$ cm²

Area of the shaded section $= 2\pi r^2 + 4.5\pi r^2 - 0.5\pi r^2$
$= 6\pi r^2$ cm²

Area of the unshaded section $= 0.5\pi r^2 + 4.5\pi r^2 - 2\pi r^2$
$= 3\pi r^2$ cm²

Difference in area between the shaded section and the unshaded section
$= 6\pi r^2 - 3\pi r^2$
$= \mathbf{3\pi r^2}$ **cm²**

Must-Know Concept:
Area of a semicircle $= \frac{1}{2}\pi r^2$

24. (a) P(marble is yellow) $= \frac{\mathbf{10 - n}}{\mathbf{10}}$

(b) P(both marbles are yellow) $= \left(\frac{10 - n}{10}\right)\left(\frac{9 - n}{9}\right)$
$= \frac{90 - 10n - 9n + n^2}{90}$
$= \frac{\mathbf{n^2 - 19n + 90}}{\mathbf{90}}$

Must-Know Concept:
As the draws are without replacement, remember to minus one from the total number after every draw.

(c) (i) $\frac{n^2 - 19n + 90}{90} = \frac{1}{15}$
$n^2 - 19n + 90 = 6$
$\mathbf{n^2 - 19n + 84 = 0}$ (shown)

(ii) $n^2 - 19n + 84 = 0$
$(n - 12)(n - 7) = 0$
$n - 12 = 0$ or $n - 7 = 0$
$n = 12$ (rejected) $\quad n = 7$
$\therefore$ number of yellow marbles $= 10 - 7$
$= \mathbf{3}$

Paper 2

1. (a) $9x^2 - 16y^2 = (3x)^2 - (4y)^2$
$= \mathbf{(3x - 4y)(3x + 4y)}$

Must-Know Concept:
Factorise the expression using $a^2 - b^2 = (a + b)(a - b)$.

(b) (i) $\frac{15xy}{12} \div \frac{9x^2}{4y} = \frac{5xy}{4} \times \frac{4y}{9x^2}$
$= \frac{\mathbf{5y^2}}{\mathbf{9x}}$

Must-Know Concept:
$\frac{15xy}{12} \div \frac{9x^2}{4y} = \frac{15xy}{12} \times \frac{4y}{9x^2}$

(ii) $\frac{6}{2x - 3} - \frac{1}{x + 2} = \frac{6(x + 2) - (2x - 3)}{(2x - 3)(x + 2)}$
$= \frac{6x + 12 - 2x + 3}{(2x - 3)(x + 2)}$
$= \frac{\mathbf{4x + 15}}{\mathbf{(2x - 3)(x + 2)}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Since there are no common term between both the denominators, the denominator will remain as $(2x - 3)(x + 2)$.

(c) $\frac{9}{x - 4} = 2x - 1$
$(2x - 1)(x - 4) = 9$
$2x^2 - 8x - x + 4 = 9$
$2x^2 - 9x - 5 = 0$
$(2x + 1)(x - 5) = 0$
$2x + 1 = 0$ or $x - 5 = 0$
$x = \mathbf{-0.5}$ or $x = \mathbf{5}$

Must-Know Concept:
Perform cross multiplication and solve for x.

(d) (i) $x^2 - 9x + 17 = \left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 17$
$= \mathbf{(x - 4.5)^2 - 3.25}$

Must-Know Concept:
Complete the square method.

(ii)
$$x^2 - 9x + 17 = 0$$
$$(x - 4.5)^2 - 3.25 = 0$$
$$(x - 4.5)^2 = 3.25$$
$$x - 4.5 = \pm\sqrt{3.25}$$
$$x = 4.5 \pm \sqrt{3.25}$$
$= \mathbf{2.70}$ or $\mathbf{6.30}$ (2 d.p.)

2. (a) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= \mathbf{-a + b}$
$= \mathbf{b - a}$

(ii) $\overrightarrow{BD} = \overrightarrow{BM} + \overrightarrow{MD}$
$= -(\mathbf{b} - 2\mathbf{a}) + 3\mathbf{b} - 2\mathbf{a}$
$= -\mathbf{b} + 2\mathbf{a} + 3\mathbf{b} - 2\mathbf{a}$
$= \mathbf{2b}$

(b) $\overrightarrow{CD} = \overrightarrow{CM} + \overrightarrow{MD}$
$= \overrightarrow{MA} + \overrightarrow{MD}$
$= \overrightarrow{MB} + \overrightarrow{BA} + \overrightarrow{MD}$
$= \mathbf{b} - 2\mathbf{a} - (-\mathbf{a} + \mathbf{b}) + 3\mathbf{b} - 2\mathbf{a}$
$= \mathbf{b} - 2\mathbf{a} + \mathbf{a} - \mathbf{b} + 3\mathbf{b} - 2\mathbf{a}$
$= \mathbf{3b - 3a}$ (shown)

(c) Since $\overrightarrow{CD} = 3\mathbf{b} - 3\mathbf{a} = 3(\mathbf{b} - \mathbf{a}) = 3\overrightarrow{AB}$, $CD \parallel AB$.
$\angle OAB = \angle OCD$ (corr. $\angle$s, $CD \parallel AB$)
$\angle OBA = \angle ODC$ (corr. $\angle$s, $CD \parallel AB$)
$\angle AOB = \angle COD$ (common $\angle$)
$\therefore$ triangles OAB and OCD are similar.
(3 pairs of corr. $\angle$s are equal) (shown)

Must-Know Concept:
When all three corresponding angles of two triangles are equal, the two triangles are similar. This is known as Angle-Angle-Angle test.

(d) $\frac{\text{Area of triangle } OAB}{\text{Area of triangle } OCD} = \left(\frac{AB}{CD}\right)^2$
$= \left(\frac{1}{3}\right)^2$
$= \frac{1}{9}$

Area of triangle OAB : Area of quadrilateral $ABDC$
$= 1 : 9 - 1$
$= \mathbf{1 : 8}$

Must-Know Concept:
For similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

3. (a) Total earnings for the week
$= (32 \times \$10.50) + \left(\frac{6}{100} \times \$1350\right)$
$= \mathbf{\$417}$

(b) (i) Total amount Sue pays
$= \left(\frac{1}{5} \times \$3499\right) + (24 \times \$130)$
$= \mathbf{\$3819.80}$

(ii) Price the salesman paid
$= \frac{100}{100 - 5} \times \$342$
$= \mathbf{\$360}$

Must-Know Concept:
The amount the salesman paid for the speakers is 100% and he sold it at a loss of 5%. So, the speakers were sold at 95% of the original price.

(c) (i) Total amount John has to pay back
$= \$1500\left(1 + \frac{7.5}{100}\right)^3$
$\approx \$1863.4453$
Amount of interest John will pay
$= \$1863.4453 - \$1500$
$= \mathbf{\$363.45}$ (nearest cent)

Must-Know Concept:
The formula for compound interest is $P\left(1 + \frac{r}{100}\right)^n$.

(ii) Total cost John has to pay in pounds
$= £185 + \left(\frac{1.5}{100} \times £185\right)$
$= £187.775$
Total cost John has to pay in Singapore dollars
$= \frac{£187.775}{£0.52} \times \$1$
$= \mathbf{\$361.11}$ (nearest cent)

4. (a) When $x = 0.5$, $y = \frac{0.5^2}{5} + \frac{4}{0.5} - 3$
$= 5.05$
$\therefore p = \mathbf{5.05}$

Must-Know Concept:
Substitute $x = 0.5$ into the equation to find p.

(b), (e) (i) Refer to Appendix 5.

(c)
$$\frac{x^2}{5} + \frac{4}{x} = 3$$
$$\frac{x^2}{5} + \frac{4}{x} - 3 = 0$$
From the graph,
$x = \mathbf{1.6}$ or $x = \mathbf{2.8}$

(d) Gradient $= \frac{2.4 - 0.1}{4.9 - 3.2}$
$= \mathbf{1.35}$ (3 sig. fig.)

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient via the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

(e) (ii) $y = -1.5x + 4$

(iii) $\mathbf{(0.7, 2.95)}$ and $\mathbf{(2.7, -0.05)}$

5. (a) (i) $\angle EBC = \angle EAC$ ($\angle$s in the same segment)
$= \mathbf{68°}$

Must-Know Concept:
Note that angles in the same segment are equal.

(ii) $\angle CDE = 180° - 68°$ ($\angle$s in the opp. segments)
$= \mathbf{112°}$

Must-Know Concept:
Note that angles in opposite segments are supplementary.

(iii) $\angle COE = 2 \times 68°$ ($\angle$s at centre $= 2\angle$ at circumference)
$= 136°$
$\angle OCT = \angle OET = 90°$ (tangent $\perp$ radius)
$\angle CTE = 360° - 90° - 90° - 136°$ ($\angle$ sum of quad.)
$= \mathbf{44°}$

Must-Know Concept:
Note that an angle at centre is twice the angle at circumference. Moreover, the tangent TC is perpendicular to the radius CO and the tangent TE is perpendicular to the radius EO.

(iv) $\angle OCE = \angle OEC$ (base $\angle$s of isos. $\triangle$)
$= \frac{180° - 136°}{2}$
$= 22°$
$\angle CED = 90° - 22° - 32°$ (tangent $\perp$ radius)
$= 36°$
$\angle ECD = 180° - 112° - 36°$ ($\angle$ sum of $\triangle$)
$= 32°$
$\angle OCD = 22° + 32°$
$= \mathbf{54°}$

Must-Know Concept:
Since $OC = OE$, $\triangle OCE$ is isosceles.

(b) Perimeter of the sector $= 14.8$ cm
$r(1.7) + 2r = 14.8$
$3.7r = 14.8$
$r = 14.8 \div 3.7$
$= 4$ cm

Area of the sector $= \frac{1}{2}(4)^2(1.7)$
$= \mathbf{13.6\ cm^2}$

Must-Know Concept:
The perimeter of a sector consists of two radii and an arc length.
Arc length $= r\theta$ (where θ is in radians)

6. (a) $2\frac{40}{60}x + 1\frac{20}{60}y = 240$
$2\frac{2}{3}x + 1\frac{1}{3}y = 240$
$8x + 4y = 720$
$\mathbf{2x + y = 180}$ (shown)

Must-Know Concept:
Total distance is the summation of distance covered by each average speed.
Distance = Speed × Time

(b) $1\frac{30}{60}x + 2\frac{30}{60}y = 240 + 10.5$
$1.5x + 2.5y = 250.5$
$\mathbf{3x + 5y = 501}$

Must-Know Concept:
Calculate the total distance travelled by Hok. The difference between the distance travelled by Hok and Wei is 10.5 km.

(c) $2x + y = 180$ ---------- (1)
$3x + 5y = 501$ ---------- (2)

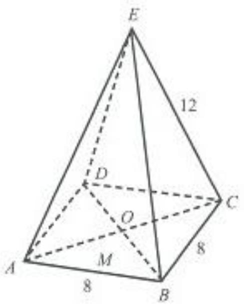
From (1), $10x + 5y = 900$ ---------- (3)
(3) − (2) : $7x = 399$
$\therefore x = \mathbf{57}$
Sub. $x = 57$ into (1): $2(57) + y = 180$
$114 + y = 180$
$\therefore y = \mathbf{66}$

Must-Know Concept:
Solve the two equations simultaneously for x and y.

(d) Additional time taken for Wei
$= \frac{240}{57} - \frac{240}{66}$
$= \frac{120}{209}$ h
$= 34\frac{94}{209}$ min
≈ 34 min 26.99 s
$=$ **34 min 27 s** (nearest second)

Must-Know Concept:
1 min = 60 s

7. (a)



Let M be the midpoint of AB and O be the centre of $ABCD$.
$AM = MO$
$= 8 \div 2$
$= 4$ cm
Using Pythagoras' Theorem,
$EM^2 = AE^2 - AM^2$
$= 12^2 - 4^2$
$= 128$
Using Pythagoras' Theorem,
$EO^2 = EM^2 - MO^2$
$= 128 - 4^2$
$= 112$
$EO = \sqrt{112}$ cm
Volume of the candle $= \frac{1}{3}(8)(8)(\sqrt{112})$
$\approx 225.771\ cm^3$
$= \mathbf{226\ cm^3}$ (3 sig. fig.)

Must-Know Concept:
Volume of a pyramid $= \frac{1}{3} \times$ Base area $\times$ Height

(b) Volume of the spherical candle
$= 225.771 \text{ cm}^3$

$\frac{4}{3}\pi r^3 = 225.771$

$r = \sqrt[3]{225.711\left(\frac{3}{4\pi}\right)}$

$\approx 3.777\ 07$ cm

$=$ **3.78 cm** (3 sig. fig.) (shown)

Must-Know Concept:
Volume of a sphere $= \frac{4}{3}\pi r^3$

(c) Volume of the empty space
$= (6)(3.777\ 07) \times (4)(3.777\ 07) \times (2)(3.777\ 07) - 6(225.771)$

$=$ **1230 cm³** (3 sig. fig.)

Must-Know Concept:
Volume of a cuboid = Length × Breadth × Height

8. (a) (i) $T_1 = 55 = 55 - 4(0)$

$T_2 = 51 = 55 - 4(1)$

$T_3 = 47 = 55 - 4(2)$

$T_4 = 43 = 55 - 4(3)$

$\vdots$

$T_n = 55 - 4(n - 1)$

$= 55 - 4n + 4$

$\therefore T_n = \mathbf{59 - 4n}$

Must-Know Concept:
Observe that the pattern in the sequence is: 4 less than the previous term.

(ii) $T_{25} = 59 - 4(25)$

$= \mathbf{-41}$

Must-Know Concept:
Substitute $n = 25$.

(b) (i) $n + 9$

(ii) $n(n + 9) = n^2 + 9n$

$(n + 1)(n + 8) = n^2 + 8n + n + 8$

$= n^2 + 9n + 8$

Difference $= n^2 + 9n + 8 - (n^2 + 9n)$

$=$ **8** (shown)

(iii) Let the sum of the four numbers be 260.

$n + n + 1 + n + 8 + n + 9 = 260$

$4n + 18 = 260$

$4n = 242$

$n = 60.5$

Since n is not an integer, the sum of the four numbers in the square cannot be 260. (shown)

9. (a) $\angle ADB = \tan^{-1}\frac{95}{60}$

$\angle BDC = \frac{180° - 40°}{2}$ (base $\angle$s of isos. $\triangle$)

$= 70°$

$\angle ADC = \tan^{-1}\frac{95}{60} + 70°$

$\approx 127.724°$

$=$ **127.7°** (1 d.p.) (shown)

Must-Know Concept:
Note that $\angle ADB$ and $\angle BDC$ sum up to $\angle ADC$.

(b) Using Pythagoras' Theorem,

$BD^2 = AD^2 + AB^2$

$= 60^2 + 95^2$

$= 12\ 625$

Using Cosine Rule,

$BD^2 = CD^2 + CB^2 - 2(CD)(CB)\cos 40°$

$BD^2 = CD^2 + CD^2 - 2(CD)(CD)\cos 40°$

$(CD = CB)$

$BD^2 = 2CD^2 - 2CD^2 \cos 40°$

$BD^2 = CD^2(2 - 2\cos 40°)$

$CD^2 = \frac{BD^2}{2 - 2\cos 40°}$

$CD = \sqrt{\frac{BD^2}{2 - 2\cos 40°}}$

$= \sqrt{\frac{12\ 625}{2 - 2\cos 40°}}$

$=$ **164 m** (3 sig. fig.)

Must-Know Concept:
Apply the Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

(c) Area of the land

$= \frac{1}{2}(95)(60) + \frac{1}{2}\left(\sqrt{\frac{12\ 625}{2 - 2\cos 40°}}\right)\left(\sqrt{\frac{12\ 625}{2 - 2\cos 40°}}\right)\sin 40°$

$= 11\ 521.725\ 61 \text{ m}^2$

$= (11\ 521.725\ 61 \div 10\ 000)$ hectares

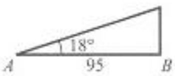
$= 1.152\ 172\ 561$ hectares

Value of the field $= 1.152\ 172\ 561 \times \$40\ 000$

$=$ **\$46 100** (3 sig. fig.)

Must-Know Concept:
Area of a triangle $= \frac{1}{2}ab\sin C$

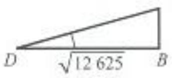
(d)



Height of the bird vertically above B

$= 95\tan 18°$

≈ 30.8674 m



Angle of elevation of the bird from D

$= \tan^{-1}\frac{30.8674}{\sqrt{12\ 625}}$

$=$ **15.4°** (1 d.p.)

Must-Know Concept:
As an intermediate step, find the height of the bird from A first.

10. (a) (i) (a) Median position $= \frac{50}{100} \times 120$

$= 60$

$\therefore$ median time $=$ **41 minutes**

Must-Know Concept:
The median is the value of the middle position of all the students.

(b) Upper quartile position $= \frac{75}{100} \times 120$

$= 90$

Upper quartile time = 52 minutes

Lower quartile position $= \frac{25}{100} \times 120$

$= 30$

Lower quartile time = 37 minutes

Interquartile range of the times

$= 52 - 37$

= **15 minutes**

Must-Know Concept:
The value of the lower quartile is at the 25% position and the value of the upper quartile is at the 75% position.

(c) From the graph,
number of men who completed the race within 1 hour = 106

Number of men who took at least 1 hour to complete the race

$= 120 - 106$

$= 14$

Percentage of the men who took at least 1 hour to complete the race

$= \frac{14}{120} \times 100\%$

$= \mathbf{11\frac{2}{3}\%}$

Must-Know Concept:
Find the number of people who took 60 minutes and less. Then, minus this value from the total number of people to find the number of people who took at least one hour.

(ii) The cumulative frequency curve shifts to the right within the interquartile range due to its higher median.

Must-Know Concept:
The values at the 25% and 75% points remain unchanged.

(b) (i) (a) P(a man aged 50 or more) $= \frac{15}{240}$

$= \mathbf{\frac{1}{16}}$

(b) P(a person aged under 30) $= \frac{35+27}{240}$

$= \mathbf{\frac{31}{120}}$

(ii) Total number of women aged under 40

$= 27 + 34$

$= 61$

P(both are women aged under 40)

$= \left(\frac{61}{240}\right)\left(\frac{60}{239}\right)$

= **0.0638** (3 sig. fig.)

Must-Know Concept:
The selections are without replacement as the same person cannot be selected twice.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2014 EXAMINATION PAPER

Paper 1

1. $\frac{27}{38} = 0.711$ (3 sig. fig.)

   $\sqrt{0.49} = 0.7$

   $0.61^{\frac{2}{3}} = 0.719$ (3 sig. fig.)

   Arranging the answers from smallest to largest

   $= \sqrt{0.49}$, **0.702**, $\frac{27}{38}$, $0.61^{\frac{2}{3}}$

   Must-Know Concept:
   Express all the numbers in decimal form to compare them.

2. Using Singapore exchange rate
   $= 500 \times 0.765$
   $= 382.50$ Swiss Francs

   Using Switzerland exchange rate
   $= 500 \div 1.294$
   ≈ 386.40 Swiss Francs

   $386.40 - 382.50 =$ **3.90 Swiss Francs** (2 d.p.)

   Must-Know Concept:
   Find the amount of Swiss Francs he will get in both countries to find the difference.

3. (a) $\frac{13.5^3}{6.48 - 2.57} = \frac{2460.375}{3.91}$
   ≈ 629.252
   $=$ **629.25** (first five digits)

   (b) **630** (2 sig. fig.)

   Must-Know Concept:
   Non-zero digits are significant.

4. $\sin\theta = 0.7420$

   $\theta = \sin^{-1} 0.7420$ or $\theta = 180° - \sin^{-1} 0.7420$
   $\approx 47.90°$ $= 180° - 47.90°$
   $=$ **47.9°** (1 d.p.) $=$ **132.1°** (1 d.p.)

   Must-Know Concept:
   Note that $\sin(180° - \theta) = \sin\theta$ where $0 < \theta < 90°$.

5. 1. **Time limit, such as per day, per week or per month, is not set.**
   2. **The intervals for the number of hours spent are not continuous.**
      E.g. Between 2h to 3h, 4h to 5h.
      Other acceptable answers:
      – **Zero hour is not offered.**
      – **Duration of each option is not constant.**

   Must-Know Concept:
   Identify possible misleading information presented in data.

6. $4 \times 64^n = 1$

   $64^n = \frac{1}{4}$

   $4^{3n} = 4^{-1}$

   $3n = -1$

   $n = -\frac{1}{3}$

   Must-Know Concept:
   Express all numbers in the same base of 4.

7. Exterior angle of a pentagon $= 360° \div 5$
   $= 72°$

   Exterior angle of a nonagon $= 360° \div 9$
   $= 40°$

   $\angle x = 72° + 40°$
   $=$ **112°**

   Must-Know Concept:
   The sum of exterior angles of any polygon = 360°.

8. Ratio of the height of the 2 bottles $= \frac{h_1}{h_2}$

   Since $\left(\frac{h_1}{h_2}\right)^3 = \frac{1000}{2000}$

   $\frac{h_1}{h_2} = \sqrt[3]{\frac{1000}{2000}}$
   $= \frac{1}{\sqrt[3]{2}}$

   $\therefore$ percentage of $\frac{h_1}{h_2} = \frac{1}{\sqrt[3]{2}} \times 100\%$
   ≈ 79.37
   $=$ **79.4%** (3 sig. fig.)

   Must-Know Concept:
   For similar figures, $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

9. (a) When $t = 0$, $B = 1000 \times 3^t$
   $= 1000 \times 3^0$
   $=$ **1000**

   (b) When $t = 1$, $B = 1000 \times 3^t$
   $= 1000 \times 3^1$
   $= 1000 \times 3$
   $= 3000$

   Increase in bacteria at the end of the first hour
   $= 3000 - 1000$
   $= 2000$

   Percentage increase in the number of bacteria at the end of first hour
   $= \frac{2000}{1000} \times 100\%$
   $=$ **200%**

   Must-Know Concept:
   Percentage increase $= \frac{\text{Increase in number of bateria}}{\text{Original number of bacteria}}$

10. (a) Median = **32 seconds**

    Must-Know Concept:
    The value of the median is the centre line in the box.

(b) Interquartile range
= Upper quartile – lower quartile
= 35 – 27.5
= **7.5 seconds**

Must-Know Concept:
The lower quartile is plotted on the left side of the box and the upper quartile is plotted on the right side of the box in the box-and-whisker diagram.

11. 15% of the cash price = $\frac{15}{100} \times \$980$
= \$147
Total amount for 12 equal monthly payments
= \$1089 – \$147
= \$942
One monthly payment = 942 ÷ 12
= **\$78.50**

Must-Know Concept:
An item bought on hire purchase always costs more than its cash price.

12. Length of $2x = 30 - (3 \times 8)$
= 6 cm
Length of $x = 6 \div 2$
= **3 cm**
Area of rectangle = 40 × 3
= 120 cm²
Length of $y = 120 \div (3 \times 8)$
= **5 cm**
Length of $z = 120 \div 8$
= **15 cm**

Must-Know Concept:
Area of a rectangle = Length × Breadth

13. Let x be the initial amount of money in the savings account.
Total amount after 3 years in term of x
$= P\left(1 + \frac{r}{100}\right)^n$
$= x\left(1 + \frac{4}{100}\right)^3$
$= 1.124864x$
$1.124864x = \$8436.48$
$x = \frac{8436.48}{1.124864}$
= **\$7500**

Must-Know Concept:
The formula for compound interest is $P\left(1 + \frac{r}{100}\right)^n$.

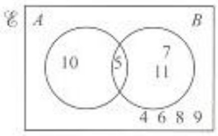
14. (a) $\overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$
$= \frac{1}{3}(\overrightarrow{OB} - \overrightarrow{OA})$
$= \frac{1}{3}(\mathbf{b} - \mathbf{a})$

(b) Given OD : DB
1 : 3

$\overrightarrow{OD} = \frac{1}{4}\mathbf{b}$
$\overrightarrow{DC} = \overrightarrow{OA} + \overrightarrow{AC} - \overrightarrow{OD}$
$= \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) - \frac{1}{4}\mathbf{b}$
$= \mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}$
$= \frac{2}{3}\mathbf{a} + \frac{1}{12}\mathbf{b}$
$\overrightarrow{DC}$ cannot be expressed as a $k(\overrightarrow{OA})$.
∴ **$\overrightarrow{DC}$ and $\overrightarrow{OA}$ are not parallel.**

Must-Know Concept:
Two vectors are parallel if **a** can be written as the scalar multiple of **b**, **a** = k**b**, where k is a scalar.

15. (a) $\mathscr{E}$ = {integers x: $4 \leq x \leq 11$}
= {4, 5, 6, 7, 8, 9, 10, 11}
A = {factors of 10}
= {5, 10}
B = {prime numbers}
= {5, 7, 11}



(b) $(A \cap B)'$ = **{4, 6, 7, 8, 9, 10, 11}**

Must-Know Concept:
The intersection of two sets is the set of all elements which are common in both sets. The complement of a set A, relative to the universal set $\mathscr{E}$, is the set that contains all the elements that are not in A but in $\mathscr{E}$.

16. (a) $36a^2b^3 \div 24a^3b^{-2} = \frac{36}{24}\,a^{(2-3)}b^{3-(-2)}$
$= \frac{3}{2}\,a^{-1}b^5$
$= \mathbf{\frac{3b^5}{2a}}$

Must-Know Concept:
Use the Law of Indices: $a^m \div a^n = a^{m-n}$.

(b) $\frac{1}{(x-5)} + \frac{7x}{(x-5)^2} = \frac{(x-5)}{(x-5)^2} + \frac{7x}{(x-5)^2}$
$= \frac{x - 5 + 7x}{(x-5)^2}$
$= \mathbf{\frac{8x-5}{(x-5)^2}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Observe that $(x-5)^2 = (x-5)(x-5)$. Hence, the common denominator between the two fractions is $(x-5)(x-5)$.

17. (a) $T_5 = 33$
$T_2 = 12$
$T_5 - T_2 = 33 - 12$
$= 21$
$21 \div 3 = 7$
$\therefore a = 12 - 7$
$= \mathbf{5}$
$\therefore b = 12 + 7$
$= \mathbf{19}$
$\therefore c = 19 + 7$
$= \mathbf{26}$

Must-Know Concept:
Observe that $T_5 = 33$ and $T_2 = 12$.

(b) $T_1 = 5 = -2 + 7(1)$
$T_2 = 12 = -2 + 7(2)$
$T_3 = 19 = -2 + 7(3)$
$\therefore$ nth term $= \mathbf{7n - 2}$

(c) Solve $7n - 2 = 109$
$7n = 109 + 2$
$= 111$
$n = 111 \div 7$
$\neq$ integer
$\therefore$ **109 is not a term of this sequence because *n* must be an integer.**

Must-Know Concept:
Equate the expression to 109. If n is a whole number, then 109 is a term of this sequence.

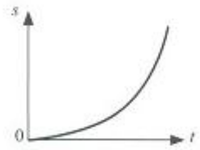
18. (a) 1 h ⟶ 31 km
3600 s ⟶ 31 000 m
1 s ⟶ 31 000 ÷ 3 600
$= 8\frac{11}{18}$
$\therefore$ 31 km/h = $\mathbf{8\frac{11}{18}}$ **m/s**

Must-Know Concept:
1 km = 1000 m
1 h = 3600 s

(b) Total distance including length of train
= 4.8 km + 220 m
= 4800 m + 220 m
= 5020 m
Time taken $= \frac{\text{Distance}}{\text{Speed}}$
$= \frac{5020}{8\frac{11}{18}}$
≈ 583 s
= **9 minutes 43 seconds**

Must-Know Concept:
$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

19. (a)



Must-Know Concept:
A distance-time graph shows the distance away from the point of origin with respect to time. Plot the graph in the form $y = kx^2$.

(b) Since $s \propto t^2$
$\Rightarrow s = kt^2$, where k is a constant
When $s = 36$, $t = 4$,
$36 = k(4)^2$
$k = \frac{36}{16}$
$= 2.25$
$\therefore s = \mathbf{2.25t^2}$

(c) When $s = 20$ m,
$20 = 2.25t^2$
$t^2 = \frac{20}{2.25}$
$t = \sqrt{8\frac{8}{9}}$
= **2.98 seconds** (3 sig. fig.)

20. (a) P(even)
$= \frac{2}{8}$
$= \mathbf{\frac{1}{4}}$

(b) P(the pointer stops on an even number both times)
= P(even) × P(even)
$= \frac{1}{4} \times \frac{1}{4}$
$= \mathbf{\frac{1}{16}}$

(c) P(her score is not 2)
= 1 – P(her score is 2)
= 1 – [P(pointer stops at 1) × P(pointer stops at 1)]
$= 1 - (\frac{3}{8} \times \frac{3}{8})$
$= 1 - \frac{9}{64}$
$= \mathbf{\frac{55}{64}}$

Must-Know Concept:
Find the probability that her score is 2 first, then minus the probability from 1 to find the probability that her score is not 2.

21. (a) (i) $\mathbf{D} = \begin{pmatrix} \mathbf{2.6} \\ \mathbf{4.5} \\ \mathbf{6.3} \end{pmatrix}$

(ii) $\mathbf{E} = \mathbf{CD}$

$= \begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 2.6 \\ 4.5 \\ 6.3 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{151.9} \\ \mathbf{155.4} \end{pmatrix}$

Must-Know Concept:
The product of a 2×3 matrix and a 3×1 matrix gives a resulting 2×1 matrix.

(iii) The elements of **E** represent **the total mass, in grams, of the coins saved last month by Lin and Hadi respectively**.

(b) $\text{F} = \begin{pmatrix} \mathbf{10} \\ \mathbf{20} \\ \mathbf{100} \end{pmatrix}$

22. Area of the pendant $= [2 \times \frac{1}{2} \times (2.1 \times 1)] - \pi r^2$

$= 2.1 - \pi\left(\frac{0.8}{2}\right)^2$

$= (2.1 - 0.16\pi)$

$\approx 1.597 \text{ cm}^2$ (3 d.p.)

Volume of the pendant $= 1.597 \times 0.3$

$= 0.4791 \text{ cm}^3$

$1 \text{ cm}^3 = 19.3$ g

$0.4792 \text{ cm}^3 = 19.3 \times 0.4791$

$= 9.246\ 63$ g

Value of the gold in the pendant

$= 9.246\ 63 \text{ g} \times \$69.65$

$=$ **\$644.03** (nearest cent)

Must-Know Concept:
Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height
Area of a circle $= \pi r^2$
Volume of a prism = Cross-sectional area × Length

23. (a) Total surface area of the cuboid

$= (2 \times 9 \times 8) + (2 \times 8 \times 5) + (2 \times 9 \times 5)$

$= 144 + 80 + 90$

$= \mathbf{314 \text{ cm}^2}$

Must-Know Concept:
Total surface area of a cuboid = 2 × (length × breadth) + 2 × (breadth × height) + 2 × (length × height)

(b)

5	385
7	77
11	11
	1

$\therefore$ 385 = **5 cm** by **7 cm** by **11 cm**

Must-Know Concept:
Volume of a cuboid = Length × Breadth × Height
Therefore, prime factorisation is a relevant concept in this question.

(c) Largest possible cube $= 7^3$

$= 343$

$385 - 343 = 42$

$\therefore$ He has **42** cubes left over.

Must-Know Concept:
All sides of a cube have the same length.
Volume of a cube $= (\text{Length})^3$

24. (a) Length of DC

$= (20x - 3) - [(2x + 6) + (4x + 3) + (5x - 3)]$

$= 20x - 3 - 11x - 6$

$= 9x - 9$

$= \mathbf{9(x - 1) \text{ cm}}$

Must-Know Concept:
Perimeter of the trapezium $= AB + BC + CD + AD$

(b) Since $AD = BC$,

$2x + 6 = 5x - 3$

$9 = 3x$

$x = 3$

Perimeter of the trapezium $= 20x - 3$

$= 20(3) - 3$

$= \mathbf{57 \text{ cm}}$

Must-Know Concept:
Solve for x after equating AD and BC. Substitute the value of x into the expression for perimeter.

(c) Length of $AB = 4x + 3$

$= 4(3) + 3$

$= 15$ cm

Length of $DC = 9(x - 1)$

$= 9(3 - 1)$

$= 18$ cm

Length of $AD = 2x + 6$

$= 2(3) + 6$

$= 12$ cm

$DX = CY$

$= \frac{18 - 15}{2}$

$= 1.5$ cm

$AX = BY$

$= \sqrt{12^2 - 1.5^2}$

≈ 11.9 cm

Area of trapezium $= \frac{18 + 15}{2} \times 11.9$

$\approx \mathbf{196 \text{ cm}^2}$ (3 sig. fig.)

Must-Know Concept:
Area of trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) × Height

Paper 2

1. (a) $\frac{4 - 5x}{2} > \frac{4x + 1}{4}$

$\frac{4 - 5x}{1} > \frac{4x + 1}{2}$

$2(4 - 5x) > 4x + 1$

$8 - 10x > 4x + 1$

$7 > 14x$

$x < \frac{7}{14}$

$\mathbf{x < \frac{1}{2}}$

Must-Know Concept:
Perform cross multiplication across the inequality. Do take note of the signs of the terms that are multiplied across. Solve for x.

(b) (i) $W = \frac{1}{2}m(v^2 - u^2)$
$= \frac{1}{2}(3)(10^2 - 4^2)$
$= \mathbf{126}$

Must-Know Concept:
Substitute $m = 3$, $u = 4$ and $v = 10$ into the given equation.

(ii) $W = \frac{1}{2}m(v^2 - u^2)$
$2W = m(v^2 - u^2)$
$\frac{2W}{m} = v^2 - u^2$
$u^2 = v^2 - \frac{2W}{m}$
$\mathbf{u = \pm\sqrt{v^2 - \frac{2W}{m}}}$

Must-Know Concept:
Make u the subject by shifting terms that have u to the left-hand side and terms that do not have u to the right-hand side, and simplify the equation.

(c) $\frac{8xy + 2x^2}{x^2 - 16y^2} = \frac{2x(4y + x)}{(x - 4y)(x + 4y)}$
$= \mathbf{\frac{2x}{x - 4y}}$

Must-Know Concept:
Factorise the numerator and denominator. Cancel out the common terms.

(d) $\frac{4}{x+3} + \frac{3}{x-2} = 1$
$4(x - 2) + 3(x + 3) = (x - 2)(x + 3)$
$4x - 8 + 3x + 9 = x^2 - 2x + 3x - 6$
$0 = x^2 - 6x - 7$
$0 = (x - 7)(x + 1)$
$\therefore x = \mathbf{7}$ or $x = \mathbf{-1}$

2. (a) 41 199.8 GWh $= 41\ 199.8 \times 10^9$ Wh
$= 4.119\ 98 \times 10^{13}$ Wh
$= 4.119\ 98 \times 10^{10}$ kWh
$= \mathbf{4.12 \times 10^{10}}$ **kWh** (3 sig. fig.)

Must-Know Concept:
Standard form refers to $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

(b) $\frac{16\ 693.0 \text{ GWh}}{41\ 199.8 \text{ GWh}} \times 100\% \approx 40.517$
$= \mathbf{40.5\%}$ (3 sig. fig.)

(c) $\frac{7304.5 \text{ GWh}}{5.077 \times 10^6} = \frac{7304.5 \times 10^9 \text{ Wh}}{5.077 \times 10^6}$
$= \frac{7304.5 \times 10^6 \text{ kWh}}{5.077 \times 10^6}$
≈ 1438.74 kWh
$= \mathbf{1439}$ **kWh** (nearest kWh)

(d) $17\ 202.3 - 13\ 628.0 = 3574.3$ GWh
$\frac{3574.3}{13\ 628.0} \times 100\% \approx 26.2276$
$= \mathbf{26.2\%}$ (3 sig. fig.)

Must-Know Concept:
Percentage increase $= \frac{\text{Increase}}{\text{Original amount}} \times 100\%$

(e) Let the amount of domestic electricity consumption in 2000 be x GWh.
$\frac{7304.5 - x}{x} \times 100\% = 27.6\%$
$\frac{7304.5 - x}{x} = 0.276$
$7304.5 - x = 0.276x$
$1.276x = 7304.5$
$x \approx 5\ 724.53$
$= \mathbf{5725}$ **GWh** (nearest GWh)

3. (a) $y = \frac{x}{5}(2 + 4x - x^2)$
$p = \frac{-3}{5}[2 + 4(-3) - (-3)^2]$
$= \mathbf{11.4}$

Must-Know Concept:
Substitute $x = -3$ into the equation to find p.

(b) Refer to Appendix 6.

(c) From the graph,
$x = \mathbf{-1.5}$ or $x = \mathbf{1.7}$ or $x = \mathbf{3.85}$

Must-Know Concept:
Solutions of the curve are the x-intercepts.

(d) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{10 - 2}{-3 - (-1.5)}$
$= \mathbf{-5.33}$ (3 sig. fig.)

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient via the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

(e) (i) $3y = 2x - 6$

x	−3	0	3
y	−4	−2	0

(ii) From the graph, the coordinates of the point where the line intersects the curve are **(4.2, 0.8)**.

4. (a) $\angle BFC = 180° - 97°$ (adj. $\angle$s on a str. line)
$= 83°$
$\angle ABF = \angle BFC$ (alt. $\angle$s, AB // DC)
$= \mathbf{83°}$

(b) $\angle BFC = \angle EFD$ (vert. opp. $\angle$s)
$\angle BCF = \angle EDF$ (alt. $\angle$s, BC // DE)
$\angle CBF = \angle DEF$ (alt. $\angle$s, BC // DE)
$\therefore$ **$\triangle BCF$ is similar to $\triangle EDF$** (AAA) (shown)

Must-Know Concept:
Vertically opposite angles are equal.
When there are parallel lines, look out for alternate, corresponding or interior angles.

(c) $\triangle EAB$

Must-Know Concept:
Two triangles are similar if there are three pairs of equal corresponding angles.

(d) (i) Since $\triangle BCF$ is similar to $\triangle EAB$

$\frac{AB}{CF} = \frac{AE}{BC}$

$= \frac{3+2}{3}$

$= \frac{5}{3}$

$\therefore$ the ratio $AB : CF$ is **5 : 3**.

(ii) Since $\triangle EDF$ is similar to $\triangle EAB$

$\frac{\text{Area of } \triangle EDF}{\text{Area of } \triangle EAB} = \left(\frac{2}{5}\right)^2$

$\frac{9.72}{\text{Area of } \triangle EAB} = \frac{4}{25}$

Area of $\triangle EAB = \frac{25 \times 9.72}{4}$

$= 60.75 \text{ cm}^2$

Since $\triangle BCF$ is similar to $\triangle EDF$

$\frac{\text{Area of } \triangle BCF}{\text{Area of } \triangle EDF} = \left(\frac{3}{2}\right)^2$

$\frac{\text{Area of } \triangle BCF}{9.72} = \frac{9}{4}$

Area of $\triangle BCF = \frac{9 \times 9.72}{4}$

$= 21.87 \text{ cm}^2$

Total area of the shape $ABCFE$

$= 60.75 + 21.87$

$= \mathbf{82.62 \text{ cm}^2}$

Must-Know Concept:
For similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

5. (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} -5 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-6} \\ \mathbf{-7} \end{pmatrix}$

(ii) $|\overrightarrow{AB}| = \sqrt{(-6)^2 + (-7)^2}$

$= \sqrt{85}$

$= \mathbf{9.22 \text{ units}}$ (3 sig. fig.)

Must-Know Concept:
The formula for the magnitude of a vector is $\sqrt{x^2 + y^2}$.

(iii) $\overrightarrow{BA} = -\overrightarrow{AB}$

$= -\begin{pmatrix} -6 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

Since $\overrightarrow{BA} = 2\overrightarrow{AC}$,

$\overrightarrow{AC} = \frac{1}{2}\overrightarrow{BA}$

$= \frac{1}{2}\begin{pmatrix} 6 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 3.5 \end{pmatrix}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= \begin{pmatrix} 1 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 3.5 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 8.5 \end{pmatrix}$

$\therefore$ the coordinates of point C are **(4, 8.5)**.

(b) (i) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= \begin{pmatrix} 8 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ -5 \end{pmatrix}$

$\therefore$ the coordinates of point Q are (12, –5).

Gradient $PQ = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \frac{-5-(-2)}{12-8}$

$= \frac{-3}{4}$

Using $y = mx + c$,
since the coordinates of point P are (8, –2),

$-2 = \frac{-3}{4}(8) + c$

$c = -2 + \frac{3}{4}(8)$

$= 4$

$\therefore$ the equation of line PQ is

$\mathbf{y = \frac{-3}{4}x + 4}$.

Must-Know Concept:
Find the gradient of the line PQ and substitute the gradient and given point in the form $y = mx + c$.

(ii) $y = \frac{-3}{4}x + 4$ ------ ①

$2y - 4x = 19$ ------ ②

Substitute ① into ②:

$2\left(\frac{-3}{4}x + 4\right) - 4x = 19$

$\frac{-3}{2}x + 8 - 4x = 19$

$\frac{-11}{2}x = 11$

$x = -2$

$y = \frac{-3}{4}(-2) + 4$

$= 5.5$

$\therefore$ the coordinates of the point of intersection of PQ and RS are **(–2, 5.5)**.

Must-Know Concept:
Solve simultaneous equations of both lines to find the point of intersection of PQ and RS.

6. (a) $\tan a = \frac{1}{12}$

$a = \tan^{-1}\frac{1}{12}$

$= \mathbf{4.76°}$ (3 sig. fig.) (shown)

Must-Know Concept:
Taking a as the reference angle, we know that the ratio of the opposite side to the adjacent side is 1 : 12.

(b) Volume of the prism

$= \left[\frac{1}{2} \times (0.28 \times 12) \times 0.28\right] \times 1.95$

$= 0.917\ 28\ m^3$

$1\ m^3 \longrightarrow 2300$ kg

$0.917\ 28\ m^3 \longrightarrow 2300 \times 0.917\ 28$

$= 2110$ kg (3 sig. fig.)

$\therefore$ the mass of the completed ramp is **2110 kg**.

Must-Know Concept:
Volume of a prism = Cross-sectional area × Length

(c) Length of ramp $= \sqrt{(0.28 \times 12)^2 + (0.28)^2}$

≈ 3.3716 m

300 mm = 0.3 m

The total length of the handrail

$= 3.3716 + (0.3 \times 2)$

$= 3.9716$

$=$ **3.97 m** (3 sig. fig.)

7. (a) (i) $\angle BAC = \angle BDC$ ($\angle$s in the same segment)

$=$ **48°**

Must-Know Concept:
Note that angles in the same segment are equal.

(ii) $\angle BCA = 180° - 90° - 48°$ ($\angle$ sum of $\triangle$)

$=$ **42°**

(iii) $\angle AEO = 90°$ ($\perp$ from centre bisect chord)

$\angle EOA = 180° - 90° - 48°$ ($\angle$ sum of $\triangle$)

$= 42°$

$\angle AOD = 180° - 42°$ (adj. $\angle$s on a str. line)

$=$ **138°**

(iv) $\angle FAD = \frac{180° - 138°}{2}$ (base $\angle$s of isos $\triangle$)

$= 21°$

$\angle FBC = \angle FAD$ ($\angle$s in the same segment)

$= 21°$

$\angle AFB = 21° + 42°$ (ext. $\angle$ of $\triangle$)

$=$ **63°**

Must-Know Concept:
Note that the exterior angle of a triangle = sum of its interior opposite angles.

(b) (i) Let $\angle AOB$ be θ,

Perimeter of mirror = arc DC + arc AB + AD + BC

$235 = 50\theta + 20\theta + 30 + 30$

$70\theta = 175$

$\theta = 2.5$

$\therefore \angle AOB =$ **2.5 rad**

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians)
In this question, we have formed an equation to solve for the unknown angle.

(ii) Area of the mirror

= Area of sector ODC − Area of sector OAB

$= \frac{1}{2}(50)^2(2.5) - \frac{1}{2}(20)^2(2.5)$

$=$ **2625 cm²**

Must-Know Concept:
Area of a sector $= \frac{1}{2}r^2\theta$ (where θ is in radians)

8. (a) Speed $= \frac{\text{Distance}}{\text{Time}}$

$= \frac{40}{\frac{x}{60}}$

$= \mathbf{\frac{2400}{x}}$ **km/h**

Must-Know Concept:
Speed $= \frac{\text{Distance}}{\text{Time}}$

(b) $90 - 40 = 50$ km

Speed $= \frac{\text{Distance}}{\text{Time}}$

$= \frac{50}{\frac{x+15}{60}}$

$= \mathbf{\frac{3000}{x+15}}$ **km/h**

Must-Know Concept:
Speed for the remaining part of journey $= \frac{(90 - 40 =)\ 50\ \text{km}}{\text{Time taken for this part}}$

(c) $\frac{2400}{x} - \frac{3000}{x+15} = 9$

$2400(x + 15) - 3000(x) = 9x(x + 15)$

$2400x + 36\ 000 - 3000x = 9x^2 + 135x$

$9x^2 + 735x - 36\ 000 = 0$

$\mathbf{3x^2 + 245x - 12\ 000 = 0}$ (shown)

Must-Know Concept:
The difference in speed for (a) and (b) is 9.

(d) $3x^2 + 245x - 12\ 000 = 0$

$x = \frac{-245 \pm \sqrt{245^2 - 4(3)(-12\ 000)}}{2(3)}$

$= \frac{-245 \pm \sqrt{204\ 025}}{2(3)}$

≈ 34.449 or -116.115

$=$ **34.45** or **−116.12** (2 d.p.)

Must-Know Concept:
Solve for x using the quadratic equation.

(e) Since $t > 0$, $t = 34.449$ min,

average speed for the whole journey

$= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{90}{\frac{x}{60} + \frac{x+15}{60}}$

$= \frac{5400}{2x + 15}$

$= \frac{5400}{2(34.449) + 15}$

≈ 64.3647 km/h

$=$ **64.4 km/h** (3 sig. fig.)

Must-Know Concept:
Average speed $= \frac{\text{Total distance}}{\text{Total time taken}}$

9. (a) (i) $\frac{AC}{\sin \angle ABC} = \frac{AB}{\sin \angle ACB}$

$\frac{3.8}{\sin \angle ABC} = \frac{4.6}{\sin 78°}$

$\sin \angle ABC = \frac{3.8 \sin 78°}{4.6}$

$\angle ABC \approx 53.90°$

$= \mathbf{53.9°}$ (1 d.p.)

Must-Know Concept:
Apply the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

(ii) Area of flower bed

$= \frac{1}{2}(AB)(AC) \sin \angle BAC$

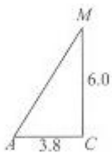
$= \frac{1}{2}(4.6)(3.8) \sin (180° - 78° - 53.9°)$

≈ 6.505

$= \mathbf{6.51\ m^2}$

Must-Know Concept:
Area of a triangle $= \frac{1}{2}ab \sin C$

(b) (i)

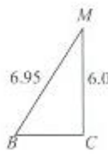


$AM = \sqrt{6^2 + 3.8^2}$

≈ 7.102

$= \mathbf{7.10\ m}$ (2 d.p.) (shown)

(ii)



$\sin \angle MBC = \frac{6}{6.95}$

$\angle MBC \approx 59.69°$

$= 59.7°$ (1 d.p.)

$\therefore$ the angle of elevation of M from B is **59.7°**.

Must-Know Concept:
Since M is vertically above C, $\triangle MBC$ is a right-angled triangle.

(iii) $(AB)^2 = (AM)^2 + (BM)^2 - 2(AM)(BM) \cos \angle AMB$

$(4.6)^2 = (7.10)^2 + (6.95)^2 - 2(7.10)(6.95) \cos \angle AMB$

$\cos \angle AMB = \frac{(4.6)^2 - (7.10)^2 - (6.95)^2}{-2(7.10)(6.95)}$

$\approx 38.20°$

$= \mathbf{38.2°}$ (1 d.p.)

Must-Know Concept:
Apply the Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$

10. (a) (i)

Time (t minutes)	Cumulative frequency
$t < 40$	15
$t < 50$	47
$t < 60$	77
$t < 70$	93
$t < 80$	100

When $t < 60$,
percentage of males who ran faster than 10 km/h

$= \frac{77}{100} \times 100\%$

$= \mathbf{77\%}$

(ii) (a) Mean time

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{15(35) + 32(45) + 30(55) + 16(65) + 7(75)}{100}$

$= \frac{5180}{100}$

$= \mathbf{51.8\ min}$ (3 sig. fig.)

Must-Know Concept:
Use the formula $\frac{\Sigma ft}{\Sigma f}$ to find the mean since we are given the frequency and values of t.

(b) Standard deviation

$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$= \sqrt{\frac{280\,900}{100} - \left(\frac{5180}{100}\right)^2}$

≈ 11.214

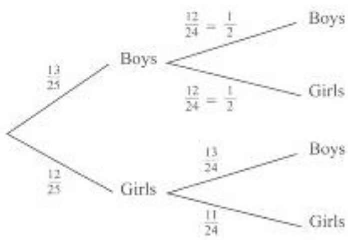
$= \mathbf{11.2\ min}$ (3 sig. fig.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma ft^2}{\Sigma f} - (\bar{t})^2}$ to find the standard deviation since we have the required values.

(iii) The males ran faster than the females as their mean time was shorter.
The females were more consistent in terms of their running speed as their standard deviation was lesser than that of the males which means their speeds were closer to the average mean speed.

Must-Know Concept:
Compare both data using their mean times and standard deviations as these values are found in previous parts.

(b) (i)



Must-Know Concept:
As the selections are without replacement, remember to minus one from the total number after every selection.

(ii) (a) P(two girls are selected)

$= \frac{12}{25} \times \frac{11}{24}$

$= \mathbf{\frac{11}{50}}$

(b) P(one boy and one girl are selected)

= P(boy then girl) + P(girl then boy)

$= (\frac{13}{25} \times \frac{1}{2}) + (\frac{12}{25} \times \frac{13}{24})$

$= \frac{13}{50} + \frac{13}{50}$

$= \mathbf{\frac{13}{25}}$

Must-Know Concept:
Use the tree diagram in (i) to find the required probabilities.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2013 EXAMINATION PAPER

Paper 1

1. (a) $5p - 3(p-2) = 5p - 3p + 6$
$= \mathbf{2p + 6}$

Must-Know Concept:
Expand 3 into the term $(p-2)$. Group and simplify the like terms together.

(b) $(2x-1)(x+3) = 0$
$2x - 1 = 0$ or $x + 3 = 0$
$x = \frac{1}{2}$ or $x = -3$

Must-Know Concept:
$(a+b)(c-d) = ac - ad + bc - bd$

2. (a) $x = \mathbf{7.5\text{ cm}}$

Must-Know Concept:
Note that all four lengths of a rhombus are equal.

(b) $y° = \frac{180° - 25° - 25°}{2}$ (base ∠ of isos. △)
$= \frac{130°}{2}$
$= 65°$
$y = \mathbf{65}$

Must-Know Concept:
Note that the diagonals of a rhombus are perpendicular to each other.

3. (a) When $y = 0$,
$2x - 5(0) = 20$
$2x = 20$
$x = 10$
∴ the coordinates of point A are **(10, 0)**.

Must-Know Concept:
The y-coordinate of all the points lying on the x-axis is 0.

(b) When $x = 0$,
$2(0) - 5y = 20$
$-5y = 20$
$y = -4$
∴ the coordinates of point B are **(0, −4)**.

Must-Know Concept:
The x-coordinate of all the points lying on the y-axis is 0.

4. Difference between Brian's and Anne's money = 7 – 4
= 3 units

3 units = \$42
1 unit = \$42 ÷ 3
= \$14
9 units = 9 × \$14
= \$126
Cheryl had **\$126**.

Must-Know Concept:
Find the difference in units between Brian and Anne. Equate that difference to \$42 and find the amount that represents 1 unit. Multiply the amount by the number of units that Cheryl has.

5. (a) $8 \times 16^{\frac{1}{4}} = 2^n$
$2^3 \times (2^4)^{\frac{1}{4}} = 2^n$
$2^3 \times 2 = 2^n$
$2^4 = 2^n$
$\therefore n = \mathbf{4}$

Must-Know Concept:
Express all numbers in the same base of 2.

(b) $\frac{1}{9} = 3^k$
$\frac{1}{3^2} = 3^k$
$3^{-2} = 3^k$
$\therefore k = \mathbf{-2}$

Must-Know Concept:
Express all numbers in the same base of 3.

6. Total surface area of the solid
$= \pi(3r)^2 + 2\pi(3r)^2 + 2\pi r(2r)$
$= 9\pi r^2 + 18\pi r^2 + 4\pi r^2$
$= \mathbf{31\pi r^2}$

Must-Know Concept:
When finding total surface area, count the number of surfaces carefully.
Area of a circle = πr^2
Curved surface area of a hemisphere = $2\pi r^2$
Curved surface area of a cylinder = $2\pi rh$

7. (a) $294 = \mathbf{2 \times 3 \times 7^2}$

2	294
3	147
7	49
7	7
	1

(b) $8 = 2^3$
$20 = 2^2 \times 5$
Number $= 2^3 \times 5$
$= 40$
Factors of 40 = **1, 2, 4, 5, 8, 10, 20 and 40**

Must-Know Concept:
Find the lowest common multiple of the given numbers.

8. Flight time = 5572 ÷ 725
$= 7\frac{497}{725}$
= **7 hours 41 minutes** (nearest minute)

Must-Know Concept:
Time = $\frac{\text{Distance}}{\text{Speed}}$

9. (a) The integers are **−2, −1, 0, 1, 2, 3 and 4**.

Must-Know Concept:
Split the inequality into 2 inequalities, and solve each of them individually. List out all the integers that lie within the range of the inequalities.

(b) $-5 < 2x - 3$ and $2x - 3 \leqslant 7$

$2x > -2$ $\quad$ $2x \leqslant 10$

$x > -1$ $\quad$ $x \leqslant 5$

$\therefore \mathbf{-1 < x \leqslant 5}$

Must-Know Concept:
Split the inequality into 2 inequalities, and solve each of them individually.

10. (a) $x^2 - 8x + 14 = x^2 - 8x + 16 - 2$
$= x^2 - 2(4)x + 4^2 - 2$
$= \mathbf{(x-4)^2 - 2}$

Must-Know Concept:
Complete the square method.

(b) Minimum value = **−2**

(c) Equation of the line of symmetry:
$\mathbf{x = 4}$

Must-Know Concept:
The line of symmetry is a line passing through the minimum point.

11. Fraction of yellow marbles $= 70\% \times \left(1 - \frac{3}{8}\right)$
$= \frac{70}{100} \times \frac{5}{8}$
$= \frac{7}{16}$

Fraction of green marbles $= 1 - \frac{3}{8} - \frac{7}{16}$
$= \frac{3}{16}$

$\frac{3}{16} \longrightarrow 12$ marbles

$\frac{1}{16} \longrightarrow 12 \div 3$
$= 4$ marbles

$\frac{16}{16} \longrightarrow 16 \times 4$
$= 64$ marbles

64 marbles are in the bag altogether.

Must-Know Concept:
Fraction of yellow marbles = 70% × Fraction of marbles that are not red.
Find the fraction of green marbles and equate it to 12. Solve for the number of marbles in the bag altogether.

12. (a) $y = x^3 + 5$

(b) $y = 5 - x^2$

(c) $y = 5^x$

Must-Know Concept:
Observe the shapes of the graphs.

13. (a) $\angle DBC = 180° - 53° - 39°$ ($\angle$ sum of $\triangle$)
$= 88°$

Since $\angle DBC \neq 90°$, ***BD* is not a diameter of the circle.**

Must-Know Concept:
When a tangent to the circle meets its radius, there is a right angle.

(b) (i) $\angle ACD = \angle ABD$ ($\angle$ in the same segment)
$= \mathbf{42°}$

Must-Know Concept:
Note that angles in the same segment are equal.

(ii) $\angle CDA = 180° - 37° - 42°$ ($\angle$ sum of $\triangle$)
$= 101°$
$\angle CDE = 180° - 101°$ (adj. $\angle$s on a str. line)
$= \mathbf{79°}$

14. (a) $\mathscr{E}$ = {integers x: $2 \leqslant x \leqslant 12$}
= {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
A = {prime numbers}
= {2, 3, 5, 7, 11}
B = {multiples of 4}
= {4, 8, 12}
B' = **{2, 3, 5, 6, 7, 9, 10, 11}**

Must-Know Concept:
The complement of a set A, relative to the universal set $\mathscr{E}$, is the set that contains all the elements that are not in A but in $\mathscr{E}$.

(b) $A \cap B'$ = **{2, 3, 5, 7, 11}**

Must-Know Concept:
The intersection of two sets is the set of all elements which are common in both sets.

(c) $A \cup B$ = {2, 3, 4, 5, 7, 8, 11, 12}
$(A \cup B)'$ = **{6, 9, 10}**

Must-Know Concept:
The union of two sets is the set of all elements which are either in one of the set or both.

15. (a) $\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 29(5) + 10(3) + 5(2) \\ 30(5) + 6(3) + 8(2) \end{pmatrix}$
$= \begin{pmatrix} \mathbf{185} \\ \mathbf{184} \end{pmatrix}$

Must-Know Concept:
The product of a 2 × 3 matrix and a 3 × 1 matrix gives a resulting 2 × 1 matrix.

(b) It represents **the points gained by Teresa and Robert respectively.**

16. (a) Using Pythagoras' Theorem,
$h = \sqrt{13^2 - 5^2}$
$= \sqrt{144}$
$= \mathbf{12\ cm}$ (shown)

Must-Know Concept:
Make use of Pythagoras' Theorem: $c^2 = a^2 + b^2$

(b) Volume of the solid $= \frac{1}{3}\pi(5^2)(12) + \frac{2}{3}\pi(5^3)$

$= \mathbf{576\ cm^3}$ (3 sig. fig.)

Must-Know Concept:
Volume of a cone $= \frac{1}{3}\pi r^2 h$
Volume of a hemisphere $= \frac{2}{3}\pi r^3$

17. (a) $12x^2y \div 3xy^{-5} = \frac{12}{3}x^{2-1}y^{1-(-5)}$

$= \mathbf{4xy^6}$

Must-Know Concept:
Use the Law of Indices: $a^m \div a^n = a^{m-n}$.

(b) $\frac{3x}{(2x-1)^2} - \frac{2}{2x-1} = \frac{3x - 2(2x-1)}{(2x-1)^2}$

$= \frac{3x - 4x + 2}{(2x-1)^2}$

$= \mathbf{\frac{-x+2}{(2x-1)^2}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Observe that $(2x-1)^2 = (2x-1)(2x-1)$. Hence, the common denominator between the two fractions is $(2x-1)(2x-1)$.

18. (a) $3xy - 6ay - 4x + 8a = 3y(x - 2a) - 4(x - 2a)$

$= \mathbf{(x - 2a)(3y - 4)}$

Must-Know Concept:
Group similar terms together and factorise the common factors out.

(b) $3x^2 + 10x - 8 = \mathbf{(3x - 2)(x + 4)}$

19. (a) $f \propto \sqrt{T}$

$\Rightarrow f = k\sqrt{T}$, where k is a constant

When $f = 360$ and $T = 64$,

$360 = k\sqrt{64}$

$8k = 360$

$k = 45$

$\therefore f = \mathbf{45\sqrt{T}}$

Must-Know Concept:
Frequency, f Hz, is proportional to the square root of the tension, T newtons, $f = k\sqrt{T}$.

(b) When $f = 540$,

$540 = 45\sqrt{T}$

$\sqrt{T} = 12$

$T = \mathbf{144\ N}$

(c) Let T_1 and T_2 be the tensions in the strings.

$\frac{45\sqrt{T_1}}{45\sqrt{T_2}} = \frac{2}{1}$

$\frac{\sqrt{T_1}}{\sqrt{T_2}} = \frac{2}{1}$

$\frac{T_1}{T_2} = \frac{4}{1}$

The ratio of the tensions in the strings is **4 : 1**.

Must-Know Concept:
Observe that $T = \frac{f^2}{k}$. Hence, the ratio of the tensions in the strings would be the square of the ratio of the frequencies of the notes.

20. (a) Using Cosine Rule,

$BC^2 = BD^2 + DC^2 - 2(BD)(DC)\cos\angle BDC$

$125^2 = 58^2 + 84^2 - 2(58)(84)\cos\angle BDC$

$\cos\angle BDC = \frac{58^2 + 84^2 - 125^2}{2(58)(84)}$

$\angle BDC = \cos^{-1}\frac{58^2 + 84^2 - 125^2}{2(58)(84)}$

$= \mathbf{122.3°}$ (1 d.p.)

Must-Know Concept:
Given all three lengths in $\triangle ABD$, we can apply the Cosine Rule to solve for the required angle.

(b) Using Cosine Rule,

$DC^2 = BD^2 + BC^2 - 2(BD)(BC)\cos\angle CBD$

$84^2 = 58^2 + 125^2 - 2(58)(125)\cos\angle CBD$

$\cos\angle CBD = \frac{58^2 + 125^2 - 84^2}{2(58)(125)}$

$\angle CBD = \cos^{-1}\frac{58^2 + 125^2 - 84^2}{2(58)(125)}$

$= 34.6°$ (1 d.p.)

$\angle ABC = 55° + 34.6°$

$= 89.6°$

$\angle BCD = 180° - 122.3° - 34.6°$

$= 23.1°$

Bearing of D from C

$= 360° - 89.6° - 23.1°$ (alt. $\angle$s, // lines, $\angle$s at a point)

$= \mathbf{247.3°}$

Must-Know Concept:
When calculating bearings, always move in a clockwise direction to identify the required angle.

21. (a) $1370 = \mathbf{1.37 \times 10^3}$

Must-Know Concept:
Standard form refers to $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

(b) Percentage increase $= \frac{2.48 \times 10^4 - 8.69 \times 10^3}{8.69 \times 10^3} \times 100\%$

$= \mathbf{185\%}$ (3 sig. fig)

Must-Know Concept:
Percentage increase $= \frac{\text{Increase in volume flowing}}{\text{Original volume of water flowing}} \times 100\%$

(c) Volume of water flowing over the waterfall in one hour

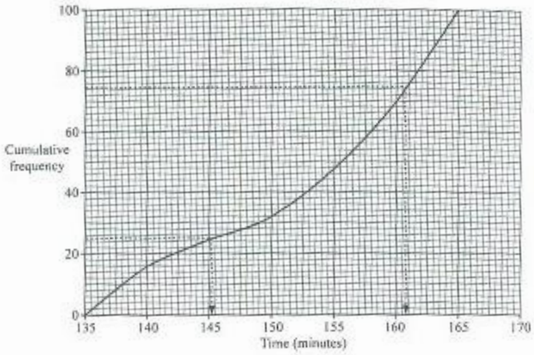
$= 1.82 \times 10^4 \times 3600$

$= 6.552 \times 10^7$

$= \mathbf{6.6 \times 10^7}$ **litres** (2 sig. fig.)

Must-Know Concept:
3600 s = 1 h
Convert the rate of water flow per second to per hour.

22. (a)



Year	Lower quartile	Median	Upper quartile	Interquartile range
2011	**145.25**	155.5	**160.75**	**15.5**
2012	155.5	**160.5**	165.5	10

Must-Know Concept:
Read the 2011 data from the cumulative frequency graph. The value of the lower quartile is at the 25% position and the value of the upper quartile is at the 75% position. Read the 2012 data from the box-and-whisker plot. The value of the median is the centre line in the box.

(b)

Statement	Agree/disagree	Reason
The runners in 2012 were quicker on average	**Disagree**	**The median time for 2011 is shorter than that for 2012.**
The times of the first 25 runners were closer together in 2011	**Agree**	**The lower quartile in 2011 is shorter than that for 2012.**

23. (a) $x + 41 = 180 - (5y - 8) - (2x - 18)$ .... ①

$2x - 18 = 5y - 8$ .... ②

From ①,

$x + 41 = 206 - 5y - 2x$

$\mathbf{3x + 5y = 165}$ .... ③

From ②,

$\mathbf{2x - 5y = 10}$ .... ④

Must-Know Concept:
ABC is an isosceles triangle, hence the base angles are equal. Since *ABC* is a triangle, all interior angles add up to 180°.

(b) ③ + ④:

$5x = 175$

$x = 35$

Substitute $x = 35$ into ③:

$3(35) + 5y = 165$

$105 + 5y = 165$

$5y = 60$

$y = 12$

$\therefore x = 35, y = 12$

$\angle A = x + 41$
$= 35 + 41$
$= \mathbf{76°}$

$\angle B = 2x - 18$
$= 2(35) - 18$
$= \mathbf{52°}$

$\angle C = 5y - 8$
$= 5(12) - 8$
$= \mathbf{52°}$

24. (a) Acceleration after 5 s $= \frac{16 - 0}{10 - 0}$

$= \mathbf{1.6\ m/s^2}$

Must-Know Concept:
In a speed-time graph, the gradient of the graph is the acceleration.

(b) Total distance travelled between the two road junctions

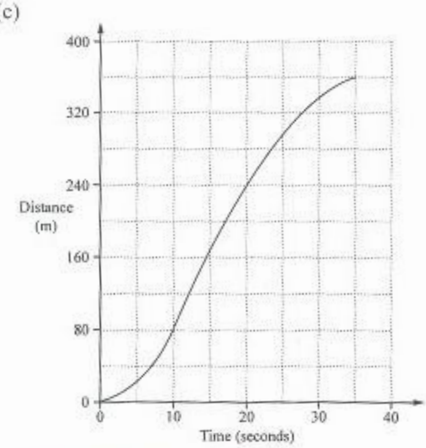
$= \frac{1}{2}(10)(16) + (20 - 10)(16) + \frac{1}{2}(35 - 20)(16)$

$= 80 + 160 + 120$

$= \mathbf{360\ m}$

Must-Know Concept:
In a speed-time graph, the area under the graph is the distance travelled.

(c)



Must-Know Concept:
A distance-time graph shows the distance away from the point of origin with respect to time.

Paper 2

1. (a) (i) $s = (0)(15) + \frac{1}{2}(0.6)(15)^2$

$= \mathbf{67\frac{1}{2}}$

Must-Know Concept:
Substitute $u = 0$, $a = 0.6$ and $t = 15$ to find s.

(ii) $s = ut + \frac{1}{2}at^2$

$\frac{1}{2}at^2 = s - ut$

$at^2 = 2(s - ut)$

$a = \frac{2(s - ut)}{t^2}$

Must-Know Concept:
Make a the subject by shifting terms that have a to the left-hand side and terms that do not have a to the right-hand side, and simplify the equation.

(b) (i) $18p^2 - 8 = 2(9p^2 - 4)$
$= \mathbf{2(3p + 2)(3p - 2)}$

Must-Know Concept:
Factorise the common term '2' out and further factorise the expression via $a^2 - b^2 = (a + b)(a - b)$.

(ii) $\frac{18p^2 - 8}{6p^2 - 14p - 12} = \frac{2(3p + 2)(3p - 2)}{2(3p^2 - 7p - 6)}$

$= \frac{(3p + 2)(3p - 2)}{(3p + 2)(p - 3)}$

$= \mathbf{\frac{3p - 2}{p - 3}}$

Must-Know Concept:
Factorise the numerator and denominator. Cancel out any common terms and simplify.

(c) $\frac{6}{3 - 2x} - \frac{4}{2 - x} = \frac{6(2 - x) - 4(3 - 2x)}{(3 - 2x)(2 - x)}$

$= \frac{12 - 6x - 12 + 8x}{(3 - 2x)(2 - x)}$

$= \mathbf{\frac{2x}{(3 - 2x)(2 - x)}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Since there are no common term between both the denominators, the denominator would become $(3 - 2x)(2 - x)$.

2. (a) (i) Deposit $= \frac{1}{5} \times \$78\ 500$
$= \$15\ 700$
Total amount David pays for the car
$= \$15\ 700 + 36 \times \$1900$
$= \$15\ 700 + \$68\ 400$
$= \mathbf{\$84\ 100}$

(ii) Value of the car at the end of three years
$= \$78\ 500\left(1 - \frac{10}{100}\right)^3$
$= \$57\ 226.50$
Reduction in the value of the car
$= \$78\ 500 - \$57\ 226.50$
$= \$21\ 273.50$
Overall percentage reduction in the value of the car $= \frac{\$212\ 73.50}{\$78\ 500} \times 100\%$
$= \mathbf{27.1\%}$

Must-Know Concept:
Percentage reduction $= \frac{\text{Reduction}}{\text{Original value}} \times 100\%$

(b) Volume of fuel David would use in one year
$= \frac{12\ 000}{100} \times 4.5$
$= 540\ l$
Amount David would expect to spend on fuel in one year $= 540 \times \$1.95$
$= \mathbf{\$1053}$

(c) Amount of each monthly payment
$= (\frac{100 + 7}{100} \times \$565) \div 12$
$= (\frac{107}{100} \times \$565) \div 12$
$= \mathbf{\$50.38}$ (nearest cent)

3. (a) (i) $\angle ABD = \angle BDC$ (alt. $\angle$s, $AB \parallel DC$)
$= 41°$
$\angle ADB = 180° - 34° - 41°$ ($\angle$ sum of $\triangle$)
$= \mathbf{105°}$

Must-Know Concept:
When there are parallel lines, look out for alternate, corresponding or interior angles.

(ii) Using Sine Rule,

$\frac{62}{\sin 105°} = \frac{BD}{\sin 34°}$

$BD = \frac{62 \sin 34°}{\sin 105°}$

$= \mathbf{35.9\ mm}$ (3 sig. fig)

Must-Know Concept:
Apply the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

(iii) Area of trapezium $ABCD$
$=$ Area of $\triangle ABD$ + Area of $\triangle BCD$
$= \frac{1}{2} \times 62 \times 35.9 \times \sin 41° + \frac{1}{2} \times 55 \times 35.9 \times \sin 41°$
≈ 1377.82
$= \mathbf{1380\ mm^2}$ (3 sig. fig)

Must-Know Concept:
Note that the area of trapezium $ABCD$ is made up of two triangles – $\triangle ABD$ and $\triangle BCD$.

(b) $\frac{A_s}{A_L} = \left(\frac{CD_s}{CD_L}\right)^2$

$= \left(\frac{55}{88}\right)^2$

$= \left(\frac{5}{8}\right)^2$

$= \frac{25}{64}$

$\frac{1377.82}{A_L} = \frac{25}{64}$

$A_L = \frac{64(1377.82)}{25}$

$= 3530\ mm^2$ (3 sig. fig)

The area of the enlarged logo is $\mathbf{3530\ mm^2}$.

Must-Know Concept:
For similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

4. (a) (i) $T_4 = 4^2 + 8$
$= \mathbf{24}$

(ii) $T_n = \boldsymbol{n^2 + 2n}$

(iii) $T_{50} = 50^2 + 2(50)$
$= \mathbf{2600}$

Must-Know Concept:
Substitute $n = 50$.

(b) $P_1 = -9 + 4(1) = -5$
$P_2 = -9 + 4(2) = -1$
$P_3 = -9 + 4(3) = 3$
$P_4 = -9 + 4(4) = 7$
$P_n = \boldsymbol{-9 + 4n}$

Must-Know Concept:
Observe that the pattern of the sequence is: 4 more than the previous term.

(c)
$$\frac{P_n}{T_n} = \frac{1}{5}$$
$$\frac{-9+4n}{n^2+2n} = \frac{1}{5}$$
$$5(-9+4n) = n^2 + 2n$$
$$-45 + 20n = n^2 + 2n$$
$$n^2 - 18n + 45 = 0$$
$$(n-15)(n-3) = 0$$
$n = \mathbf{15}$ or $n = \mathbf{3}$

Must-Know Concept:
Equate the value of P_n and T_n to 1 and 5 respectively. Solve for n.

5. (a) $p = 80 + 16(6) - 5(6)^2$
$= \mathbf{-4}$

Must-Know Concept:
Substitute $t = 6$ into the equation to find p.

(b) Refer to Appendix 7.

(c) (i) From the graph,
maximum height of the stone above sea level = **93 m**

(ii) From the graph,
the time that the stone was more than 85 m above the sea level is $0.35 < t < 2.85$.
Hence, the length of time that the stone was more than 85 m above sea level
$= 2.85 - 0.35$
$=$ **2.5 s**

(iii) From the graph,
the time taken for the stone to hit the water
$=$ **5.9 s**

(d) Gradient of the curve at (4, 64) $= \frac{88-40}{3-5}$
$= \mathbf{-24\ m/s}$

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient via the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

6. (a) (i) $\overrightarrow{OL} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{OM} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 4 \end{pmatrix}$
$= \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$\therefore$ the coordinates of point M are **(–3, 6)**.

Must-Know Concept:
Translation means the movement of a point by the same distance in a given direction. A translation of $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$ means the point shifts 6 units to the left along the x-axis and 4 units up along the y-axis.

(ii) Gradient of $LM = \frac{4}{-6}$
$= -\frac{2}{3}$

Equation of line LM:
$$y - 2 = -\frac{2}{3}(x-3)$$
$$y = -\frac{2}{3}x + 2 + 2$$
$$\boldsymbol{y = -\frac{2}{3}x + 4}$$

Must-Know Concept:
Find the gradient and substitute the values in the form $(y - y_1) = m(x - x_1)$.

(iii)
$$\overrightarrow{MN} = 2\,\overrightarrow{ML}$$
$$\overrightarrow{ON} - \overrightarrow{OM} = 2(\overrightarrow{OL} - \overrightarrow{OM})$$
$$\overrightarrow{ON} = 2\overrightarrow{OL} - 2\overrightarrow{OM} + \overrightarrow{OM}$$
$$= 2\overrightarrow{OL} - \overrightarrow{OM}$$
$$= 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} 9 \\ -2 \end{pmatrix}$$

$\therefore$ the coordinates of point N are **(9, –2)**.

(b) (i) (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= \mathbf{b - a}$

(b) $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB}$
$= \frac{2}{3}(\mathbf{b} - \mathbf{a})$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a})$
$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$
$= \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$

(c) $\overrightarrow{AF} = -\overrightarrow{FA}$
$= -\overrightarrow{OB}$
$= -\mathbf{b}$

$\overrightarrow{AQ} = \frac{2}{3}\overrightarrow{AF}$
$= -\frac{2}{3}\mathbf{b}$

(ii) $\overrightarrow{QX} = \frac{1}{2}\overrightarrow{QP}$
$= \frac{1}{2}(\overrightarrow{QA} + \overrightarrow{AP})$
$= \frac{1}{2}\left[\frac{2}{3}\mathbf{b} + \frac{2}{3}(\mathbf{b} - \mathbf{a})\right]$
$= \frac{1}{3}\mathbf{b} + \frac{1}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$
$= \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$
$= \mathbf{\frac{1}{3}(2b - a)}$

7. (a) (i) Interior angle of a regular 20-sided polygon
$= \frac{(20-2) \times 180°}{20}$
$= \frac{3240°}{20}$
$= \mathbf{162°}$

Must-Know Concept:
Each interior angle of an n-sided regular polygon $= \frac{(n-2) \times 180°}{n}$

(ii) $(n-2) \times 180 = 4(120) + (n-4)p$
$180n - 360 = 480 + (n-4)p$
$(n-4)p = 180n - 840$
$p = \mathbf{\frac{180n - 840}{n-4}}$

Must-Know Concept:
Sum of interior angles of an n-sided regular polygon $= (n-2) \times 180°$

(b) (i) $\mathbf{\triangle OAE \equiv \triangle ODE}$

Must-Know Concept:
Two triangles are congruent if they have the same shape and the same size.

(ii) (a) $\angle AOE = 180° - 90° - 23°$ (∠ sum of △)
$= \mathbf{67°}$

Must-Know Concept:
The tangent AE is perpendicular to the radius AO.

(b) $\angle AOD = \angle AOE + \angle DOE$
$= 67° + 67°$
$= 134°$

$\angle ODA = \frac{180° - 134°}{2}$ (base ∠ of isos. △)
$= \mathbf{23°}$

Must-Know Concept:
Since $OA = OD$, $\triangle AOD$ is isosceles.
When solving questions on circles, keep a lookout for isosceles triangles.

(c) $\angle ACD = 134° \div 2$ (∠ at centre = 2∠ at circumference)
$= 67°$
$\angle ADC = 180° - 67° - 26° - 23°$ (∠ sum of △)
$= \mathbf{64°}$

Must-Know Concept:
Note that an angle at centre is twice the angle at circumference.

(d) $\angle ABC = 180° - 64°$ (∠s in opp. segments)
$= \mathbf{116°}$

Must-Know Concept:
Note that angles in opposite segments are supplementary.

8. (a) (i) $AO = 1.20 \div 2$
$= 0.6$ m

In $\triangle ABO$,
$\sin 20° = \frac{AO}{BO}$
$= \frac{0.6}{BO}$
$BO = \frac{0.6}{\sin 20°}$
$= 1.75$ m (3 sig. fig)

The radius of the sector OBC is **1.75 m**.

Must-Know Concept:
$\triangle AOB$ is a right-angled triangle. Taking $\angle ABO$ as the reference angle, AO is the opposite side while BO is the hypotenuse.

(ii) $\angle BOA = 180° - 20° - 90°$ (∠ sum of △)
$= 70°$
$\angle BOC = 180° - 70° - 70°$ (adj. ∠s on a str. line)
$= 40°$
$= \frac{40°}{180°} \times \pi$
$= \mathbf{\frac{2}{9}\pi \text{ rad}}$

Must-Know Concept:
$180° = \pi$ rad

(iii) In $\triangle ABO$,
$\tan 20° = \frac{AO}{AB}$
$= \frac{0.6}{AB}$
$AB = \frac{0.6}{\tan 20°}$
≈ 1.648

Total perimeter of the window frame
$= 1.648 + 1.20 + 1.648 + 1.75\left(\frac{2}{9}\pi\right)$
$= \mathbf{5.72}$ **m** (3 sig. fig)

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians)

(b) Area of the window

$= 2\left(\frac{1}{2} \times 0.6 \times 1.648\right) + \frac{1}{2}(1.75)^2\left(\frac{2}{9}\pi\right)$

$\approx 2.0578 \text{ m}^2$

Cost of manufacturing the window

$= 2.0578 \times \$78.50$

$=$ **\$161.54** (nearest cent)

Must-Know Concept:
Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height
Area of a sector $= \frac{1}{2}r^2\theta$ (where θ is in radians)

9. (a) Width $= \frac{x}{2}$ **cm**

Height $=$ **$(x-3)$ cm**

(b) Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= x^2 + \left(\frac{x}{2}\right)^2$

$= x^2 + \frac{x^2}{4}$

$= \frac{5x^2}{4}$ (shown)

Must-Know Concept:
$ABCD$ is a rectangle, hence $\angle ABC = 90°$. Use Pythagoras' Theorem to solve for AC, $AC^2 = AB^2 + BC^2$.

(c) Using Pythagoras' Theorem,

$AG^2 = AC^2 + CG^2$

$12^2 = \frac{5x^2}{4} + (x-3)^2$

$144 = \frac{5x^2}{4} + x^2 - 6x + 9$

$576 = 5x^2 + 4x^2 - 24x + 36$

$9x^2 - 24x - 540 = 0$

$\mathbf{3x^2 - 8x - 180 = 0}$ (shown)

Must-Know Concept:
AGC is a right-angled triangle.

(d) $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-180)}}{2(3)}$

$=$ **9.193** or **−6.527** (3 d.p.)

Must-Know Concept:
Solve for x using the quadratic equation.

(e) Since $x > 0$,
volume of the cuboid

$= 9.193 \times \frac{9.193}{2} \times (9.193 - 3)$

$=$ **262 cm³** (3 sig. fig)

Must-Know Concept:
Volume of the cuboid = Length × Breadth × Height

10. (a) (i)

1st counter (columns); 2nd counter (rows)

	1	2	3	4	5
1		(2,1)	(3,1)	(4,1)	(5,1)
2	(1,2)		(3,2)	(4,2)	(5,2)
3	(1,3)	(2,3)		(4,3)	(5,3)
4	(1,4)	(2,4)	(3,4)		(5,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	

Must-Know Concept:
The diagonal of the table is left blank as it is not possible to draw two counters of the same number, since the draws are without replacement.

(ii) (a) P(both counters have numbers less than 3) $= \frac{2}{20}$

$= \mathbf{\frac{1}{10}}$

(b) P(neither counter has an even number)

$= \frac{6}{20}$

$= \mathbf{\frac{3}{10}}$

(c) P(sum of the numbers is 10) = **0**

(d) P(product of the numbers is less than 6)

$= \frac{8}{20}$

$= \mathbf{\frac{2}{5}}$

Must-Know Concept:
Use the possibility diagram in (a) to identify the combinations that fulfill the questions' requirements.

(b) (i) Percentage of the students who are less than 170 cm tall

$= \frac{6}{14} \times 100\%$

$=$ **42.9%** (3 sig. fig.)

(ii) Median height

= average of the 7th and 8th students' height

$= \frac{171 + 172}{2}$

$=$ **171.5 cm**

Must-Know Concept:
As there is an even number of values, the median is the average of the two middle values when arranged in ascending order.

(iii) Mean height

$$= \frac{156 + 160 + 162 + 164 + 167 + 168 + 171 + 172 + 172 + 174 + 178 + 181 + 183 + 190}{14}$$

$$= 171\frac{2}{7} \text{ cm}$$

Standard deviation of the heights

$$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$$

$$= \sqrt{\frac{411\,908}{14} - \left(171\frac{2}{7}\right)^2}$$

= **9.12 cm** (3 sig. fig)

Must-Know Concept:

Find the mean height first, then use the formula $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ to find the standard deviation.

(iv) The median of the heights will be reduced by 5 cm, but the standard deviation of the heights will remain the same.

Must-Know Concept:

Standard deviation measures the spread of the values, therefore the error in the values will not affect the standard deviation.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2012 EXAMINATION PAPER

Paper 1

1. (a) $\frac{11.83^2}{18.52 - 2.79} = \mathbf{8.8969}$

(b) $\frac{11.83^2}{18.52 - 2.79} = \mathbf{8.9}$ (2 sig. fig.)

Must-Know Concept:
Non-zero digits are significant.

2. (a) Exterior $\angle = 180° - 140°$ (adj. $\angle$s on a str. line)
$= 40°$

Number of sides $= \frac{360°}{40°}$
$= \mathbf{9}$

Must-Know Concept:
One method to solve the problem is to find the exterior angle of the polygon first.

(b) Exterior $\angle = \frac{360°}{10}$
$= \mathbf{36°}$

Must-Know Concept:
The sum of exterior angles of any n-sided polygon is 360°.

3. (a) P(red marble) $= \frac{\text{Number of red marbles}}{\text{Total number of marbles}}$
$= \frac{10}{18}$
$= \mathbf{\frac{5}{9}}$

(b) Let x be the number of blue marbles that must be placed in the bag.
$\frac{5+x}{18+x} = \frac{1}{2}$
$2(5+x) = 18 + x$
$10 + 2x = 18 + x$
$2x - x = 18 - 10$
$x = \mathbf{8}$

Must-Know Concept:
The number of blue marbles and total number of marbles will increase at the same time.

4. (a) Range $= 67 - 32$
$= \mathbf{35}$

Must-Know Concept:
Range is the difference between the maximum value and the minimum value.

(b) Median position $= \frac{n+1}{2}$
$= \frac{16+1}{2}$
$= 8.5$
= average of 8th and 9th data

$\therefore$ median $= \frac{42+45}{2}$
$= \mathbf{43.5}$

Must-Know Concept:
As there is an even number of values, the median is the average of the two middle values when arranged in ascending order.

5. $y = ka^x$.........①
Substitute (0, 5) into ①:
$5 = ka^0$
$k = \mathbf{5}$

Substitute (6, 320) and $k = 5$ into ①:
$320 = 5a^6$
$64 = a^6$
$2^6 = a^6$
$\therefore a = \mathbf{2}$

Must-Know Concept:
Substitute the two pairs of coordinates into the equation to find k and a.

6. 1.5 l = 1500 ml = 1500 cm^3
Height of the container $= \frac{1500}{\pi(6.8)^2}$
$= \mathbf{10.3\ cm}$ (3 sig. fig.)

Must-Know Concept:
Volume of a cylinder $= \pi r^2 h$

7. (a) 5, 9, 13, 17, 21, 25, 29, 33
+4 +4 +4 +4 +4 +4 +4

8th term = **33**

Must-Know Concept:
Observe that the pattern of the sequence is: 4 more than the previous term.

(b) nth term $= 5 + 4(n-1)$
$= 5 + 4n - 4$
$= \mathbf{1 + 4n}$

(c) $1 + 4n = 205$
$4n = 205 - 1$
$= 204$
$n = \mathbf{51}$

Must-Know Concept:
Equate the expression found in (b) to 205 and solve for n.

8. $\frac{2x-3}{6} + \frac{x+2}{3} = \frac{5}{2}$

$\frac{2x-3+2(x+2)}{6} = \frac{5}{2}$

$\frac{2x-3+2x+4}{6} = \frac{5}{2}$

$\frac{4x+1}{6} = \frac{5}{2}$

$2(4x+1) = 5(6)$

$8x + 2 = 30$

$8x = 30 - 2$

$= 28$

$x = \mathbf{3.5}$

Must-Know Concept:
Convert the denominators of the fractions on the left-hand side such that it becomes a single fraction. Perform cross multiplication and solve for x.

9. (a)

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

$1008 = 2^4 \times 3^2 \times 7$

$\therefore x = \mathbf{4}$ and $y = \mathbf{2}$

(b) Smallest positive integer

$k = 2 \times 3$

$= \mathbf{6}$

Must-Know Concept:
For $\frac{1350}{k}$ to be a square number, the power of each of its prime factors must be a multiple of 2.

10. (a) 4 cm : 1 km

4 cm : 100 000 cm

1 : **25 000**

(b) 4 cm : 1 km

16 cm² : 1 km²

Actual area of the lake $= \frac{236}{16}$

$= \mathbf{14.75\ km^2}$

Must-Know Concept:
Convert the scale to cm² : km². The scale becomes 1 cm² : n^2 km².

11. (a) $2(3x+2y) - 5(x-2y) = 6x + 4y - 5x + 10y$

$= \mathbf{x + 14y}$

Must-Know Concept:
Expand 2 and 5 to the terms $(3x+2y)$ and $(x-2y)$ respectively. Group and simplify the like terms together.

(b) $\frac{5}{(x+2)^2} - \frac{1}{(x+2)} = \frac{5-(x+2)}{(x+2)^2}$

$= \frac{5-x-2}{(x+2)^2}$

$= \mathbf{\frac{3-x}{(x+2)^2}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Observe that $(x+2)^2 = (x+2)(x+2)$. Hence, the common denominator between the two fractions is $(x+2)(x+2)$.

12. (a) Mean height of the girls

$= \frac{\Sigma fh}{\Sigma f}$

$= \frac{4(145) + 9(155) + 11(165) + 6(175)}{4+9+11+6}$

$= \frac{4840}{30}$

$= \mathbf{161\ cm}$ (3 sig. fig.)

Must-Know Concept:
Use the formula $\frac{\Sigma fh}{\Sigma f}$ to find the mean since we are given the frequency and values of h.

(b) Sum of fh^2

$= 4(145)^2 + 9(155)^2 + 11(165)^2 + 6(175)^2$

$= 783\ 550$

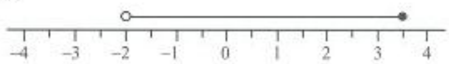
Standard deviation $= \sqrt{\frac{\Sigma fh^2}{\Sigma f} - (\bar{h})^2}$

$= \sqrt{\frac{783\ 550}{30} - \left(\frac{4840}{30}\right)^2}$

$= \mathbf{9.48\ cm}$ (3 sig. fig.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma fh^2}{\Sigma f} - (\bar{h})^2}$ to find the standard deviation since we have the required values.

13. (a)



Must-Know Concept:
Split the inequality into 2 inequalities, and solve each of them individually. Closed circles are used for inequalities that are 'less / more than or equals to', whereas open circles are used for inequalities that are 'less / more than'.

(b) $-9 \leqslant 4x - 3 < 9$

$-9 + 3 \leqslant 4x < 9 + 3$

$-6 \leqslant 4x < 12$

$\mathbf{-1.5 \leqslant x < 3}$

Must-Know Concept:
Split the inequality into 2 inequalities, and solve each of them individually.

14. (a) Refer to Appendix 8.

Must-Know Concept:
When calculating bearings, always move in a clockwise direction to identify the required angle.

(b) Actual distance of the aeroplane from B
$= 4.8 \times 10$
$= \mathbf{48\ km}$

(c) Bearing of the aeroplane from $B = 360° - 28°$
$= \mathbf{332°}$

15. (a) $\angle OBA = \angle OAB$ (base $\angle$s of isos. $\triangle$)
$= 35°$
$\angle AOB = 180° - 35° - 35°$ ($\angle$ sum of $\triangle$)
$= \mathbf{110°}$

Must-Know Concept:
Since $OA = OB$, $\triangle AOB$ is isosceles.
When solving questions on circles, keep a lookout for isosceles triangles.

(b) $\angle ACB = \frac{1}{2}(\angle AOB)$ ($\angle$ at centre $= 2\angle$ at circumference)
$= \frac{1}{2}(110°)$
$= \mathbf{55°}$

Must-Know Concept:
Note that an angle at centre is twice the angle at circumference.

(c) $\angle OAT = \angle OBT$ (tangent $\perp$ radius)
$= 90°$
$\angle ATB = 360° - 90° - 110° - 90°$ ($\angle$ sum of quad.)
$= \mathbf{70°}$

Must-Know Concept:
The tangent TA is perpendicular to the radius AO and the tangent TB is perpendicular to the radius BO.

16. (a) $x^2y \times x^5y^{-2} = x^{2+5}y^{1-2}$
$= x^7y^{-1}$
$= \mathbf{\frac{x^7}{y}}$

(b) $\left(\frac{64}{x^{15}}\right)^{-\frac{1}{3}} = \left(\frac{x^{15}}{64}\right)^{\frac{1}{3}}$
$= \frac{x^5}{\sqrt[3]{64}}$
$= \mathbf{\frac{x^5}{4}}$

Must-Know Concept:
Use the Law of Indices: $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^{x}$.

17. (a) Total number of pupils who chose to go to college
$= 180 - 72 - 42$
$= 66$
Number of females who chose to go to college
$= 66 - 34$
$= \mathbf{32}$

(b) Angle representing the males choosing polytechnic
$= \frac{28}{105} \times 360°$
$= \mathbf{96°}$

(c) Males
Percentage of males who chose employment
$= \frac{43}{105} \times 100\%$
$= 40\frac{20}{21}\%$

Females
Total number of females
$= 180 - 105$
$= 75$

Percentage of females who chose employment
$= \frac{29}{75} \times 100\%$
$= 38\frac{2}{3}\%$

$40\frac{20}{21}\% - 38\frac{2}{3}\% = 2.29\%$ (3 sig. fig.)
2.29% more males chose employment.

Must-Know Concept:
Express intermediate answers in exact form to retain accuracy, only express your answer in 3 significant figures in the last step.

18. (a) $xy - 3x + 2y - 6 = x(y-3) + 2(y-3)$
$= \mathbf{(x+2)(y-3)}$

Must-Know Concept:
Group similar terms together and factorise the common factors out.

(b) $6x^2 - 15x - 9 = 3(2x^2 - 5x - 3)$
$= \mathbf{3(2x+1)(x-3)}$

19. (a) Deceleration for the last 20 s = –gradient
$= -\frac{18-0}{80-100}$
$= \mathbf{0.9\ m/s^2}$

Must-Know Concept:
In a speed-time graph, the gradient of the graph is the acceleration or deceleration.

(b) Total distance $= \frac{1}{2}(30-0)(18) + (80-30)(18) + \frac{1}{2}(100-80)(18)$
$= \mathbf{1350\ m}$

Must-Know Concept:
In a speed-time graph, the area under the graph is the distance travelled.

(c) 1 s $\longrightarrow$ 18 m
3600 s $\longrightarrow$ 3600×18
$= 64\ 800$ m
$= 64.8$ km
18 m/s = **64.8 km/h**

20. (a) $y = 10 - 2x$ .... ①

Substitute $P(0, y)$ into ①:

$y = 10 - 2(0)$

$= 10$

Substitute $Q(x, 0)$ into ①:

$0 = 10 - 2x$

$2x = 10$

$x = 5$

$\therefore$ the coordinates of P are **(0, 10)** and the coordinates of Q are **(5, 0)**.

Must-Know Concept:
The x-coordinate of all the points lying on the y-axis is 0. The y-coordinate of all the points lying on the x-axis is 0.

(b) Length of PQ

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-5)^2 + (10-0)^2}$

$= \sqrt{25 + 100}$

$=$ **11.2 units** (3 sig. fig.)

21. (a) Length of arc $PAQ = 16 \times 2.5$

$=$ **40 cm**

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians)

(b) Let r be the radius of the base of the cone.

$40 = 2\pi r$

$r = \frac{40}{2\pi}$

$=$ **6.37 cm** (3 sig. fig.)

Must-Know Concept:
A sector of a circle can be used to form a cone. The radius of the sector becomes the slant height of the cone, while the arc length of the sector becomes the circumference of the cone.

(c) π rad $= 180°$

1 rad $= \frac{180°}{\pi}$

2.5 rad $= 2.5 \times \frac{180°}{\pi}$

$=$ **143.2°** (1 d.p.)

Must-Know Concept:
π rad $= 180°$

22. (a) In $\triangle BCD$, $\tan \angle BCD = \frac{BD}{CD}$

$= \frac{8.5}{12}$

$\angle BCD = \tan^{-1} \frac{8.5}{12}$

$=$ **35.3°** (1 d.p.)

Must-Know Concept:
Taking $\angle BCD$ as the reference angle, BD is the opposite side while CD is the adjacent side.

(b) In $\triangle ABD$, $\sin 48° = \frac{BD}{AB}$

$= \frac{8.5}{AB}$

$AB = \frac{8.5}{\sin 48°}$

$=$ **11.4 cm** (3 sig. fig.)

Must-Know Concept:
Taking $\angle BAD$ as the reference angle, BD is the opposite side while AB is the hypotenuse.

23. (a) Since $\triangle ABX$ is similar to $\triangle CDX$,

$\frac{12}{CD} = \frac{8}{12}$

$8CD = 144$

$CD =$ **18 cm**

Must-Know Concept:
The ratios of corresponding lengths in similar triangles are the same.

(b) Since $\triangle ABX$ is similar to $\triangle CDX$,

$\frac{6}{XD} = \frac{8}{12}$

$72 = 8\,XD$

$XD = 9$ cm

$\therefore AD = AX + XD$

$= 8 + 9$

$=$ **17 cm**

(c) Since $\triangle ABX$ is similar to $\triangle CDX$,

$\frac{\text{Area of } \triangle ABX}{\text{Area of } \triangle CDX} = \left(\frac{8}{12}\right)^2$

$\frac{21.3}{\text{Area of } \triangle CDX} = \frac{4}{9}$

Area of $\triangle CDX = 21.3 \div \frac{4}{9}$

$=$ **47.9 cm²** (3 sig. fig.)

Must-Know Concept:
For similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

24. (a) (i) $M \propto r^2$

$\Rightarrow M = kr^2$, where k is a constant

When $r = 4$, $M = 720$,

$720 = k(4)^2$

$k = \frac{720}{16}$

$= 45$

$\therefore$ **$M = 45r^2$**

(ii) When $r = 6$,

$M = 45(6)^2$

$=$ **1620 grams**

Must-Know Concept:
M is directly proportional to r^2, $M = kr^2$.

(b) $\frac{M_A}{M_B} = \left(\frac{r_A}{r_B}\right)^2$

$= \left(\frac{150}{100}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

$\therefore M_A : M_B = \mathbf{9 : 4}$

Must-Know Concept:
Since A's radius is 50% greater than B's, that would mean that A's radius is 1.5 times of B. Hence, the mass of A would be ($1.5^2 =$) 2.25 times of B.

25. (a) (i) $\overrightarrow{OM} = \frac{2}{3}\overrightarrow{OC}$

$= \frac{2}{3}(3\mathbf{a} + 3\mathbf{b})$

$= \mathbf{2a + 2b}$

(ii) $\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM}$

$= -6\mathbf{a} + 2\mathbf{a} + 2\mathbf{b}$

$= \mathbf{-4a + 2b}$

(iii) $\overrightarrow{ON} = \frac{1}{2}\overrightarrow{OB}$

$= \frac{1}{2}(6\mathbf{b})$

$= \mathbf{3b}$

$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$

$= \mathbf{-6a + 3b}$

(b) $\overrightarrow{AM} = \frac{2}{3}(-6\mathbf{a} + 3\mathbf{b})$

$= \frac{2}{3}\overrightarrow{AN}$

$\therefore$ **A, M and N lie in a straight line.**

Must-Know Concept:
Two vectors are parallel if **a** can be written as the scalar multiple of **b**, **a** = k**b**, where k is a scalar.

(c) $\overrightarrow{AM} = \frac{2}{3}\overrightarrow{AN}$

$\frac{\overrightarrow{AM}}{\overrightarrow{AN}} = \frac{2}{3}$

$\therefore AM : AN = \mathbf{2 : 3}$

Paper 2

1. (a) $3x^2 - 48y^2 = 3(x^2 - 16y^2)$

$= \mathbf{3(x + 4y)(x - 4y)}$

Must-Know Concept:
Factorise the common term '3' out and further factorise the expression using $a^2 - b^2 = (a + b)(a - b)$.

(b) $\frac{2x}{5} - \frac{x-1}{15} = \frac{6x}{15} - \frac{x-1}{15}$

$= \frac{6x - x + 1}{15}$

$= \mathbf{\frac{5x+1}{15}}$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction. Since $15 = 5 \times 3$, hence the common denominator is 15.

(c) $\frac{x}{2y} \div \frac{3x^2y}{4} = \frac{x}{2y} \times \frac{4}{3x^2y}$

$= \frac{4x}{6x^2y^2}$

$= \mathbf{\frac{2}{3xy^2}}$

Must-Know Concept:
$\frac{x}{2y} \div \frac{3x^2y}{4} = \frac{x}{2y} \times \frac{4}{3x^2y}$

(d) (i) $V = \frac{4\pi}{3}(a^3 - b^3)$

When $a = 2.5$ and $b = 1.9$,

$V = \frac{4\pi}{3}(2.5^3 - 1.9^3)$

$= \mathbf{36.7}$ (3 sig. fig.)

Must-Know Concept:
Substitute $a = 2.5$ and $b = 1.9$ into the given equation.

(ii) $V = \frac{4\pi}{3}(a^3 - b^3)$

$\frac{3V}{4\pi} = a^3 - b^3$

$a^3 = \frac{3V}{4\pi} + b^3$

$= \frac{3V + 4\pi b^3}{4\pi}$

$a = \sqrt[3]{\frac{3V + 4\pi b^3}{4\pi}}$

Must-Know Concept:
Make a the subject by shifting terms that have a to the left-hand side and terms that do not have a to the right-hand side, and simplify the equation.

(e) $\frac{2x - 7}{3x + 2} = 4$

$2x - 7 = 4(3x + 2)$

$= 12x + 8$

$2x - 12x = 8 + 7$

$-10x = 15$

$x = -\frac{15}{10}$

$= \mathbf{-1\frac{1}{2}}$

Must-Know Concept:
Perform cross multiplication and solve for x.

2. (a) (i) $\angle ABC = 180° - 60°$ (int. $\angle$s, $AB // DC$)

$= \mathbf{120°}$

Must-Know Concept:
The sum of interior angles between two parallel lines is 180°.

(ii) $\angle BAD = \angle BCD$ (opp. ∠s of //gram)
$= 60°$
$\angle DAE = 60° - 18°$
$= \mathbf{42°}$

Must-Know Concept:
Note that opposite angles in a parallelogram are equal.

(b) Area of parallelogram $ABCD$

$= 2 \times \text{Area of } \triangle ABD$

$= 2 \times \left[\frac{1}{2} \times AD \times \left(AB \times \sin \angle BAD\right)\right]$

$= 2 \times \left[\frac{1}{2} \times 15.2 \times \left(10.6 \times \sin 60°\right)\right]$

$= \mathbf{140\ cm^2}$ (3 sig. fig.)

(c) $\angle AEB = 180° - 120° - 18°$ (∠ sum of △)
$= 42°$

Using Sine Rule,

$$\frac{AB}{\sin \angle AEB} = \frac{AE}{\sin \angle ABE}$$

$$\frac{10.6}{\sin 42°} = \frac{AE}{\sin 120°}$$

$$AE = \frac{10.6 \sin 120°}{\sin 42°}$$

$= \mathbf{13.72\ cm}$ (4 sig. fig.) (shown)

Must-Know Concept:
Apply the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

(d) Using Cosine Rule,

$DE^2 = AE^2 + AD^2 - 2(AE)(AD) \cos \angle EAD$
$= 13.72^2 + 15.2^2 - 2(13.72)(15.2) \cos 42°$

$DE = \sqrt{13.72^2 + 15.2^2 - 2(13.72)(15.2) \cos 42°}$
$= \mathbf{10.5\ cm}$ (3 sig. fig.)

Must-Know Concept:
Apply the Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$

3. (a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{8} \\ \mathbf{-4} \end{pmatrix}$$

(ii) Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1-(-3)}{-5-3}$

$= \frac{4}{-8}$

$= -\frac{1}{2}$

$y = -\frac{1}{2}x + c$ .... ①

Substitute $A(-5, 1)$ into ①:

$1 = -\frac{1}{2}(-5) + c$

$1 = \frac{5}{2} + c$

$c = -1\frac{1}{2}$

∴ equation of line AB is $\mathbf{y = -\frac{1}{2}x - 1\frac{1}{2}}$.

Must-Know Concept:
Find the gradient and y-intercept of the line AB and substitute the values in the form $y = mx + c$.

(b) (i) (a) $4y - 3x + 8 = 0$

$4y = 3x - 8$

$y = \frac{3}{4}x - 2$

∴ gradient of line $l = \mathbf{\frac{3}{4}}$.

(b) ∴ y-intercept of line $l = \mathbf{-2}$.

Must-Know Concept:
Express the equation of the line in the form $y = mx + c$.

(ii) $y = \frac{3}{4}x - 2$ .... ①

$3x + 2y = 5$ .... ②

Substitute ① into ②:

$3x + 2\left(\frac{3}{4}x - 2\right) = 5$

$3x + \frac{6}{4}x - 4 = 5$

$\frac{12x + 6x}{4} = 5 + 4$

$\frac{18x}{4} = 9$

$18x = 36$

$x = 2$

Substitute $x = 2$ into ①:

$y = \frac{3(2)}{4} - 2$

$= -\frac{1}{2}$

∴ the coordinates of C are $\mathbf{(2, -\frac{1}{2})}$.

Must-Know Concept:
Solve simultaneous equations of both lines to find the point of intersection C.

4\. (a) (i) $£1 = \$2.10$

$£200 = \$2.10 \times 200$

$= \$420$

$\therefore$ she received **\$420**.

(ii) $€1 = £0.93$

$£1 = €\frac{1}{0.93}$

Since $£1 = \$2.10$ and $£1 = €\frac{1}{0.93}$,

$\therefore \$2.10 = €\frac{1}{0.93}$

$\$1 = €\frac{1}{1.953}$

$\$500 = €\frac{1}{1.953} \times 500$

$= €256$ (nearest euro).

$\therefore$ he received **€256**.

Must-Know Concept:
Convert all the exchange rate of given currency to base unit 1 first to easily find the required amount.

(b) $107\% \longrightarrow \$295$

$1\% \longrightarrow \left(\frac{295}{107}\right)$

$100\% \longrightarrow 100 \times \$\left(\frac{295}{107}\right)$

$= \$275.70$ (nearest cent)

$\therefore$ the price of the camera before tax is **\$275.70**.

Must-Know Concept:
The price of the camera including tax is 107%. So, the original price of the camera before tax is 100%.

(c) (i) 1 light year $= 9.46 \times 10^{15}$ m

8.6 light years $= 8.6 \times 9.46 \times 10^{15}$ m

$= 81.356 \times 10^{15}$ m

$= 8.1356 \times 10^{16}$ m

$= 8.1356 \times 10^{16} \times 10^{-3}$ km

$= 8.1356 \times 10^{13}$ km

$\therefore$ the distance of Sirius from the Sun is **8.1356×10^{13} km**.

(ii) 1 year = 365 days

$= 365 \times 24$ h

$= 8760$ h

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$= \frac{3.97 \times 10^{13}}{60\,000}$ h

$= \frac{3.97 \times 10^{13}}{6 \times 10^{4}}$ h

$= \frac{3.97 \times 10^{9}}{6 \times 8760}$ years

$=$ **75 500 years** (3 sig. fig.)

Must-Know Concept:
Let 1 year be 365 days. Then, 1 year = 8760 hours.

5\. (a) (i) $\angle ADE = \angle ABE$ ($\angle$s in the same segment)

$=$ **62°**

$\angle BAE = 90°$ ($\angle$ in a semicircle)

$\angle AEB = 180° - 90° - 62°$ ($\angle$ sum of $\triangle$)

$=$ **28°**

$\angle BAD = 180° - 110°$ ($\angle$s in opp. segment)

$=$ **70°**

Must-Know Concept:
Note that angles in the same segment are equal and angles in opposite segments are supplementary.

(ii) (a) $\angle DAE = 90° - 70°$

$= 20°$

$\angle AFE = 180° - 20° - 28°$ ($\angle$ sum of $\triangle$)

$= 132°$

$\angle BFD = \angle AFE$ (vert. opp. $\angle$s)

$=$ **132°**

Must-Know Concept:
Note that vertically opposite angles are equal.

(b) $\angle ADB = \angle AEB$ ($\angle$s in the same segment)

$= 28°$

$\angle DBF = 180° - 28° - 132°$ ($\angle$ sum of $\triangle$)

$= 20°$

$\angle BDC = \angle DBF$ (alt. $\angle$s, $BE // CD$)

$= 20°$

$\angle DBC = 180° - 110° - 20°$ ($\angle$ sum of $\triangle$)

$=$ **50°**

Must-Know Concept:
Since BE is parallel to CD, keep a lookout for alternate, corresponding or interior angles.

(b) Volume of the solid = Area of sector × Height

$= \left(\frac{76°}{360°} \times \pi \times 6^2\right) \times 4$

$=$ **95.5 cm³** (3 sig. fig.)

Must-Know Concept:
Area of a sector $= \frac{\theta}{360} \times \pi r^2$ (where θ is in degrees)
Volume of a prism = Cross-sectional area × Length

6\. (a) (i) $\mathscr{E}$ = {integers x: $1 \leqslant x \leqslant 15$}

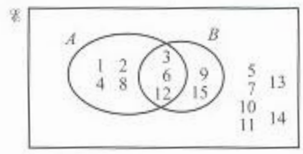
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

A = {factors of 24}

= {1, 2, 3, 4, 6, 8, 12}

B = {multiples of 3}

= {3, 6, 9, 12, 15}



(ii) $A \cap B$ = {3, 6, 12}

$\therefore$ n($A \cap B$) = **3**

Must-Know Concept:
The number of elements in a set A is written as n(A). The intersection of two sets is the set of all elements which are common in both sets.

(iii) $A' \cup B = \{3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15\}$

Must-Know Concept:
The complement of a set A, relative to the universal set $\mathscr{E}$, is the set that contains all the elements that are not in A but in $\mathscr{E}$. The union of two sets is the set of all elements which are either in one of the set or both.

(b) (i) $\mathbf{M} = 5\mathbf{L}$

$$= 5\begin{pmatrix} 25 & 10 \\ 20 & 30 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{125} & \mathbf{50} \\ \mathbf{100} & \mathbf{150} \end{pmatrix}$$

Must-Know Concept:
When a matrix is multiplied by a scalar, then every element in the matrix is multiplied by the scalar.

(ii) $\mathbf{N} = \begin{pmatrix} \mathbf{3} \\ \mathbf{2.5} \end{pmatrix}$

(iii) $\mathbf{P} = \mathbf{MN}$

$$= \begin{pmatrix} 125 & 50 \\ 100 & 150 \end{pmatrix}\begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$$

$$= \begin{pmatrix} 125(3) + 50(2.5) \\ 100(3) + 150(2.5) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{500} \\ \mathbf{675} \end{pmatrix}$$

Must-Know Concept:
The product of a 2 × 2 matrix and a 2 × 1 matrix gives a resulting 2 × 1 matrix.

(iv) The elements of **P** represent the **total fee in dollars taken in a week by the playgroup from the boys and girls**.

(v) 100% ⟶ \$500 + \$675
= \$1175

1% ⟶ $\$\left(\frac{1175}{100}\right)$

105% ⟶ $105 \times \$\left(\frac{1175}{100}\right)$
= \$1234 (nearest dollars)

∴ the total fees taken in a week by the playgroup after the increase is **\$1234**.

7. (a) $p = \frac{6^3}{20} + \frac{10}{6} - 5$

$= \mathbf{7.47}$ (2 d.p.)

Must-Know Concept:
Substitute $x = 6$ into the equation to find p.

(b) Refer to Appendix 9.

(c) $\frac{x^3}{20} + \frac{10}{x} = 5$

$\frac{x^3}{20} + \frac{10}{x} - 5 = 0$

From the graph,
$x = \mathbf{2.2}$ or $x = \mathbf{3.5}$

Must-Know Concept:
$y = \frac{x^3}{20} + \frac{10}{x} - 5 = 0$

(d) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4.2 - (-1)}{6 - 3}$

$= \mathbf{1.73}$ (3 sig. fig.)

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient using the formula, gradient $= \frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

(e) From the graph, the coordinates of the minimum point are **(2.80, –0.42)**.

8. (a) Number of minutes the pump would take to empty the full tank

$= \frac{\mathbf{2000}}{\boldsymbol{x}}$

Must-Know Concept:
Time $= \frac{\text{Amount of water}}{\text{Rate}}$

(b) Number of minutes the large pump would take to empty the full tank

$= \frac{\mathbf{2000}}{\boldsymbol{x} + \mathbf{100}}$

Must-Know Concept:
Time $= \frac{\text{Amount of water}}{\text{Rate}}$

(c) $\frac{2000}{x} - \frac{2000}{x + 100} = 20$

$2000(x + 100) - 2000x = 20x(x + 100)$
$2000x + 200\,000 - 2000x = 20x^2 + 2000x$
$200\,000 = 20x^2 + 2000x$
$20x^2 + 2000x - 200\,000 = 0$
$\mathbf{x^2 + 100x - 10\,000 = 0}$ (shown)

Must-Know Concept:
The difference between the time taken for the small pump and the time taken for the large pump is 20.

(d) $x^2 + 100x - 10\,000 = 0$

$x = \frac{-100 \pm \sqrt{100^2 - 4(1)(-10\,000)}}{2(1)}$

$= \mathbf{61.8}$ or $\mathbf{-161.8}$ (1 d.p.)

Must-Know Concept:
Solve for x using the quadratic equation.

(e) Since the rate is positive, $x = 61.8$

Time taken to empty the full tank using the large pump $= \frac{2000}{x+100}$

$= \frac{2000}{61.8+100}$

≈ 12.36 min

$=$ **12 min 22 s** (nearest second)

Must-Know Concept:
1 min = 60 s

9. (a) $\angle AOB = 360° \div 6$ ($\angle$s at a point)

$= 60°$

$\angle ABO = \frac{180° - 60°}{2}$ (base $\angle$ of isos. $\triangle$)

$= 60°$

$MB = 4 \div 2$

$= 2$ cm

In $\triangle OMB$,

$\tan 60° = \frac{OM}{MB}$

$= \frac{OM}{2}$

$OM = 2 \tan 60°$

Area of the hexagon $ABCDEF$

$= 6 \times$ Area of $\triangle AOB$

$= 6 \times \left(\frac{1}{2} \times AB \times OM\right)$

$= 6 \times \left(\frac{1}{2} \times 4 \times 2 \tan 60°\right)$

≈ 41.569

$=$ **41.6 cm²** (3 sig. fig.) (shown)

Must-Know Concept:
To find the area of the hexagon, note that it is made up of 6 congruent triangles.
Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height

(b) Total surface area of the pyramid

$=$ Area of the hexagon $ABCDEF$ $+$ $6 \times$ Area of $\triangle AXB$

$= 41.569 + 6 \times \left(\frac{1}{2} \times AB \times MX\right)$

$= 41.569 + 6 \times \left(\frac{1}{2} \times 4 \times 10\right)$

$=$ **162 cm²** (3 sig. fig.)

Must-Know Concept:
The total surface area of a pyramid includes the area of all its faces.

(c) Using Pythagoras' Theorem,

$XM^2 = OX^2 + OM^2$

$10^2 = OX^2 + (2 \tan 60°)^2$

$OX = \sqrt{100 - (2 \tan 60°)^2}$

$=$ **9.38 cm** (3 sig. fig.)

Must-Know Concept:
Make use of Pythagoras' Theorem: $c^2 = a^2 + b^2$

(d) Volume of the pyramid

$= \frac{1}{3} \times$ Base area $\times$ Height

$= \frac{1}{3} \times$ Area of the hexagon $ABCDEF \times OX$

$= \frac{1}{3} \times 41.569 \times \sqrt{100 - (2 \tan 60°)^2}$

≈ 129.98

$=$ **130 cm³** (3 sig. fig.)

Must-Know Concept:
Volume of a pyramid $= \frac{1}{3} \times$ Base area $\times$ Height

(e) $\frac{V_S}{V_L} = \left(\frac{l_S}{l_L}\right)^3$

$\frac{129.98}{V_L} = \left(\frac{4}{9}\right)^3$

$V_L = 129.98 \div \frac{64}{729}$

$=$ **1480 cm³** (3 sig. fig.)

Must-Know Concept:
For similar figures, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

10. (a) (i) (a) Median position $= \frac{50}{100} \times 80$

$= 40$

$\therefore$ median mark $=$ **37.4**

Must-Know Concept:
The median is the value of the middle position of all the students.

(b) Upper quartile position $= \frac{75}{100} \times 80$

$= 60$

Upper quartile mark $= 41$

Lower quartile position $= \frac{25}{100} \times 80$

$= 20$

Lower quartile mark $= 24$

Interquartile range

$=$ Upper quartile mark $-$ Lower quartile mark

$= 41 - 24$

$=$ **17**

Must-Know Concept:
The value of the lower quartile is at the 25% position and the value of the upper quartile is at the 75% position.

(ii) From the graph, when mark $= 35$,

number of students who failed the test $= 34$

$\therefore$ number of students who passed the test

$= 80 - 34$

$=$ **46**

Must-Know Concept:
Find the number of students who scored 35 marks and below. Then, minus this value from the total number of students to find the number of students who passed the test.

(iii) The group of students scored **better** in the **Physics test** than the Chemistry test because **the median mark for the Physics test (37.4) is higher than that in the Chemistry test (34)**.

The group of students scored **more consistently** in the **Physics test** than the Chemistry test because **the interquartile range for the Physics test in smaller**.

Must-Know Concept:
Compare both distributions using their median values and interquartile ranges as these values are found in previous parts.

(b) (i) (a) P(a right-handed male)

$= \frac{6}{6+5+3+2}$

$= \mathbf{\frac{3}{8}}$

(b) P(a female) $= \frac{5+2}{6+5+3+2}$

$= \mathbf{\frac{7}{16}}$

(ii) (a) P(first person is a male)

$= \frac{6+3}{6+5+3+2}$

$= \frac{9}{16}$

P(they are both males)
= P(first person is a male) × P(second person is a male)

$= \frac{9}{16} \times \frac{8}{15}$

$= \mathbf{\frac{3}{10}}$

(b) P(first person is not a right-handed female)

= 1 – P(first person is a right-handed female)

$= 1 - \frac{5}{6+3+5+2}$

$= 1 - \frac{5}{16}$

$= \frac{11}{16}$

P(neither of them is a right-handed female)

= P(first person is not a right-handed female) × P(second person is not a right-handed female)

$= \frac{11}{16} \times \frac{10}{15}$

$= \mathbf{\frac{11}{24}}$

Must-Know Concept:
The selections are without replacement as the same student cannot be selected twice.

O LEVEL MATHEMATICS OCTOBER/NOVEMBER 2011 EXAMINATION PAPER

Paper 1

1. $\frac{4.51}{19.6-3.91^2} = \frac{4.51}{19.6-15.2881}$
$= \frac{4.51}{4.3119}$
$=$ **1.05** (3 sig. fig.)

Must-Know Concept:
Non-zero digits and zeros between non-zero digits are significant.

2. (a) Difference between the two temperatures
$= 32°C - (-7°C)$
$=$ **39°C**

(b) Mean of the lowest temperatures
$= \frac{(-3°C) + 25°C + (-7°C)}{3}$
$= \frac{15}{3}°C$
$=$ **5°C**

Must-Know Concept:
Mean $= \frac{\text{Sum of lowest temperatures}}{\text{Number of days}}$

3. $120° - 105° \longrightarrow 25$ students
$15° \longrightarrow 25$ students
$1° \longrightarrow \frac{25}{15}$
$= \frac{5}{3}$ students
$360° \longrightarrow 360 \times \frac{5}{3}$
$=$ **600 students**

Must-Know Concept:
The difference in the number of votes is represented by the difference in the angles.

4. (a) $\begin{pmatrix} 59 & 15 & 101 \\ 102 & 3 & 72 \end{pmatrix}\begin{pmatrix} 10 \\ 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 59(10)+15(7)+101(5) \\ 102(10)+3(7)+72(5) \end{pmatrix}$
$= \begin{pmatrix} \mathbf{1200} \\ \mathbf{1401} \end{pmatrix}$

Must-Know Concept:
The product of a 2×3 matrix and a 3×1 matrix gives a resulting 2×1 matrix.

(b) The ticket sales for the first match was \$1200.
The ticket sales for the second match was \$1401.

5. x ¢ $\longrightarrow 1\ l$
1 ¢ $\longrightarrow \frac{1}{x}\ l$
$\$y = 100y$ ¢
$100y$ ¢ $\longrightarrow 100y \times \frac{1}{x}$
$= \mathbf{\frac{100y}{x}\ l}$

Must-Know Concept:
Convert both terms to a common unit (cents or dollars). Divide the amount John paid by the cost to find the number of litres that he buys.

6. (a) $\frac{2}{3}$ bottles $\longrightarrow$ 1 glass
1 bottle $\longrightarrow 1 \div \frac{2}{3}$
$= \frac{3}{2}$ glasses
12 bottles $\longrightarrow 12 \times \frac{3}{2}$
$=$ **18 glasses**

Must-Know Concept:
1 bottle can fill $\frac{3}{2}$ glasses.

(b) 6 m = 600 cm
Percentage $= \frac{3}{600} \times 100\%$
$=$ **0.5%**

Must-Know Concept:
Convert 3 centimetres to metres or 6 metres to centimetres.

7. (a) $\sqrt[3]{p} = \sqrt[3]{2^3 \times 3^6}$
$= (2^3 \times 3^6)^{\frac{1}{3}}$
$= (2^3)^{\frac{1}{3}} \times (3^6)^{\frac{1}{3}}$
$= 2^1 \times 3^2$
$= 2 \times 9$
$=$ **18**

(b) LCM of p, q and r $= \mathbf{2^3 \times 3^6 \times 5 \times 7}$

(c) Greatest number that will divide p, q and r exactly
$=$ HCF of p, q and r
$= 2 \times 3$
$=$ **6**

Must-Know Concept:
The greatest number that will divide p, q and r exactly is the highest common factor of p, q and r.

8. (a) $\frac{5c}{2} \div \frac{20c^2}{d} = \frac{5c}{2} \times \frac{d}{20c^2}$
$= \frac{5cd}{40c^2}$
$= \mathbf{\frac{d}{8c}}$

Must-Know Concept:
$\frac{5c}{2} \div \frac{20c^2}{d} = \frac{5c}{2} \times \frac{d}{20c^2}$

(b) $6x^2 + 14x - 12 = 2(3x^2 + 7x - 6)$
$= \mathbf{2(3x - 2)(x + 3)}$

9. (a) 1 : 25 000

1 cm : 25 000 cm

1 cm : $\frac{25\ 000}{100 \times 1000}$ km

1 cm : 0.25 km

$\therefore n =$ **0.25**

Must-Know Concept:
1 km = 100 000 cm

(b) 1 cm : 0.25 km

30 cm : 30 × 0.25 km

30 cm : 7.5 km

$\therefore$ the actual distance between the two towns is **7.5 km**.

(c) 1 cm : 0.25 km

1^2 cm^2 : 0.25^2 km^2

1 cm^2 : 0.0625 km^2

$\frac{1}{0.0625}$ cm^2 : 1 km^2

$\frac{1}{0.0625} \times 2.5$ cm^2 : 2.5 km^2

40 cm^2 : 2.5 km^2

$\therefore$ the area of the lake on the map is **40 cm²**.

Must-Know Concept:
Convert the scale to cm^2 : km^2. The scale becomes 1 cm^2 : n^2 km^2.

10. (a) $\angle ABC = \angle ACN$ (given)

$\angle BAC = \angle CAN$ (common)

$\angle ACB = \angle ANC$ (sum of $\triangle$)

$\therefore \triangle ABC$ is similar to $\triangle ACN$ (AA).

Must-Know Concept:
One method to show that two triangles are similar is to show that there are three pairs of equal corresponding angles.

(b) $\frac{AC}{AN} = \frac{AB}{AC}$

$\frac{AC}{4} = \frac{4+5}{AC}$

$\frac{AC}{4} = \frac{9}{AC}$

$AC^2 = 36$

$AC = \pm\sqrt{36}$

= **6 m** or –6 (rejected as AC cannot be negative)

Must-Know Concept:
The ratios of corresponding lengths in similar figures are the same.

11. (a) Since $AD \perp DC$, $\angle ADC = 90°$

$\cos \angle ACD = \frac{CD}{AC}$

$= \frac{4}{x}$

$\cos p° = \cos (180° - \angle ACD)$

$= -\cos \angle ACD$

$= -\frac{4}{x}$

Must-Know Concept:
Note that $\cos (180° - \theta) = -\cos \theta$ where $0 < \theta < 90°$.

(b) Area of trapezium $ABCD$ = 5 × Area of $\triangle ADC$

$\frac{1}{2} \times (AB + CD) \times AD = 5 \times \frac{1}{2} \times CD \times AD$

$AB + CD = 5 \times CD$

$AB + 4 = 5 \times 4$

$AB = 20 - 4$

= **16 cm**

Must-Know Concept:
Note that the trapezium $ABCD$ and triangle ADC have the same height.

12. (a) Least possible mass of the glass block = **42.5 g**

(b) Greatest possible mass of the glass block = $43.4\dot{9}$ g

Least possible volume of the glass block = 14.5 cm^3

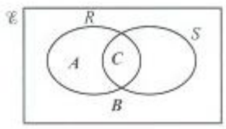
Greatest possible mass of 1 cm^3 of the glass

= 43.49 ÷ 14.5

= **3.00 g** (3 sig. fig.)

Must-Know Concept:
Find the greatest possible mass of the glass block and the least possible volume of the glass block.

13.



Must-Know Concept:
Element C is in the intersection of the Venn Diagram as it is both a right-angled triangle (according to Pythagoras' Theorem) and a triangle with three unequal sides.

14. (a) (i) $\angle MDC = 180° - \angle DMC - \angle DCM$ ($\angle$ sum of $\triangle$)

$= 180° - 28° - x°$

$=$ **$(152 - x)°$**

Must-Know Concept:
Angles in a triangle sum up to 180°.

(ii) Method 1:

$\angle NBC = 180° - \angle BNC - \angle BCN$ ($\angle$ sum of $\triangle$)

$= 180° - 22° - x°$

$=$ **$(158 - x)°$**

Method 2:

$\angle NBC = 180° - \angle MDC$ ($\angle$s in opp. segments)

$= 180° - (152 - x)°$

$=$ **$(28 + x)°$**

(b) Method 1:

$\angle MDC + \angle NBC = 180°$ ($\angle$s in opp. segments)
$152° - x° + 158° - x° = 180°$
$310° - 2x° = 180°$
$2x° = 310° - 180°$
$= 130°$
$x = \mathbf{65}$

Method 2:

$\angle BNC + \angle NBC + \angle BCN = 180°$ ($\angle$ sum of $\triangle$)
$22° + 28° + x° + x° = 180°$
$2x° + 50° = 180°$
$2x° = 130°$
$x = \mathbf{65}$

Must-Know Concept:
Note that angles in opposite segments are supplementary.

15. (a) Area of the minor sector $AOB = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 4^2 \times 2.5$
$= \mathbf{20\ cm^2}$

Must-Know Concept:
Area of a sector $= \frac{1}{2}r^2\theta$ (where θ is in radians)

(b) (i) Reflex $\angle AOB = \mathbf{(2\pi - 2.5)}$ **radians** ($\angle$s at a point)

Must-Know Concept:
$360° = 2\pi$ rad

(ii) Length of arc $ACB = r\theta$
$= 4(2\pi - 2.5)$
$= \mathbf{(8\pi - 10)\ cm}$

Must-Know Concept:
Arc length $= r\theta$ (where θ is in radians)

16. (a) $3(a - 4) = 1 - (3 - a)$
$3a - 12 = 1 - 3 + a$
$3a - a = 1 - 3 + 12$
$2a = 10$
$a = \mathbf{5}$

Must-Know Concept:
Expand '3' to $(a - 4)$. Group and simplify like terms together.

(b) $y = 3x - 7$..........①
$x = y + 4$..........②
Substitute ① into ②,
$x = (3x - 7) + 4$
$x - 3x = -7 + 4$
$-2x = -3$
$x = 1\frac{1}{2}$
Substitute $x = 1\frac{1}{2}$ into ①,
$y = 3\left(1\frac{1}{2}\right) - 7$
$= 4\frac{1}{2} - 7$
$= -2\frac{1}{2}$

$\therefore x = \mathbf{1\frac{1}{2}}, y = \mathbf{-2\frac{1}{2}}$

17. (a) (i) $\angle BCD = \angle FDE$ (corr. $\angle$s, FD // BC)
$= \mathbf{44°}$

Must-Know Concept:
Since AB is parallel to EDC, look out for corresponding angles.

(ii) $\angle BDF = 56°$ (alt. $\angle$s, FD // BC)
$\angle BDC = 180° - 44° - 56°$ (adj. $\angle$s on a str. line)
$= 80°$
$\angle ABD = \angle BDC$ (alt. $\angle$s, AB // EC)
$= \mathbf{80°}$

Must-Know Concept:
Solve for $\angle BDF$ first.

(iii) $\angle AFD = 360° - 101° - 80° - 56°$ ($\angle$ sum of quad.)
$= \mathbf{123°}$

Must-Know Concept:
The sum of angles in a quadrilateral is 360°.

(b) AF **is not** parallel to BD because
$\angle FAB + \angle ABD = 101° + 80° = 181° \neq 180°$.
If AF is parallel to BD, these two $\angle$s must be supplementary.

Must-Know Concept:
Note that the sum of interior angles between two parallel lines must be 180°.

18. (a) (i) $7^{12} \div 7^3 = 7^{12-3}$
$= \mathbf{7^9}$

(ii) $\frac{1}{49} = \frac{1}{7^2}$
$= \mathbf{7^{-2}}$

(iii) $\sqrt[4]{7} = \mathbf{7^{\frac{1}{4}}}$

(b) $3^{-2} \times 3^k = 1$
$3^{-2+k} = 3^0$
$\therefore -2 + k = 0$
$k = \mathbf{2}$

Must-Know Concept:
Express 1 as a base of 3. Recall that $a^0 = 1$.

19. (a) Substitute $x = 8$ into $y = 6 - x$,
$y = 6 - 8$
$= -2$
$\therefore A\mathbf{(8, -2)}$.
Substitute $x = 8$ into $2y = 3x + 2$,
$2y = 3(8) + 2$
$= 26$
$y = 13$
$\therefore B\mathbf{(8, 13)}$.

Must-Know Concept:
The x-coordinate of both points A and B are 8 as they lie on the line $x = 8$.

(b) Comparing $y = 6 - x$ with $y = mx + c$, the gradient m is **–1**.

Must-Know Concept:
Express the equation of the line in the form $y = mx + c$.

(c) $\sqrt{(0-8)^2 + [k-(-2)]^2} = \sqrt{(0-8)^2 + (k-13)^2}$

$$\begin{aligned} 64 + (k+2)^2 &= 64 + (k-13)^2 \\ (k+2)^2 &= (k-13)^2 \\ k^2 + 4k + 4 &= k^2 - 26k + 169 \\ k^2 - k^2 + 4k + 26k &= 169 - 4 \\ 30k &= 165 \\ k &= \mathbf{5.5} \end{aligned}$$

Must-Know Concept:
Find the lengths between points A and K and between points B and K.

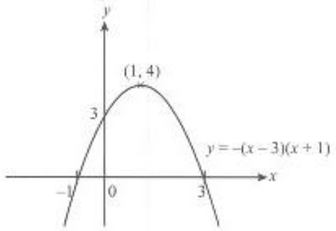
20. (a), (b), (c)

Refer to Appendix 10.

Must-Know Concept:
Since $BC = 9$ cm, an arc 9 cm from B can be constructed using a compass. Similarly, since $AC = 10$ cm, an arc 10 cm from A can be constructed using a compass. The intersection of the two arcs would therefore be the point C.
The perpendicular bisector of AB is a line that cuts AB into two equal lengths and meets AB at 90°. The bisector of $\angle ABC$ is a line that cuts $\angle ABC$ into two equal angles.
In general, any point that lies on the perpendicular bisector of a line XY is equidistant from the points X and Y, and any point that lies on the angle bisector of an angle $\angle XYZ$ is equidistant from XY and YZ.

21. (a) $y = -(x-3)(x+1)$
Since the coefficient of $x^2 < 0$, the curve is '∩' shaped.
When $y = 0$,
$-(x-3)(x+1) = 0$
$x - 3 = 0$ or $x + 1 = 0$
$x = 3$ or $x = -1$
$\therefore$ x-intercepts are 3 and –1.

When $x = 0$,
$y = -(0-3)(0+1)$
$= -(-3)(1)$
$= 3$
$\therefore$ y-intercept is 3.



Must-Know Concept:
Label the x- and y-intercepts and turning points on the sketch.

(b) Equation of the line of symmetry is
$x = \frac{-1+3}{2}$
$\mathbf{x = 1}$

Must-Know Concept:
The line of symmetry is the vertical line passing through the turning point.

(c) Substitute $x = 1$ into $y = -(x-3)(x+1)$,
$y = -(1-3)(1+1)$
$= -(-2)(2)$
$= 4$

$\therefore$ the coordinates of the maximum point are **(1, 4)**.

22. (a) 1.34×10^{-7} seconds $= 134 \times 10^{-9}$ seconds
$= 134$ nanoseconds
$\therefore k = \mathbf{134}$

Must-Know Concept:
1 nanosecond = 1×10^{-9} seconds

(b) (i) $6.1 \times 10^7 - 4.8 \times 10^6 = 61 \times 10^6 - 4.8 \times 10^6$
$= (61 - 4.8) \times 10^6$
$= 56.2 \times 10^6$
$= 5.62 \times 10^7$
$\therefore$ $\mathbf{5.6 \times 10^7}$ more people lived in the United Kingdom than in Singapore.

Must-Know Concept:
Express your answer in 1 decimal place as the figures provided in the question are in 1 decimal place.

(ii) Population in 1851 $= \frac{100}{273} \times (6.1 \times 10^7)$
$\approx 2.234 \times 10^7$
$= \mathbf{2.2 \times 10^7}$

Must-Know Concept:
'Increased by 173%' implies that the population in United Kingdom is (100% + 173% =) 273% in 2009.

23. (a) Upper quartile position $= \frac{75}{100} \times 1200$
$= 900$
Upper quartile = 32
Lower quartile position $= \frac{25}{100} \times 1200$
$= 300$
Lower quartile = 18
Interquartile range
= Upper quartile – Lower quartile
= 32 – 18
= **14**

Must-Know Concept:
The value of the lower quartile is at the 25% position and the value of the upper quartile is at the 75% position.

(b) Number of candidates that gained at least 75% of the marks for Paper 2
$= 1200 - 1060$
$=$ **140**

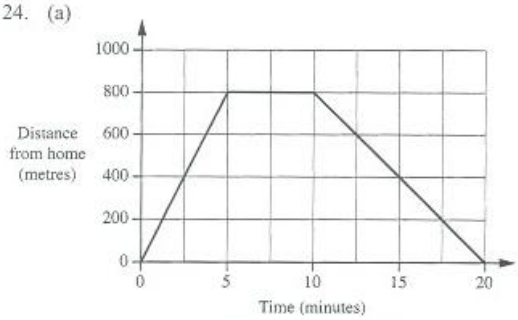
(c) Marks obtained if the student had taken Paper 2
$=$ **43**

Must-Know Concept:
The student is the 1000th student. At the 1000th student, the mark obtained is 43.

(d) Paper 1 was more difficult because **its median mark, 26, is lower than the median mark, 31, of Paper 2.**

Must-Know Concept:
Compare using the median mark of both papers.

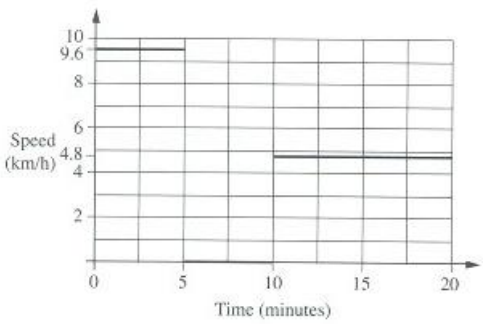
24. (a)



Must-Know Concept:
A distance-time graph shows the distance away from the point of origin with respect to time. When an object is travelling at constant speed, the graph will show a straight line with positive gradient or negative gradient.

(b) Ali's speed $= \dfrac{\text{Distance}}{\text{Time}}$
$= \dfrac{0.8 \text{ km}}{\frac{5}{60} \text{ h}}$
$=$ **9.6 km/h**

(c) Ali's speed on the way home $= \dfrac{0.8}{\frac{10}{60} \text{ h}}$
$= 4.8$ km/h



Must-Know Concept:
In a speed-time graph, a horizontal line represents a constant speed.

Paper 2

1. (a)
$$\frac{2x+7}{3} = \frac{3-x}{5}$$
$$5(2x+7) = 3(3-x)$$
$$10x + 35 = 9 - 3x$$
$$10x + 3x = 9 - 35$$
$$13x = -26$$
$$x = \mathbf{-2}$$

Must-Know Concept:
Perform cross multiplication and solve for x.

(b)
$$xy = 2(x+3)$$
$$xy = 2x + 6$$
$$xy - 2x = 6$$
$$x(y-2) = 6$$
$$x = \mathbf{\frac{6}{y-2}}$$

Must-Know Concept:
Make x the subject by shifting terms that have x to the left-hand side and terms that do not have x to the right-hand side, and simplify the equation.

(c)
$$\frac{4}{x-2} + \frac{2}{2x+1} = \frac{4(2x+1)}{(x-2)(2x+1)} + \frac{2(x-2)}{(x-2)(2x+1)}$$
$$= \frac{4(2x+1) + 2(x-2)}{(x-2)(2x+1)}$$
$$= \frac{8x + 4 + 2x - 4}{(x-2)(2x+1)}$$
$$= \mathbf{\frac{10x}{(x-2)(2x+1)}}$$

Must-Know Concept:
Convert the two denominators into a common denominator, to become a single fraction.

(d)
$$\frac{4x^2 - y^2}{2x^2 + xy} = \frac{(2x+y)(2x-y)}{x(2x+y)}$$
$$= \mathbf{\frac{2x-y}{x}}$$

Must-Know Concept:
Observe that $4x^2 - y^2 = (2x-y)(2x+y)$. Simplify $2x^2 + xy$ by factorising out the common term x. Cancel out the common factor between the numerator and denominator.

2. (a) (i) Fuel consumption $= \frac{100}{250} \times 15.75$
$=$ **6.3** litres per 100 km

(ii) (a) Distance she can travel
$= \frac{60}{8.2} \times 100$
$=$ **732 km** (3 sig. fig.)

(b) Petrol used $= \frac{120}{100} \times 8.2$
$= 9.84$ litres
Cost of petrol $= 9.84 \times \$1.65$
$=$ **\$16.24** (nearest cent)

Must-Know Concept:
Calculate the amount of petrol required for 120 km, then multiply with the cost per litre.

(b) (i) Total amount of money he inherits

$= \frac{3+4+5}{5} \times \$1000$

$= \mathbf{\$2400}$

(ii) Total amount of money in his account after 5 years

$= 1000 \times (1 + \frac{3.5}{100})^5$

$= \mathbf{\$1187.69}$ (nearest cent)

Must-Know Concept:
The formula for compound interest is $P(1 + \frac{r}{100})^n$.

3. (a) Interior $\angle$ of regular polygon $= \frac{(n-2) \times 180°}{n}$

$162° = \frac{(n-2) \times 180°}{n}$

$162°n = 180°n - 360°$

$180°n - 162°n = 360°$

$18°n = 360°$

$n = 20$

$\therefore$ the polygon has **20** sides.

Must-Know Concept:
The sum of interior angles of an n-sided regular polygon $= (n-2) \times 180°$.

(b) (i) (a) $\angle PAO = 90°$(tangent $\perp$ radius)

$\angle AOP = 180° - 90° - 36°$($\angle$ sum of $\triangle$)

$= \mathbf{54°}$

Must-Know Concept:
The tangent PA is perpendicular to the radius AO.

(b) $\angle BOP = \angle AOP$

$= 54°$ (PO bisects $\angle AOB$)

$\angle OBC = \frac{180° - 54°}{2}$ (base $\angle$s of isos. $\triangle$)

$= \frac{126°}{2}$

$= \mathbf{63°}$

Must-Know Concept:
Since $OC = OB$, $\triangle OBC$ is isosceles.
When solving questions on circles, keep a lookout for isosceles triangles.

(c) Reflex $\angle AOB = 360° - 54° - 54°$ ($\angle$s at a point)

$= 252°$

$\angle ACB = 252° \div 2$ ($\angle$ at the centre $= 2\angle$ at circumference)

$= \mathbf{126°}$

(ii) $\tan 36° = \frac{6}{AP}$

$AP = \frac{6}{\tan 36°}$

≈ 8.25829 cm

Area of quadrilateral $AOBP$

= Area of $\triangle OAP$ + Area of $\triangle OBP$

$= \left(\frac{1}{2} \times 6 \times 8.25829\right) + \left(\frac{1}{2} \times 6 \times 8.25829\right)$

$= 2\left(\frac{1}{2} \times 6 \times 8.25829\right)$

$= \mathbf{49.5\ cm^2}$ (3 sig. fig.)

Must-Know Concept:
Solve for AP in order to find the area of triangle OAP.
Area of a right-angled triangle $= \frac{1}{2} \times$ Base $\times$ Height

4. (a) Width of the block $= (x-3)$ cm
Height of the block $= (x+1)$ cm
Total surface area

$= 2x(x-3) + 2x(x+1) + 2(x-3)(x+1)$

$= 2x^2 - 6x + 2x^2 + 2x + 2(x^2 + x - 3x - 3)$

$= 4x^2 - 4x + 2x^2 + 2x - 6x - 6$

$= \mathbf{(6x^2 - 8x - 6)\ cm^2}$

Must-Know Concept:
Find the width and the height of the block in terms of x.

(b) $6x^2 - 8x - 6 = 500$

$6x^2 - 8x - 6 - 500 = 0$

$6x^2 - 8x - 506 = 0$

$\mathbf{3x^2 - 4x - 253 = 0}$ (shown)

Must-Know Concept:
Equate the expression in (a) to 500.

(c) $3x^2 - 4x - 253 = 0$

$a = 3$, $b = -4$ and $c = -253$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-253)}}{2(3)}$

$= \frac{4 \pm \sqrt{16 + 3036}}{6}$

$= \frac{4 + \sqrt{3052}}{6}$ or $\frac{4 - \sqrt{3052}}{6}$

$= \mathbf{9.87}$ or $\mathbf{-8.54}$ (2 d.p.)

Must-Know Concept:
Solve for x using the quadratic formula.

(d) Since length cannot be negative, $x = 9.87$

$\therefore$ height of the block $= x + 1$

$= 9.87 + 1$

$= \mathbf{10.87\ cm}$

5. (a) (i) $T_{20} = \frac{20(20+1)}{4}$
$= \frac{20(21)}{4}$
$= \frac{420}{4}$
$= \mathbf{105}$

Must-Know Concept:
Substitute $n = 20$.

(ii) $33 = \frac{n(n+1)}{4}$
$132 = n^2 + n$
$n^2 + n - 132 = 0$
$(n - 11)(n + 12) = 0$
$n - 11 = 0$ or $n + 12 = 0$
$n = 11$ or $n = -12$ (rejected as the term of the sequence cannot be negative)

$\therefore$ the **11th** term of the sequence has value 33.

Must-Know Concept:
Equate the expression to 33 and solve for n.

(b) (i) $\frac{2p+1}{2} = p + \frac{1}{2}$
$\therefore 2p + 1$ is not divisible by 2, hence it is an odd number.

Must-Know Concept:
2 × even number = even number since 2 is an even number.

(ii) Next odd number $= 2p + 1 + 2$
$= \mathbf{2p + 3}$

Must-Know Concept:
An odd number is always 2 more than the previous odd number.

(iii) $(2p + 1)^2 = (2p)^2 + 2(2p)(1) + 1^2$
$= \mathbf{4p^2 + 4p + 1}$
$(2p + 3)^2 = (2p)^2 + 2(2p)(3) + 3^2$
$= \mathbf{4p^2 + 12p + 9}$

(iv) $(2p + 3)^2 - (2p + 1)^2$
$= (4p^2 + 12p + 9) - (4p^2 + 4p + 1)$
$= 4p^2 - 4p^2 + 12p - 4p + 9 - 1$
$= 8p + 8$
$= 8(p + 1)$
Since p is a positive integer, $8(p + 1)$ is a multiple of 8.

Must-Know Concept:
'Multiple of 8' implies that the expression is divisible by 8.

6. (a) (i) Using Cosine Rule,
$LB^2 = AB^2 + AL^2 - 2(AB)(AL)\cos\angle BAL$
$= 400^2 + 250^2 - 2(400)(250)\cos 65°$
$= 160\,000 + 62\,500 - 200\,000\cos 65°$
$= 222\,500 - 200\,000\cos 65°$
$LB \approx 371.45$
$= \mathbf{371\ m}$ (3 sig. fig.)

Must-Know Concept:
Apply the Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

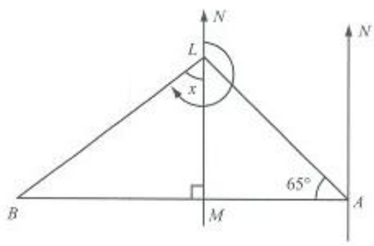
(ii) Area of $\triangle LAB = \frac{1}{2} \times AB \times AL \times \sin\angle BAL$
$= \frac{1}{2} \times 400 \times 250 \times \sin 65°$
$= \mathbf{45\ 300\ m^2}$ (3 sig. fig.)

Must-Know Concept:
Area of a triangle $= \frac{1}{2}ab\sin C$

(iii) Using Sine Rule,
$\frac{LA}{\sin\angle LBA} = \frac{LB}{\sin\angle LAB}$
$\frac{250}{\sin\angle LBA} = \frac{371.45}{\sin 65°}$
$\sin\angle LBA = \frac{250\sin 65°}{371.45}$
$\angle LBA \approx 37.642°$
$= \mathbf{37.6°}$ (1 d.p.)

Must-Know Concept:
Apply the Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

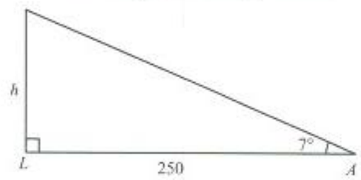
(iv)



$\angle LMB = 90°$ (B due west of A)
$\angle x = 180° - 90° - 37.6°$ ($\angle$ sum of $\triangle$)
$= 52.4°$
$\therefore$ bearing of B from $L = 52.4° + 180°$
$= \mathbf{232.4°}$

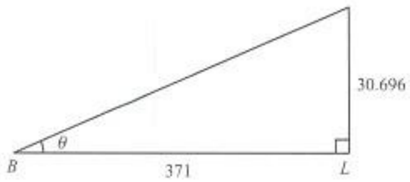
Must-Know Concept:
When calculating bearings, always move in a clockwise direction to identify the required angle.

(b) Let h be the height of the lighthouse.



$\tan 7° = \frac{h}{AL}$
$= \frac{h}{250}$
$h = 250\tan 7°$
≈ 30.696 m

Let θ be the angle of elevation of the top of the lighthouse from B.



$\tan\theta = \frac{h}{BL}$

$= \frac{30.696}{371}$

$\theta \approx 4.7298°$

$= \mathbf{4.7°}$ (1 d.p.)

Must-Know Concept:
Find the height of the lighthouse first.

7. (a) (i) Let V be the quantity of paint and h be the depth of the container.

$V \propto h^2$

$\Rightarrow V = kh^2$, where k is a constant

When $V = 150$, $h = 50$,

$150 = k(50^2)$

$k = \frac{150}{50^2}$

$= 0.06$

$\therefore V = 0.06h^2$

When $h = 70$,

$V = 0.06(70^2)$

$= 294$

$\therefore$ **294 m*l*** of paint is needed when the depth is 70 cm.

(ii) When $V = 54$,

$54 = 0.06h^2$

$h^2 = \frac{54}{0.06}$

$= 900$

$h = \pm\sqrt{900}$

$= 30$ or -30 (rejected as the depth of the container cannot be negative)

$\therefore$ the depth of the container is **30 cm** when the quantity of paint needed is 54 m*l*.

Must-Know Concept:
The quantity of paint needed is proportional to the square of the depth of the container, quantity of paint needed $= k \times (\text{depth of the container})^2$

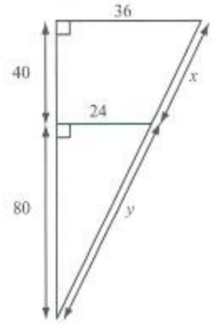
(b) (i) Using Pythagoras' Theorem,

$y^2 = 80^2 + 24^2$

$= 6976$

$y = \sqrt{6976}$

≈ 83.522



Using Pythagoras' Theorem,

$(x + y)^2 = (40 + 80)^2 + 36^2$

$= 15\,696$

$x + y = \sqrt{15\,696}$

≈ 125.284

$x = 125.284 - 83.522$

$= \mathbf{41.8}$ (3 sig. fig.) (shown)

(ii) Total surface area of the outside of the pot

$= \pi(24)^2 + \pi(36)(125.284) - \pi(24)(83.522)$

$= \mathbf{9680\ cm^2}$ (3 sig. fig.)

(iii) $\frac{V_S}{V_L} = \left(\frac{h_S}{h_L}\right)^3$

$\frac{1}{2} = \left(\frac{40}{h_L}\right)^3$

$\frac{40}{h_L} = \sqrt[3]{\frac{1}{2}}$

$h_L = \frac{40}{\sqrt[3]{\frac{1}{2}}}$

$= 50.4$ cm (3 sig. fig.)

$\therefore$ the height of the larger pot is **50.4 cm**.

Must-Know Concept:
For similar figures, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

8. (a) (i) $\vec{OP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\vec{OQ} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\vec{PQ} = \vec{PO} + \vec{OQ}$

$= -\vec{OP} + \vec{OQ}$

$= -\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{-4} \\ \mathbf{-2} \end{pmatrix}$

(ii) $|\overrightarrow{PQ}| = \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{20}$

$=$ **4.47 units** (3 sig. fig.)

Must-Know Concept:
The formula for the magnitude of a vector is $\sqrt{x^2 + y^2}$.

(iii) $\overrightarrow{PL} = \frac{1}{2}\overrightarrow{PQ}$

$= \frac{1}{2}\begin{pmatrix}-4\\-2\end{pmatrix}$

$= \begin{pmatrix}\mathbf{-2}\\\mathbf{-1}\end{pmatrix}$

(b) (i) (a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\overrightarrow{OA} + \overrightarrow{OB}$

$= \mathbf{-a + b}$

(b) $\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}(-\mathbf{a} + \mathbf{b})$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= \mathbf{a} + \frac{1}{2}(-\mathbf{a} + \mathbf{b})$

$= \mathbf{a} - \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

(c) Given $\overrightarrow{OB} = 2\overrightarrow{BC}$,

$\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$

$= \frac{3}{2}\mathbf{b}$

$\overrightarrow{OD} = 3\overrightarrow{OA}$

$= 3\mathbf{a}$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= -\overrightarrow{OC} + \overrightarrow{OD}$

$= -\frac{3}{2}\mathbf{b} + 3\mathbf{a}$

$= \mathbf{3a - \frac{3}{2}b}$

(ii) $\overrightarrow{CY} = \frac{1}{3}\overrightarrow{CD}$

$= \frac{1}{3}(3\mathbf{a} - \frac{3}{2}\mathbf{b})$

$= \mathbf{a} - \frac{1}{2}\mathbf{b}$

$\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CY}$

$= \frac{3}{2}\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{b}$

$= \mathbf{a + b}$

(iii) $\overrightarrow{OX} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$= \frac{1}{2}\overrightarrow{OY}$

∴ *O*, *X* and *Y* are collinear.
***X* is the midpoint of *OY*.**

Must-Know Concept:
Two vectors are parallel if **a** can be written as the scalar multiple of **b**, **a** = *k***b**, where *k* is a scalar. Since *O*, *X* and *Y* lie on the same line and $\overrightarrow{OX} = \frac{1}{2}\overrightarrow{OY}$, then *X* is the midpoint of *OY*.

9. (a) $p = \frac{1}{5}(-2)^2(-2-4)$

$=$ **−4.8**

Must-Know Concept:
Substitute $x = -2$ to find value of p.

(b) Refer to Appendix 11.

(c) $\frac{1}{5}x^2(x-2) = -1$

From the graph, $x =$ **−1, 1.4** or **3.6**

Must-Know Concept:
Solutions of the curve are the x-intercepts.

(d) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3.1 - (-3.3)}{5 - 3}$

$=$ **3.2**

Must-Know Concept:
Select two points (x_1, y_1) and (x_2, y_2) on the tangent to find the gradient using the formula $\frac{y_1 - y_2}{x_1 - x_2}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

(e) (i) $y = 4 - 2x$

x	0	2	5
y	4	0	−6

(ii) From the graph, $x =$ **2.9**

Must-Know Concept:
Solve the two equation simultaneously and compare the equation to $x^3 - 4x^2 + Ax + B = 0$

(iii) $y = \frac{1}{5}x^2(x-4)$ .... ①

$y = 4 - 2x$ .... ②

Substitute ① into ②,

$\frac{1}{5}x^2(x-4) = 4 - 2x$

$x^2(x-4) = 20 - 10x$

$x^3 - 4x^2 + 10x - 20 = 0$

Compare with $x^3 - 4x^2 + Ax + B = 0$,

$A =$ **10**, $B =$ **−20**

10. (a) (i) Percentage of students who received less than 20 emails in a week

$= \frac{8 + 13}{8 + 13 + 25 + 30 + 18 + 6} \times 100\%$

$=$ **21%**

(ii) (a) Mean number of emails received in a week,

$= \frac{\Sigma fn}{\Sigma f}$

$= \frac{8(5) + 13(15) + 25(25) + 30(35) + 18(45) + 6(55)}{100}$

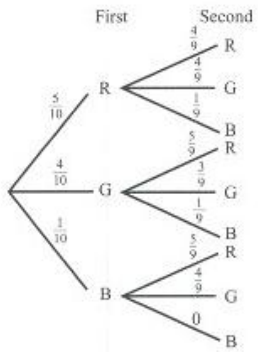
$=$ **30.5**

Must-Know Concept:
Use the formula $\frac{\Sigma fn}{\Sigma f}$ to find the mean since we are given the frequency and values of n.

(b) Sum of fn^2
$= 8(5)^2 + 13(15)^2 + 25(25)^2 + 30(35)^2 + 18(45)^2 + 6(55)^2$
$= 110\ 100$
Standard deviation
$= \sqrt{\frac{\Sigma fn^2}{\Sigma f} - (\bar{n})^2}$
$= \sqrt{\frac{110\ 100}{100} - (30.5)^2}$
$=$ **13.1** (3 sig. fig.)

Must-Know Concept:
Use the formula $\sqrt{\frac{\Sigma fn^2}{\Sigma f} - (\bar{n})^2}$ to find the standard deviation since we have the required values.

(b) (i)



Must-Know Concept:
As the draws are without replacement, remember to minus one from the total number after every draw.

(ii) (a) P(two blue sweets taken) $= \left(\frac{1}{10}\right)(0)$
$=$ **0**

(b) P(both sweets are the same colour)
$=$ P(RR) + P(GG) + P(BB)
$= \left(\frac{5}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) + 0$
$= \mathbf{\frac{16}{45}}$

Must-Know Concept:
Use the tree diagram in (i) to find the required probabilities.

(c) Method 1:
P(one taken is blue)
$= 1 -$ P(none taken is blue)
$= 1 -$ P(RR) $-$ P(RG) $-$ P(GR) $-$ P(GG)
$= 1 - \left(\frac{5}{10}\right)\left(\frac{4}{9}\right) - \left(\frac{5}{10}\right)\left(\frac{4}{9}\right) - \left(\frac{4}{10}\right)\left(\frac{5}{9}\right) - \left(\frac{4}{10}\right)\left(\frac{3}{9}\right)$
$= 1 - \frac{4}{5}$
$= \mathbf{\frac{1}{5}}$

Method 2:
P(one taken is blue)
$=$ P(RB) + P(GB) + P(BR) + P(BG)
$= \left(\frac{5}{10}\right)\left(\frac{1}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{1}{9}\right) + \left(\frac{1}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{1}{10}\right)\left(\frac{4}{9}\right)$
$= \mathbf{\frac{1}{5}}$