1 LINEAR EXPRESSIONS, EQUATIONS AND SIMPLE INEQUALITIES

LEARNING OBJECTIVES

In this topic, we will learn to:

- simplify linear expressions with fractional coefficients
- solve linear equations with fractional coefficients
- solve linear inequalities in one variable and represent the solution on a number line
- solve real-world problems by applying linear inequalities

1.1 LINEAR EXPRESSIONS WITH FRACTIONAL COEFFICIENTS

1. $\frac{1}{3}x + \frac{3}{4}y$ and $\frac{x-4}{2}$ are examples of linear expressions with fractional coefficients. They can be written as $\frac{x}{3} + \frac{3y}{4}$ and $\frac{1}{2}(x-4)$ respectively.

2. To add or subtract fractional coefficients, convert both fractions to equivalent fractions with the same denominator which is the lowest common multiple (LCM) of both denominators.

WORKED EXAMPLE 1

Simplify

(a) $\frac{1}{5}x + \frac{1}{4}y - \frac{1}{10}x - \frac{1}{6}y$,

(b) $\frac{1}{2}(2x + 6y) - 3x$

Worked Solution:

(a)
$$\begin{aligned}\frac{1}{5}x + \frac{1}{4}y - \frac{1}{10}x - \frac{1}{6}y &= \frac{1}{5}x - \frac{1}{10}x + \frac{1}{4}y - \frac{1}{6}y \\ &= \frac{2}{10}x - \frac{1}{10}x + \frac{3}{12}y - \frac{2}{12}y \\ &= \frac{1}{10}x + \frac{1}{12}y\end{aligned}$$

Note:
Add or subtract like terms only.

(b)
$$\begin{aligned}\frac{1}{2}(2x + 6y) - 3x &= \frac{1}{2}(2x) + \frac{1}{2}(6y) - 3x \\ &= x + 3y - 3x \\ &= 3y - 2x\end{aligned}$$

Note:
Use the distributive law.

1

WORKED EXAMPLE 2

Simplify the following.

(a) $\frac{x-y}{2}+\frac{2x+y}{4}$ (b) $\frac{m+3n}{3}-\frac{3n-2m}{2}$ (c) $\frac{a-4b}{2}+\frac{a}{3}-\frac{b}{6}$

Worked Solution:

(a)
$$\frac{x-y}{2}+\frac{2x+y}{4}=\frac{(x-y)\times 2}{2\times 2}+\frac{2x+y}{4}$$
$$=\frac{2(x-y)}{4}+\frac{2x+y}{4}$$
$$=\frac{2(x-y)+(2x+y)}{4}$$
$$=\frac{2x-2y+2x+y}{4}$$
$$=\frac{\mathbf{4x-y}}{\mathbf{4}}$$

(b)
$$\frac{m+3n}{3}-\frac{3n-2m}{2}=\frac{(m+3n)\times 2}{3\times 2}-\frac{(3n-2m)\times 3}{2\times 3}$$
$$=\frac{2(m+3n)}{6}-\frac{3(3n-2m)}{6}$$
$$=\frac{2(m+3n)-3(3n-2m)}{6}$$
$$=\frac{2m+6n-9n+6m}{6}$$
$$=\frac{\mathbf{8m-3n}}{\mathbf{6}}$$

(c)
$$\frac{a-4b}{2}+\frac{a}{3}-\frac{b}{6}=\frac{(a-4b)\times 3}{2\times 3}+\frac{a\times 2}{3\times 2}-\frac{b}{6}$$
$$=\frac{3(a-4b)}{6}+\frac{2a}{6}-\frac{b}{6}$$
$$=\frac{3a-12b}{6}+\frac{2a}{6}-\frac{b}{6}$$
$$=\frac{3a-12b+2a-b}{6}$$
$$=\frac{\mathbf{5a-13b}}{\mathbf{6}}$$

Student's common mistake:

In (b), $\frac{m+3n}{3}-\frac{3n-2m}{2}=\frac{2(m+3n)}{6}-\frac{3(3n-2m)}{2}$ is wrong! Remember to multiply the denominator of $\frac{3n-2m}{2}$ by 3 to obtain the common denominator, which is 6.

1.2 LINEAR EQUATIONS WITH FRACTIONAL COEFFICIENTS AND FRACTIONAL EQUATIONS

1. Solving a linear equation with fractional coefficients is similar to solving one with integer coefficients.

2. To solve a fractional equation, the equation needs to be transformed into a linear equation by multiplication.

WORKED EXAMPLE 3

Solve the following equations.

(a) $\frac{1}{2}x - 9 = 5 - 2x$

(b) $\frac{3}{4}x - \frac{3}{2} = \frac{1}{2}x$

Worked Solution:

(a)
$$\frac{1}{2}x - 9 = 5 - 2x$$
$$\frac{1}{2}x + 2x = 5 + 9$$
$$\frac{5}{2}x = 14$$
$$x = 14 \times \frac{2}{5}$$
$$x = \mathbf{5\frac{3}{5}}$$

(b)
$$\frac{3}{4}x - \frac{3}{2} = \frac{1}{2}x$$
$$\frac{3}{4}x - \frac{1}{2}x = \frac{3}{2}$$
$$\frac{1}{4}x = \frac{3}{2}$$
$$x = \frac{3}{2} \times 4$$
$$x = \mathbf{6}$$

WORKED EXAMPLE 4

Solve the following equations.

(a) $\frac{2x+1}{3} = 7$

(b) $\frac{x-3}{4} = \frac{2x+3}{3}$

Worked Solution:

(a)
$$\frac{2x+1}{3} = 7$$
Multiply by 3 throughout,
$$2x + 1 = 21$$
$$2x = 20$$
$$x = \mathbf{10}$$

(b)
$$\frac{x-3}{4} = \frac{2x+3}{3}$$
Multiply by 12 throughout,
$$3(x - 3) = 4(2x + 3)$$
$$3x - 9 = 8x + 12$$
$$5x = -21$$
$$x = \mathbf{-\frac{21}{5}}$$

WORKED EXAMPLE 5

Solve the following equations.

(a) $\frac{x-2}{2}+\frac{x+1}{4}=3$ (b) $\frac{m+3}{2}-\frac{3-m}{3}=-2$ (c) $\frac{a-4}{2}+\frac{2a}{3}=5$

Worked Solution:

(a) $\frac{x-2}{2}+\frac{x+1}{4}=3$

Multiply by 4 throughout,

$2(x-2)+(x+1)=12$

$2x-4+x+1=12$

$3x-3=12$

$3x=15$

$x=\mathbf{5}$

(b) $\frac{m+3}{2}-\frac{3-m}{3}=-2$

Multiply by 6 throughout,

$3(m+3)-2(3-m)=-12$

$3m+9-6+2m=-12$

$5m+3=-12$

$5m=-15$

$m=\mathbf{-3}$

(c) $\frac{a-4}{2}+\frac{2a}{3}=5$

Multiply by 6 throughout,

$3(a-4)+2(2a)=30$

$3a-12+4a=30$

$7a=42$

$a=\mathbf{6}$

WORKED EXAMPLE 6

Solve the following equations.

(a) $\frac{9}{2x-3}=3$ (b) $\frac{y+2}{2y-3}=\frac{2}{5}$

Worked Solution:

(a) $\frac{9}{2x-3}=3$

$3(2x-3)=9$

$2x-3=3$

$2x=6$

$x=\mathbf{3}$

(b) $\frac{y+2}{2y-3}=\frac{2}{5}$

$2(2y-3)=5(y+2)$

$4y-6=5y+10$

$y=\mathbf{-16}$

1.3 LINEAR INEQUALITIES

1. An inequality is similar to an equation except that an inequality sign ($<$, $>$, $\leq$ or $\geq$) is in the place of an equal sign.

2. When we multiply or divide by a negative number on both sides of an inequality, the inequality sign is reversed.

WORKED EXAMPLE 7

Solve each inequality and represent the solution on a number line.

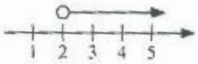
(a) $5x > 10$ (b) $-6x \geq 24$

Worked Solution:

(a) $5x > 10$

$\frac{5x}{5} > \frac{10}{5}$

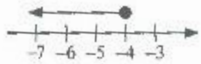
$x > 2$



(b) $-6x \geq 24$

$\frac{-6x}{-6} \leq \frac{24}{-6}$

$x \leq -4$



WORKED EXAMPLE 8

Find the smallest integer x satisfying $5x > 6$.

Worked Solution:

$5x > 6$

$\frac{5x}{5} > \frac{6}{5}$

$x > 1\frac{1}{5}$

The smallest integer x is **2**.

WORKED EXAMPLE 9

A milk manufacturer is repackaging its milk drink to produce a 170-millimetre milk drink for toddlers. If the manufacturer produces 500 litres of milk per day, what is the maximum number of packets of milk it can produce?

Worked Solution:

Let x be the number of packets of milk.

$170x \leq 500\ 000$

$\frac{170x}{170} \leq \frac{500\ 000}{170}$

$x \leq 2941.2$ (5 sig. fig.)

The maximum number of packets of milk it can produce is **2941**.

PRACTICE QUESTIONS

1. Simplify the following.

(a) $\frac{2}{3}x - \frac{1}{7}y + x + \frac{3}{5}y$

(b) $b - \frac{2}{3}(6a - 3b)$

(c) $\frac{f+2}{5} - \frac{1}{3}$

(d) $\frac{x+y}{2} + \frac{3x}{5}$

(e) $\frac{2x-2y}{3} - \frac{x}{6}$

(f) $\frac{a-b}{2} + \frac{a-1}{3}$

(g) $\frac{x+2y}{2} - \frac{y-x}{4}$

(h) $\frac{4m-1}{4} - \frac{m-n}{3}$

2. Simplify the following.

(a) $\frac{4a-b}{3} + \frac{a}{3} - \frac{b}{6}$

(b) $\frac{x}{3} + \frac{x-y}{4} - \frac{y}{6}$

(c) $\frac{m+n}{2} - \frac{2n-m}{3} - \frac{2n+m}{4}$

(d) $\frac{x}{4} + \frac{2(x-y)}{5} - \frac{x-2y}{10}$

3. Solve the following equations.

(a) $\frac{1}{2}x + 1 = 3$

(b) $7 - \frac{1}{3}y = 1$

(c) $\frac{3}{2}x - \frac{1}{2} = \frac{17}{2}$

(d) $\frac{5}{7}x - \frac{1}{7} = \frac{4}{7}$

(e) $6\frac{1}{2} = \frac{1}{2} - \frac{3}{4}x$

(f) $23 + \frac{4}{5}x = 11$

(g) $15 - 27 = \frac{1}{6}x$

(h) $\frac{9}{10}x = 2 - 6\frac{1}{2}$

(i) $\frac{11}{2} - 5\frac{1}{2}x = \frac{11}{4}$

(j) $4x + \frac{12}{5} = 2$

4. Solve the following equations.

(a) $\frac{1}{3}x - \frac{1}{4}x = 5$

(b) $\frac{3}{5}x + \frac{1}{2}x = 11$

(c) $5x + 4 = \frac{13}{2}x$

(d) $23 - \frac{1}{3}x = 9 - \frac{3}{2}x$

(e) $\frac{1}{4}x - 7 = \frac{1}{5}x + 9$

(f) $\frac{3}{4}x + 17 = \frac{1}{4}x - 21$

(g) $1\frac{1}{4}x - \frac{3}{4}x - 15 = 7$

(h) $\frac{3}{5}x - 10 = x + 5$

(i) $\frac{3}{4}x + 5 = 2x + 20$

(j) $14 - \frac{7}{4}x + 12 - \frac{3}{2}x = 0$

5. Solve the following equations.

(a) $\frac{x+5}{2} = 5$

(b) $\frac{4x-1}{5} = 3$

(c) $\frac{3a+2}{5} = \frac{a+2}{3}$

(d) $\frac{y+2}{3} = \frac{3y-1}{2}$

(e) $\frac{1-p}{3} = \frac{3p+1}{5}$

(f) $\frac{1}{3}(2x-1) = \frac{1}{5}(x+2)$

6. Solve the following equations.

(a) $\frac{2-a}{4} + \frac{a-1}{3} = -1$

(b) $1 = \frac{x+3}{2} + \frac{3x}{5}$

(c) $\frac{3m-1}{3} + \frac{m}{2} = \frac{1}{4}$

(d) $\frac{2x-1}{3} = \frac{x}{6} + 2$

(e) $\frac{a-3}{2} = a - \frac{3}{5}$

(f) $\frac{f+2}{3} = 2f - \frac{1}{3}$

(g) $3x - \frac{2x-4}{3} = 2$

(h) $\frac{4-x}{5} - 2x = 1$

(i) $\frac{x}{3} + \frac{x-1}{4} = \frac{x}{6}$

(j) $\frac{a}{2} = \frac{2a}{3} - \frac{a+1}{5}$

7. Solve the following equations.

(a) $\frac{5}{2x+2} = 1$

(b) $\frac{1}{2x-1} = 3$

(c) $\frac{a+5}{a-6} = \frac{4}{5}$

(d) $\frac{2p-1}{3p-5} = \frac{4}{7}$

(e) $\frac{3}{y+6} = \frac{2}{y-2}$

(f) $\frac{2}{2n+3} = \frac{4}{n-2}$

8. In the following questions, use '<', '>' or '=' to fill in the blanks. Assume that each variable is positive.

(a) $x+2$ __ $x-1$

(b) $2y$ __ y

(c) $x-5$ __ $x-4$

(d) $2a+1$ __ $2a+3$

(e) $-2x$ __ $-x$

(f) $u-9$ __ $u+9$

(g) $x \div 3$ __ $x \div 4$

(h) $2y \times 3$ __ $2y \times (-3)$

(i) $-3(u)$ __ $3(-u)$

(j) $\frac{1}{2}$ __ $\frac{-4}{x}$

9. Solve each inequality and represent the solution on a number line.

(a) $2x > 4$ (b) $3y \leq 12$

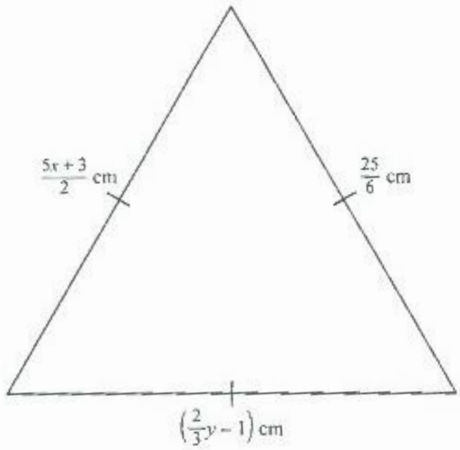
(c) $-3x \geq 9$ (d) $-4x < 8$

10. Find the largest integer x satisfying $3x < 8$.

11. The area of a triangle is 35 cm². If the base is $(x + 5)$ cm and its height is 7 cm, find the value of x.

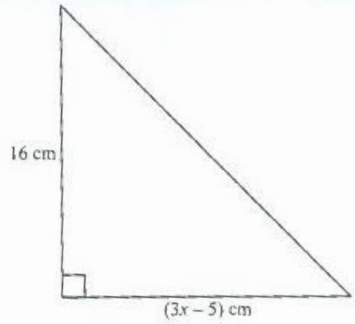
12. A triangle having a base of 15 cm and a height of $(x + 2)$ cm has an area of 60 cm². Find the value of x.

13. The diagram shows an equilateral triangle.



Find the values of x and y.

14. The diagram shows a right-angled triangle.



(a) If the triangle is an isosceles right-angled triangle, find the value of x.
(b) If the area of the right-angled triangle is 56 cm^2, find the value of x.

15. A ream of paper has 300 sheets and costs $4.50. If a bookshop owner wants to sell the sheets separately, he repacks the paper into 25 sheets per packet. What is the minimum amount he needs to price the repacked paper if he does not want to make any losses? Round off your amount to the nearest cent.

16. Betty wants to enter a lucky draw organised by a 'Everything for $3' shop. To do that, she must spend a minimum of $80 in the shop on a single receipt. Find the least number of items she must buy in order to qualify.

17. A manufacturing company has 558 staff. They plan to go on a day trip to Malaysia one weekend. The company has arranged for coaches to send them there and back. If one coach can carry a maximum of 42 passengers, how many coaches are needed in total?