SECONDARY 2 NORMAL (ACADEMIC) TOPICAL MATHS WORKED SOLUTIONS

1 LINEAR EXPRESSIONS, EQUATIONS AND SIMPLE INEQUALITIES

1. (a) $\frac{2}{3}x - \frac{1}{7}y + x + \frac{3}{5}y = \frac{2}{3}x + x + \frac{3}{5}y - \frac{1}{7}y$

$= \frac{5}{3}x + \frac{21}{35}y - \frac{5}{35}y$

$= \mathbf{\frac{5}{3}x + \frac{16}{35}y}$

(b) $b - \frac{2}{3}(6a - 3b) = b - \frac{2}{3}(6a) + \frac{2}{3}(3b)$

$= b - 4a + 2b$

$= \mathbf{3b - 4a}$

(c) $\frac{f+2}{5} - \frac{1}{3} = \frac{f+2}{5}\frac{\times 3}{\times 3} - \frac{1}{3}\frac{\times 5}{\times 5}$

$= \frac{3(f+2)}{15} - \frac{5}{15}$

$= \frac{3(f+2)-5}{15}$

$= \frac{3f+6-5}{15}$

$= \mathbf{\frac{3f+1}{15}}$

(d) $\frac{x+y}{2} + \frac{3x}{5} = \frac{x+y}{2}\frac{\times 5}{\times 5} + \frac{3x}{5}\frac{\times 2}{\times 2}$

$= \frac{5(x+y)}{10} + \frac{2(3x)}{10}$

$= \frac{5(x+y)+2(3x)}{10}$

$= \frac{5x+5y+6x}{10}$

$= \mathbf{\frac{11x+5y}{10}}$

(e) $\frac{2x-2y}{3} - \frac{x}{6} = \frac{2x-2y}{3}\frac{\times 2}{\times 2} - \frac{x}{6}$

$= \frac{2(2x-2y)}{6} - \frac{x}{6}$

$= \frac{2(2x-2y)-x}{6}$

$= \frac{4x-4y-x}{6}$

$= \mathbf{\frac{3x-4y}{6}}$

(f) $\frac{a-b}{2} + \frac{a-1}{3} = \frac{a-b}{2}\frac{\times 3}{\times 3} + \frac{a-1}{3}\frac{\times 2}{\times 2}$

$= \frac{3(a-b)}{6} + \frac{2(a-1)}{6}$

$= \frac{3(a-b)+2(a-1)}{6}$

$= \frac{3a-3b+2a-2}{6}$

$= \mathbf{\frac{5a-3b-2}{6}}$

(g) $\frac{x+2y}{2} - \frac{y-x}{4} = \frac{x+2y}{2}\frac{\times 2}{\times 2} - \frac{y-x}{4}$

$= \frac{2(x+2y)}{4} - \frac{y-x}{4}$

$= \frac{2(x+2y)-(y-x)}{4}$

$= \frac{2x+4y-y+x}{4}$

$= \mathbf{\frac{3x+3y}{4}}$

(h) $\frac{4m-1}{4} - \frac{m-n}{3} = \frac{4m-1}{4}\frac{\times 3}{\times 3} - \frac{m-n}{3}\frac{\times 4}{\times 4}$

$= \frac{3(4m-1)}{12} - \frac{4(m-n)}{12}$

$= \frac{3(4m-1)-4(m-n)}{12}$

$= \frac{12m-3-4m+4n}{12}$

$= \mathbf{\frac{8m+4n-3}{12}}$

2. (a) $\frac{4a-b}{3} + \frac{a}{3} - \frac{b}{6} = \frac{4a-b}{3}\frac{\times 2}{\times 2} + \frac{a}{3}\frac{\times 2}{\times 2} - \frac{b}{6}$

$= \frac{2(4a-b)}{6} + \frac{2a}{6} - \frac{b}{6}$

$= \frac{2(4a-b)+2a-b}{6}$

$= \frac{8a-2b+2a-b}{6}$

$= \mathbf{\frac{10a-3b}{6}}$

(b) $\frac{x}{3}+\frac{x-y}{4}-\frac{y}{6}=\frac{x\times4}{3\times4}+\frac{x-y\times3}{4\ \times3}-\frac{y\times2}{6\times2}$

$=\frac{4x}{12}+\frac{3(x-y)}{12}-\frac{2y}{12}$

$=\frac{4x+3(x-y)-2y}{12}$

$=\frac{4x+3x-3y-2y}{12}$

$=\mathbf{\frac{7x-5y}{12}}$

(c) $\frac{m+n}{2}-\frac{2n-m}{3}-\frac{2n+m}{4}$

$=\frac{m+n\times6}{2\ \times6}-\frac{2n-m\times4}{3\ \times4}-\frac{2n+m\times3}{4\ \times3}$

$=\frac{6(m+n)}{12}-\frac{4(2n-m)}{12}-\frac{3(2n+m)}{12}$

$=\frac{6(m+n)-4(2n-m)-3(2n+m)}{12}$

$=\frac{6m+6n-8n+4m-6n-3m}{12}$

$=\mathbf{\frac{7m-8n}{12}}$

(d) $\frac{x}{4}+\frac{2(x-y)}{5}-\frac{x-2y}{10}$

$=\frac{x\times5}{4\times5}+\frac{2(x-y)\times4}{5\ \times4}-\frac{x-2y\times2}{10\ \times2}$

$=\frac{5x}{20}+\frac{8(x-y)}{20}-\frac{2(x-2y)}{20}$

$=\frac{5x+8(x-y)-2(x-2y)}{20}$

$=\frac{5x+8x-8y-2x+4y}{20}$

$=\mathbf{\frac{11x-4y}{20}}$

3. (a) $\frac{1}{2}x+1=3$

$\frac{1}{2}x=3-1$

$\frac{1}{2}x=2$

$x=2\times2$

$x=\mathbf{4}$

(b) $7-\frac{1}{3}y=1$

$-\frac{1}{3}y=1-7$

$-\frac{1}{3}y=-6$

$y=-6\times(-3)$

$y=\mathbf{18}$

(c) $\frac{3}{2}x-\frac{1}{2}=\frac{17}{2}$

$\frac{3}{2}x=\frac{17}{2}+\frac{1}{2}$

$\frac{3}{2}x=\frac{18}{2}$

$\frac{3}{2}x=9$

$x=9\times\frac{2}{3}$

$x=\mathbf{6}$

(d) $\frac{5}{7}x-\frac{1}{7}=\frac{4}{7}$

$\frac{5}{7}x=\frac{4}{7}+\frac{1}{7}$

$\frac{5}{7}x=\frac{5}{7}$

$x=\mathbf{1}$

(e) $6\frac{1}{2}=\frac{1}{2}-\frac{3}{4}x$

$6\frac{1}{2}-\frac{1}{2}=-\frac{3}{4}x$

$6=-\frac{3}{4}x$

$-\frac{3}{4}x=6$

$x=6\times\left(-\frac{4}{3}\right)$

$x=\mathbf{-8}$

(f) $23+\frac{4}{5}x=11$

$\frac{4}{5}x=11-23$

$\frac{4}{5}x=-12$

$x=-12\times\frac{5}{4}$

$x=\mathbf{-15}$

(g) $15-27=\frac{1}{6}x$

$-12=\frac{1}{6}x$

$\frac{1}{6}x=-12$

$x=-12\times6$

$x=\mathbf{-72}$

(h) $\frac{9}{10}x=2-6\frac{1}{2}$

$\frac{9}{10}x=-4\frac{1}{2}$

$x=-4\frac{1}{2}\times\frac{10}{9}$

$x=\mathbf{-5}$

(i) $\frac{11}{2} - 5\frac{1}{2}x = \frac{11}{4}$

$-5\frac{1}{2}x = \frac{11}{4} - \frac{11}{2}$

$-\frac{11}{2}x = -\frac{11}{4}$

$x = -\frac{11}{4} \times \left(-\frac{2}{11}\right)$

$x = \mathbf{\frac{1}{2}}$

(j) $4x + \frac{12}{5} = 2$

$4x = 2 - \frac{12}{5}$

$4x = -\frac{2}{5}$

$x = -\frac{2}{5} \times \frac{1}{4}$

$x = \mathbf{-\frac{1}{10}}$

4. (a) $\frac{1}{3}x - \frac{1}{4}x = 5$

$\frac{4}{12}x - \frac{3}{12}x = 5$

$\frac{1}{12}x = 5$

$x = 5 \times 12$

$x = \mathbf{60}$

(b) $\frac{3}{5}x + \frac{1}{2}x = 11$

$\frac{6}{10}x + \frac{5}{10}x = 11$

$\frac{11}{10}x = 11$

$x = 11 \times \frac{10}{11}$

$x = \mathbf{10}$

(c) $5x + 4 = \frac{13}{2}x$

$5x - \frac{13}{2}x = -4$

$\frac{10}{2}x - \frac{13}{2}x = -4$

$-\frac{3}{2}x = -4$

$x = -4 \times \left(-\frac{2}{3}\right)$

$x = \mathbf{2\frac{2}{3}}$

(d) $23 - \frac{1}{3}x = 9 - \frac{3}{2}x$

$-\frac{1}{3}x + \frac{3}{2}x = 9 - 23$

$-\frac{2}{6}x + \frac{9}{6}x = -14$

$\frac{7}{6}x = -14$

$x = -14 \times \frac{6}{7}$

$x = \mathbf{-12}$

(e) $\frac{1}{4}x - 7 = \frac{1}{5}x + 9$

$\frac{1}{4}x - \frac{1}{5}x = 9 + 7$

$\frac{5}{20}x - \frac{4}{20}x = 16$

$\frac{1}{20}x = 16$

$x = 16 \times 20$

$x = \mathbf{320}$

(f) $\frac{3}{4}x + 17 = \frac{1}{4}x - 21$

$\frac{3}{4}x - \frac{1}{4}x = -21 - 17$

$\frac{1}{2}x = -38$

$x = -38 \times 2$

$x = \mathbf{-76}$

(g) $1\frac{1}{4}x - \frac{3}{4}x - 15 = 7$

$1\frac{1}{4}x - \frac{3}{4}x = 7 + 15$

$\frac{1}{2}x = 22$

$x = 22 \times 2$

$x = \mathbf{44}$

(h) $\frac{3}{5}x - 10 = x + 5$

$\frac{3}{5}x - x = 5 + 10$

$-\frac{2}{5}x = 15$

$x = 15 \times \left(-\frac{5}{2}\right)$

$x = \mathbf{-\frac{75}{2}}$

(i) $\frac{3}{4}x + 5 = 2x + 20$

$\frac{3}{4}x - 2x = 20 - 5$

$-\frac{5}{4}x = 15$

$x = 15 \times \left(-\frac{4}{5}\right)$

$x = \mathbf{-12}$

(j) $14 - \frac{7}{4}x + 12 - \frac{3}{2}x = 0$

$-\frac{7}{4}x - \frac{3}{2}x = -14 - 12$

$-\frac{7}{4}x - \frac{6}{4}x = -26$

$-\frac{13}{4}x = -26$

$x = -26 \times \left(-\frac{4}{13}\right)$

$x = \mathbf{8}$

5. (a) $\frac{x+5}{2} = 5$

Multiply by 2 throughout,

$x + 5 = 10$

$x = \mathbf{5}$

(b) $\frac{4x-1}{5} = 3$

Multiply by 5 throughout,

$4x - 1 = 15$

$4x = 16$

$x = \mathbf{4}$

(c) $\frac{3a+2}{5} = \frac{a+2}{3}$

Multiply by 15 throughout,

$3(3a + 2) = 5(a + 2)$

$9a + 6 = 5a + 10$

$4a = 4$

$a = \mathbf{1}$

(d) $\frac{y+2}{3} = \frac{3y-1}{2}$

Multiply by 6 throughout,

$3(3y - 1) = 2(y + 2)$

$9y - 3 = 2y + 4$

$7y = 7$

$y = \mathbf{1}$

(e) $\frac{1-p}{3} = \frac{3p+1}{5}$

Multiply by 15 throughout,

$3(3p + 1) = 5(1 - p)$

$9p + 3 = 5 - 5p$

$14p = 2$

$p = \mathbf{\frac{1}{7}}$

(f) $\frac{1}{3}(2x - 1) = \frac{1}{5}(x + 2)$

$\frac{2x-1}{3} = \frac{x+2}{5}$

Multiply by 15 throughout,

$5(2x - 1) = 3(x + 2)$

$10x - 5 = 3x + 6$

$7x = 11$

$x = \mathbf{\frac{11}{7}}$

6. (a) $\frac{2-a}{4} + \frac{a-1}{3} = -1$

Multiply by 12 throughout,

$3(2 - a) + 4(a - 1) = -12$

$6 - 3a + 4a - 4 = -12$

$-3a + 4a = -12 + 4 - 6$

$a = \mathbf{-14}$

(b) $1 = \frac{x+3}{2} + \frac{3x}{5}$

Multiply by 10 throughout,

$10 = 5(x + 3) + 2(3x)$

$10 = 5x + 15 + 6x$

$10 - 15 = 5x + 6x$

$11x = -5$

$x = \mathbf{-\frac{5}{11}}$

(c) $\frac{3m-1}{3} + \frac{m}{2} = \frac{1}{4}$

Multiply by 12 throughout,

$4(3m - 1) + 6m = 3$

$12m - 4 + 6m = 3$

$18m = 3 + 4$

$18m = 7$

$m = \mathbf{\frac{7}{18}}$

(d) $\frac{2x-1}{3} = \frac{x}{6} + 2$

Multiply by 6 throughout,

$2(2x - 1) = x + 12$

$4x - 2 = x + 12$

$4x - x = 12 + 2$

$3x = 14$

$x = \mathbf{\frac{14}{3}}$

(e) $\frac{a-3}{2} = a - \frac{3}{5}$

Multiply by 10 throughout,

$5(a - 3) = 10a - 6$

$5a - 15 = 10a - 6$

$5a - 10a = -6 + 15$

$-5a = 9$

$a = \mathbf{-\frac{9}{5}}$

(f) $\frac{f+2}{3} = 2f - \frac{1}{3}$

Multiply by 3 throughout,

$f + 2 = 6f - 1$

$f - 6f = -1 - 2$

$-5f = -3$

$f = \frac{-3}{-5}$

$f = \mathbf{\frac{3}{5}}$

(g) $3x - \frac{2x-4}{3} = 2$

Multiply by 3 throughout,

$9x - (2x - 4) = 6$

$9x - 2x + 4 = 6$

$7x = 6 - 4$

$7x = 2$

$x = \mathbf{\frac{2}{7}}$

(h) $\frac{4-x}{5} - 2x = 1$

Multiply by 5 throughout,

$4 - x - 10x = 5$

$-11x = 5 - 4$

$-11x = 1$

$x = \mathbf{-\frac{1}{11}}$

(i) $\frac{x}{3} + \frac{x-1}{4} = \frac{x}{6}$

Multiply by 12 throughout,

$4x + 3(x-1) = 2x$

$4x + 3x - 3 = 2x$

$7x - 2x = 3$

$5x = 3$

$x = \mathbf{\frac{3}{5}}$

(j) $\frac{a}{2} = \frac{2a}{3} - \frac{a+1}{5}$

Multiply by 30 throughout,

$15a = 10 \times (2a) - 6 \times (a+1)$

$15a = 20a - 6a - 6$

$15a = 14a - 6$

$15a - 14a = -6$

$a = \mathbf{-6}$

7. (a) $\frac{5}{2x+2} = 1$

$2x + 2 = 5$

$2x = 3$

$x = \mathbf{\frac{3}{2}}$

(b) $\frac{1}{2x-1} = 3$

$3(2x-1) = 1$

$6x - 3 = 1$

$6x = 4$

$x = \mathbf{\frac{2}{3}}$

(c) $\frac{a+5}{a-6} = \frac{4}{5}$

$4(a-6) = 5(a+5)$

$4a - 24 = 5a + 25$

$a = \mathbf{-49}$

(d) $\frac{2p-1}{3p-5} = \frac{4}{7}$

$4(3p-5) = 7(2p-1)$

$12p - 20 = 14p - 7$

$2p = -13$

$p = \mathbf{-\frac{13}{2}}$

(e) $\frac{3}{y+6} = \frac{2}{y-2}$

$2(y+6) = 3(y-2)$

$2y + 12 = 3y - 6$

$y = \mathbf{18}$

(f) $\frac{2}{2n+3} = \frac{4}{n-2}$

$4(2n+3) = 2(n-2)$

$8n + 12 = 2n - 4$

$6n = -16$

$n = \mathbf{-\frac{8}{3}}$

8. (a) $>$ (b) $>$

(c) $<$ (d) $<$

(e) $<$ (f) $<$

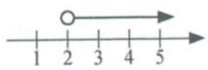
(g) $>$ (h) $>$

(i) $=$ (j) $>$

9. (a) $2x > 4$

$\frac{2x}{2} > \frac{4}{2}$

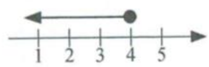
$\mathbf{x > 2}$



(b) $3y \leqslant 12$

$\frac{3y}{3} \leqslant \frac{12}{3}$

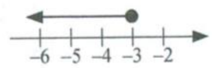
$\mathbf{y \leqslant 4}$



(c) $-3x \geqslant 9$

$\frac{-3x}{-3} \leqslant \frac{9}{-3}$

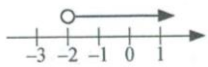
$\mathbf{x \leqslant -3}$



(d) $-4x < 8$

$\frac{-4x}{-4} > \frac{8}{-4}$

$\mathbf{x > -2}$



10. $3x < 8$

$\frac{3x}{3} < \frac{8}{3}$

$x < 2\frac{2}{3}$

The largest integer x is **2**.

11. Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (x+5) \times 7$

$= \frac{7(x+5)}{2}$

Area of triangle = 35 cm²

$\frac{7(x+5)}{2} = 35$

$7(x+5) = 35 \times 2$

$7(x+5) = 70$

$x+5 = \frac{70}{7}$

$x+5 = 10$

$x = 10-5$

$x = \mathbf{5}$

12. Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 15 \times (x+2)$

$= \frac{15(x+2)}{2}$

Area of triangle = 60 cm²

$\frac{15(x+2)}{2} = 60$

$15(x+2) = 120$

$x+2 = \frac{120}{15}$

$x+2 = 8$

$x = 8-2$

$x = \mathbf{6}$

13. For equilateral △, all sides are the same.

$\frac{5x+3}{2} = \frac{25}{6}$

$6(5x+3) = 2(25)$

$30x+18 = 50$

$30x = 50-18$

$30x = 32$

$x = \frac{32}{30}$

$x = \mathbf{1\frac{1}{15}}$

$\frac{2}{3}y - 1 = \frac{25}{6}$

$\frac{2}{3}y = \frac{25}{6} + 1$

$\frac{2}{3}y = 5\frac{1}{6}$

$y = 5\frac{1}{6} \div \frac{2}{3}$

$y = \mathbf{7\frac{3}{4}}$

14. (a) Since the triangle is an isosceles triangle,

$3x-5 = 16$

$3x = 16+5$

$3x = 21$

$x = \mathbf{7}$

(b) Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times (3x-5) \times (16)$

$= \frac{16(3x-5)}{2}$

If area of triangle = 56 cm²,

$\frac{16(3x-5)}{2} = 56$

$8(3x-5) = 56$

$3x-5 = \frac{56}{8}$

$3x-5 = 7$

$3x = 7+5$

$3x = 12$

$x = \mathbf{4}$

15. Number of packets $= \frac{300}{25}$

$= 12$

Let x be the minimum amount.

$12x \geqslant \$4.50$

$x \geqslant \$0.375$

$x = \$0.38$ (nearest cent)

The minimum amount he needs to price the repacked paper is **\$0.38**.

16. Let x be the least number of items.

$\$3 \times x \geqslant \$80$

$3x \geqslant 80$

$x \geqslant 26.67$

$x = 27$

She must buy at least **27** items to qualify for the lucky draw.

17. Let x be the number of coaches.

$42x \geqslant 558$

$x \geqslant 13.29$

$x = 14$ (2 sig. fig.)

14 coaches are needed in total.

2 LINEAR FUNCTIONS AND GRAPHS

1. Coordinates of A = **(–8, 2)**
Coordinates of B = **(–5, 2)**
Coordinates of C = **(–2, 3)**
Coordinates of D = **(0, 0)**
Coordinates of E = **(2, –3)**
Coordinates of F = **(–9, –2)**
Coordinates of G = **(–3, –1)**
Coordinates of H = **(2, 1)**

2. (a) $y = 2x + 3$
When $x = 1$,
$y = 2(1) + 3$
$= \mathbf{5}$

(b) $y = 2x + 3$
When $y = 7$,
$7 = 2x + 3$
$2x = 4$
$x = \mathbf{2}$

3. (a) $y = \frac{1}{2}x - 1$
When $x = 6$,
$y = \frac{1}{2}(6) - 1$
$= \mathbf{2}$

(b) $y = \frac{1}{2}x - 1$
When $y = 8$,
$8 = \frac{1}{2}x - 1$
$\frac{1}{2}x = 9$
$x = \mathbf{18}$

4. (a)

x	–1	0	1	2	3
y	**1**	3	**5**	7	**9**

(b) Refer to Appendix 1 (page 187).

(c) (i) When $y = 2.5$, $x = \mathbf{-0.25}$.
(ii) When $x = 1.5$, $y = \mathbf{6}$.

5. (a)

x	–3	–2	–1	0	1	2	3
y	13	**11**	9	7	**5**	3	1

(b) Refer to Appendix 2 (page 188).

(c) (i) When $y = 6$, $x = \mathbf{0.5}$.
(ii) When $x = 1.5$, $y = \mathbf{4}$.

6. (a) Gradient = 3
y-intercept = 4

(b) Gradient = 4
y-intercept = –6

(c) Gradient = –2
y-intercept = 12

(d) Gradient = $\frac{1}{2}$
y-intercept = –3

7. (a) $y = mx + c$
$y = 3x + 5$

(b) $y = mx + c$
$y = -2x + 12$

(c) $y = mx + c$
$y = -\frac{1}{3}x - 4$

(d) $y = mx + c$
$y = 1\frac{1}{3}x - 2$

8. (a) Gradient $= \frac{\text{Vertical change}}{\text{Horizontal change}}$
$= \frac{1 - 0}{1 - (-1)}$
$= \mathbf{\frac{1}{2}}$
y-intercept = **0.5**

(b) Gradient $= \frac{\text{Vertical change}}{\text{Horizontal change}}$
$= -\frac{0 - (-1.5)}{0 - (-2.5)}$
$= \mathbf{-0.6}$
y-intercept = **–1.5**

(c) Gradient $= \frac{\text{Vertical change}}{\text{Horizontal change}}$
$= -\frac{2.5 - 1}{6 - 0}$
$= \mathbf{-0.25}$
y-intercept = **2.5**

(d) Gradient $= \frac{\text{Vertical change}}{\text{Horizontal change}}$
$= \frac{3 - 1.5}{6 - 0}$
$= \mathbf{0.25}$
y-intercept = **1.5**

(e) Gradient $= \frac{\text{Vertical change}}{\text{Horizontal change}}$
$= \frac{-1-(-3.2)}{8-0}$
$= \mathbf{0.275}$

y-intercept $= \mathbf{-3.2}$

9. (a) $x = \mathbf{0.5}$ (b) $y = \mathbf{-2}$

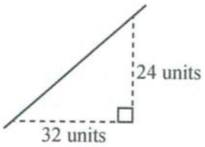
10. (a) Distance travelled in 1 min $= \frac{6}{8} = 0.75$ km
Distance travelled in 32 min $= 0.75 \times 32$
$= \mathbf{24\ km}$

(b) Refer to Appendix 3 (page 189).

(c) (i) When $t = 22$, $D = 16.5$.
Distance travelled in 22 min = **16.5 km**

(ii) When $D = 21$, $t = 28$
Time taken to travel 21 km = **28 min**

(d) (i)



Gradient $= \frac{24}{32}$
$= \mathbf{\frac{3}{4}}$

(ii) The gradient represents the **speed of the motorcyclist**.

11. Refer to Appendix 4 (page 190).

(a) (i) When $y = 10$, $t = 12\ 30$.
The time when he was 10 km away from his home was **12 30**.

(ii) When $t = 07\ 00$, $y = 3$.
He was **3 km** away from his home just before he started his journey.

(b) Speed = Gradient of the graph
$= \frac{9}{7}$
$= \mathbf{1.29\ km/h}$ (3 sig. fig.)

(c) $\mathbf{y = 1.29t + 3}$

3 SIMULTANEOUS EQUATIONS

1. (a) $x + 3y = 6$
When $x = 0$, $y = p$.
$0 + 3p = 6$
$p = \mathbf{2}$

(b) Refer to Appendix 5 (page 191).

(c) From the graph, when $y = 2.5$, $x = \mathbf{-1.5}$.

2. (a) $3x - 4y = 12$
When $x = 8$, $y = p$.
$3(8) - 4p = 12$
$4p = 12$
$p = \mathbf{3}$

(b) Refer to Appendix 6 (page 192).

(c) From the graph, when $y = 2.25$, $x = 7$.
Hence $k = \mathbf{7}$.

3. (a) Refer to Appendix 7 (page 193).
From the graph, the point of intersection is (9, 0).
Hence the solution is $x = \mathbf{9}$ and $y = \mathbf{0}$.

(b) Refer to Appendix 8 (page 194).
From the graph, the point of intersection is (2, –2).
Hence the solution is $x = \mathbf{2}$ and $y = \mathbf{-2}$.

(c) Refer to Appendix 9 (page 195).
From the graph, the point of intersection is (–2, –1).
Hence the solution is $x = \mathbf{-2}$ and $y = \mathbf{-1}$.

(d) Refer to Appendix 10 (page 196).
From the graph, the point of intersection is (6, 6).
Hence the solution is $x = \mathbf{6}$ and $y = \mathbf{6}$.

4. (a) Refer to Appendix 11 (page 197).
The two graphs coincide.
Hence the simultaneous equations have an **infinite number of solutions**.

(b) Refer to Appendix 12 (page 198).
The two graphs are parallel to each other.
Hence the simultaneous equations have **no solutions**.

5. (a) $x - y = 5$ ....(1)
$2x + y = 1$ ....(2)
(1) + (2),
$3x = 6$
$x = 2$
Substitute $x = 2$ into (1):
$2 - y = 5$
$-y = 5 - 2$
$-y = 3$
$y = -3$
$\therefore x = \mathbf{2}, y = \mathbf{-3}$

(b) $2x - y = 8$................ (1)
$x - 2y = 11$............. (2)
(2) × 2:
$2x - 4y = 22$........... (3)
(3) – (1),
$-3y = 14$
$y = -\frac{14}{3}$
Substitute $y = -\frac{14}{3}$ into (2):
$x - 2\left(-\frac{14}{3}\right) = 11$
$x + 9\frac{1}{3} = 11$
$x = 11 - 9\frac{1}{3}$
$x = \frac{5}{3}$
$\therefore x = \mathbf{\frac{5}{3}}, y = \mathbf{-\frac{14}{3}}$

(c) $x + 3y = 9$ .... (1)
$2x + 3y = 12$ .... (2)
(2) – (1),
$x = 3$
Substitute $x = 3$ into (1):
$(3) + 3y = 9$
$3y = 9 - 3$
$3y = 6$
$y = 2$
$\therefore x = \mathbf{3}, y = \mathbf{2}$

(d) $x - 3y = 2$ .... (1)
$2x + 3y = 13$ .... (2)
(1) + (2),
$3x = 15$
$x = 5$
Substitute $x = 5$ into (1):
$(5) - 3y = 2$
$-3y = 2 - 5$
$-3y = -3$
$y = 1$
$\therefore x = \mathbf{5}, y = \mathbf{1}$

(e) $2x + 3y = 7$ .... (1)
$3x - 5y = 1$............ (2)
(1) × 3,
$6x + 9y = 21$ .... (3)
(2) × 2,
$6x - 10y = 2$........... (4)
(3) – (4),
$19y = 19$
$y = 1$
Substitute $y = 1$ into (1):
$2x + 3(1) = 7$
$2x + 3 = 7$
$2x = 7 - 3$
$2x = 4$
$x = 2$
$\therefore x = \mathbf{2}, y = \mathbf{1}$

(f) $\frac{1}{2}x + 2y = -2$ ....(1)
$4x - y = 35$........................(2)
(1) × 8,
$8\left(\frac{1}{2}x\right) + 8(2y) = 8(-2)$
$4x + 16y = -16$............(3)
(3) – (2),
$17y = -51$
$y = -3$
Substitute $y = -3$ into (2):
$4x - (-3) = 35$
$4x + 3 = 35$
$4x = 35 - 3$
$4x = 32$
$x = 8$
$\therefore x = \mathbf{8}, y = \mathbf{-3}$

(g) $1.2x + 0.4y = 14$ ....(1)
$0.6x - 1.6y = -2$ ....(2)
(2) × 2,
$2(0.6x) - 2(1.6y) = 2(-2)$
$1.2x - 3.2y = -4$ ....(3)
(1) – (3),
$3.6y = 18$
$y = 5$
Substitute $y = 5$ into (1):
$1.2x + 0.4(5) = 14$
$1.2x + 2 = 14$
$1.2x = 14 - 2$
$1.2x = 12$
$x = 10$
$\therefore x = \mathbf{10}, y = \mathbf{5}$

(h) $\frac{x+1}{y} = \frac{1}{2}$

$2(x+1) = y$

$2x + 2 = y$

$2x - y = -2$..................(1)

$\frac{2y-1}{5} = \frac{x}{2}$

$2(2y-1) = 5x$

$4y - 2 = 5x$

$4y - 5x = 2$..................(2)

(1) × 4,

$8x - 4y = -8$.....................(3)

(2) + (3),

$3x = -6$

$x = -2$

Substitute $x = -2$ into (1):

$2(-2) - y = -2$

$-4 - y = -2$

$-y = -2 + 4$

$-y = 2$

$y = -2$

$\therefore x = \mathbf{-2}, y = \mathbf{-2}$

6. (a) $x + y = 15$ ....(1)

$x - y = 1$ ....(2)

From (2), make x the subject,

$x = 1 + y$ ....(3)

Substitute (3) into (1):

$(1 + y) + y = 15$

$1 + 2y = 15$

$2y = 15 - 1$

$2y = 14$

$y = 7$

Substitute $y = 7$ into (3):

$x = 1 + 7$

$x = 8$

$\therefore x = \mathbf{8}, y = \mathbf{7}$

(b) $2x + y = 4$ .... (1)

$3x + 2y = 7$ .... (2)

From (1), make y the subject,

$y = 4 - 2x$.............. (3)

Substitute (3) into (2):

$3x + 2(4 - 2x) = 7$

$3x + 8 - 4x = 7$

$3x - 4x = 7 - 8$

$-x = -1$

$x = 1$

Substitute $x = 1$ into (3):

$y = 4 - 2(1)$

$y = 4 - 2$

$y = 2$

$\therefore x = \mathbf{1}, y = \mathbf{2}$

(c) $x - y = 3$...........(1)

$2x - 3y = 1$...........(2)

From (1), make x the subject,

$x = 3 + y$ ....(3)

Substitute (3) into (2):

$2(3 + y) - 3y = 1$

$6 + 2y - 3y = 1$

$-y = 1 - 6$

$-y = -5$

$y = 5$

Substitute $y = 5$ into (3):

$x = 3 + 5$

$x = 8$

$\therefore x = \mathbf{8}, y = \mathbf{5}$

(d) $x - 3y = 8$................ (1)

$3y - 2x = -13$........... (2)

From (1), make x the subject,

$x = 8 + 3y$ .... (3)

Substitute $x = 8 + 3y$ into (2):

$3y - 2(8 + 3y) = -13$

$3y - 16 - 6y = -13$

$-3y = -13 + 16$

$-3y = 3$

$y = -1$

Substitute $y = -1$ into (3):

$x = 8 + 3(-1)$

$x = 8 - 3$

$x = 5$

$\therefore x = \mathbf{5}, y = \mathbf{-1}$

(e) $2x + 2y = 1$ ....(1)

$x - 4y = -\frac{1}{2}$ ....(2)

From (2), make x the subject,

$x = -\frac{1}{2} + 4y$ ....(3)

Substitute (3) into (1):

$2\left(-\frac{1}{2} + 4y\right) + 2y = 1$

$-1 + 8y + 2y = 1$

$10y = 2$

$y = \frac{1}{5}$

Substitute $y = \frac{1}{5}$ into (3):

$x = -\frac{1}{2} + 4\left(\frac{1}{5}\right)$

$x = \frac{3}{10}$

$\therefore x = \mathbf{\frac{3}{10}}, y = \mathbf{\frac{1}{5}}$

(f) $\frac{2}{3}x - \frac{1}{4}y = 1$ .... (1)

$x + 2y = 11$ .... (2)

From (2), make x the subject,

$x = 11 - 2y$.................. (3)

Substitute (3) into (1):

$\frac{2}{3}(11 - 2y) - \frac{1}{4}y = 1$

$\frac{22}{3} - \frac{4y}{3} - \frac{1}{4}y = 1$

$-1\frac{7}{12}y = 1 - \frac{22}{3}$

$-1\frac{7}{12}y = -6\frac{1}{3}$

$y = 4$

Substitute $y = 4$ into (3):

$x = 11 - 2(4)$

$x = 11 - 8$

$x = 3$

$\therefore$ $x = \mathbf{3}$, $y = \mathbf{4}$

(g) $\frac{x-y}{2} = 1$ .... (1)

$x + 3y = 10$ .... (2)

From (1), make x the subject,

$x - y = 2$

$x = 2 + y$.......... (3)

Substitute (3) into (2):

$(2 + y) + 3y = 10$

$2 + 4y = 10$

$4y = 10 - 2$

$4y = 8$

$y = 2$

Substitute $y = 2$ into (3):

$x = 2 + 2$

$x = 4$

$\therefore$ $x = \mathbf{4}$, $y = \mathbf{2}$

(h) $-5x - y = -13$........... (1)

$-7x - 2y = -20$........... (2)

From (1), make y the subject,

$-y = 5x - 13$

$y = 13 - 5x$................. (3)

Substitute (3) into (2):

$-7x - 2(13 - 5x) = -20$

$-7x - 26 + 10x = -20$

$3x = -20 + 26$

$3x = 6$

$x = 2$

Substitute $x = 2$ into (3):

$y = 13 - 5(2)$

$y = 13 - 10$

$y = 3$

$\therefore$ $x = \mathbf{2}$, $y = \mathbf{3}$

7. Let \$$x$ be the cost of 1 can of tuna.

Let \$$y$ be the cost of 1 can of baked beans.

$2x + y = 5.10$ .... (1)

$4x + 3y = 11.70$............. (2)

(1) × 3,

$6x + 3y = 15.30$ .... (3)

(3) – (2),

$2x = 3.60$

$x = 1.80$

Substitute $x = 1.80$ into (1):

$2(1.80) + y = 5.10$

$3.60 + y = 5.10$

$y = 5.10 - 3.60$

$y = 1.50$

The cost of 1 can of tuna is **\$1.80**.

The cost of 1 can of baked beans is **\$1.50**.

8. Let \$$x$ be the cost of 1 PDA phone.

Let \$$y$ be the cost of 1 camera phone.

$3x + 2y = 2040$ ....(1)

$x = 2y$..................(2)

Substitute (2) into (1):

$3(2y) + 2y = 2040$

$6y + 2y = 2040$

$8y = 2040$

$y = 255$

Substitute $y = 255$ into (2):

$x = 2(255)$

$x = 510$

The cost of 1 PDA phone is **\$510**.

9. Let x be the number of ostriches.

Let y be the number of giraffes.

$x + y = 12$ .... (1)

$2x + 4y = 34$ .... (2)

(1) × 2,

$2x + 2y = 24$ .... (3)

(2) – (3),

$2y = 10$

$y = 5$

Substitute $y = 5$ into (1):

$x + 5 = 12$

$x = 12 - 5$

$x = 7$

There were **7** ostriches and **5** giraffes.

10. Let x be the number of cars.
Let y be the number of motorcycles.
$x + y = 17$ .... (1)
$4x + 2y = 48$ .... (2)
(1) × 4,
$4x + 4y = 68$ .... (3)
(3) – (2),
$2y = 20$
$y = 10$
There were **10** damaged motorcycles.

11. Let x be Mary's present age.
Let y be aunt's present age.
$x + y = 68$................ (1)
$2(x + 2) = y + 2$ .... (2)
From (1), make x the subject,
$x = 68 - y$..................... (3)
Substitute (3) into (2):
$2(68 - y + 2) = y + 2$
$2(70 - y) = y + 2$
$140 - 2y = y + 2$
$140 - 2 = y + 2y$
$3y = 138$
$y = 46$
Substitute $y = 46$ into (3):
$x = 68 - 46$
$x = 22$
Mary is **22 years** old.
Her aunt is **46 years** old.

12. Let x be Gwen's present age.
Let y be Helen's present age.
$y = 3x$....................... (1)
$y - 10 = 5(x - 10)$............ (2)
Substitute (1) into (2):
$(3x) - 10 = 5(x - 10)$
$3x - 10 = 5x - 50$
$3x - 5x = -50 + 10$
$-2x = -40$
$x = 20$
Substitute $x = 20$ into (1):
$y = 3(20)$
$y = 60$
Helen is **60 years** old now.

13. Let x km/h be the speed of Car A.
Let y km/h be the speed of Car B.
1 hour later, the distance travelled by:
Car $A = x \times 1$
$= x$ km
Car $B = y \times 1$
$= y$ km
2 hours after starting, the distance travelled by:
Car $A = 2x$
Car $B = 2y$
$x - y = 20$ ....(1)
$2x + 2y = 360$ ....(2)
From (1), make x the subject,
$x = 20 + y$ ....(3)
Substitute (3) into (2):
$2(20 + y) + 2y = 360$
$40 + 2y + 2y = 360$
$4y = 360 - 40$
$4y = 320$
$y = 80$
Substitute $y = 80$ into (1):
$x - 80 = 20$
$x = 20 + 80$
$x = 100$
The speed of Car A is **100 km/h**.
The speed of Car B is **80 km/h**.

14. Let x m/s be Patrick's speed.
Let y m/s be Raymond's speed.
2 seconds into the race, the distance ran by:
Patrick $= x \times 2$
$= 2x$
Raymond $= y \times 2$
$= 2y$
$2x - 2y = 1$............(1)
$4y - 2x = 4$............(2)
(1) + (2),
$2y = 5$
$y = \frac{5}{2}$
Substitute $y = \frac{5}{2}$ into (1):
$2x - 2\left(\frac{5}{2}\right) = 1$
$2x - 5 = 1$
$2x = 6$
$x = 3$
Patrick's speed was **3 m/s**.

15. Let x cm be the length of the rectangle.
Let y cm be the breadth of the rectangle.
$x - y = 5$ .... (1)
$2x + 2y = 34$ .... (2)
From (1), make x the subject,
$x = 5 + y$ .... (3)
Substitute (3) into (2):
$2(5 + y) + 2y = 34$
$10 + 2y + 2y = 34$
$4y = 34 - 10$
$4y = 24$
$y = 6$

Substitute $y = 6$ into (3):
$x = 5 + 6$
$= 11$
Area $= 11 \times 6$
$= 66 \text{ cm}^2$
The area of the rectangle is **66 cm²**.

16. Let x cm be the length of the shorter parallel side of the trapezium.
Let y cm be the length of the longer parallel side of the trapezium.
$y - x = 3$ ....(1)
Area of trapezium
$= \frac{1}{2}(x + y)(6)$
$= 3(x + y)$
$3(x + y) = 39$ ....(2)
From (1), make y the subject,
$y = 3 + x$ ....(3)
Substitute (3) into (2):
$3(x + 3 + x) = 39$
$3(2x + 3) = 39$
$6x + 9 = 39$
$6x = 39 - 9$
$6x = 30$
$x = 5$
Substitute $x = 5$ into (3):
$y = 3 + 5$
$= 8$
The length of the shorter parallel side of the trapezium is **5 cm**.

17. Let x be the number of children.
Let y be the number of adults.
$x + y = 370$ .... (1)
$22x + 14y = 6380$ .... (2)
From (1), make x the subject,
$x = 370 - y$.................................. (3)
Substitute (3) into (2),
$22(370 - y) + 14y = 6380$
$8140 - 22y + 14y = 6380$
$-8y = -1760$
$8y = 1760$
$y = 220$
Substitute $y = 220$ into (3):
$x = 370 - 220$
$= 150$
150 children visited the zoo.

18. $2x + y + 7 = 4y$............................. (1)
$3x = 2x + y - 1$................. (2)
From (2), make x the subject,
$3x = 2x + y - 1$
$x = y - 1$.................................... (3)
Substitute (3) into (1):
$2(y - 1) + y + 7 = 4y$
$2y - 2 - 3y = -7$
$-y = -7 + 2$
$-y = -5$
$y = 5$
Substitute $y = 5$ into (3):
$x = 5 - 1$
$= 4$
The value of x is **4** and the value of y is **5**.

19. $2x + 4y = 4x - 4$........................... (1)
$3x + 4 = x + 5(y + 1)$..................... (2)
From (1), make x the subject,
$2x - 4x + 4y = -4$
$-2x + 4y = -4$
$-2(x - 2y) = -4$
$x - 2y = 2$
$x = 2y + 2$ ....(3)
Substitute (3) into (2):
$3(2y + 2) + 4 = 2y + 2 + 5(y + 1)$
$6y + 6 + 4 = 2y + 2 + 5y + 5$
$6y + 10 = 7y + 7$
$6y - 7y = 7 - 10$
$-y = -3$
$y = 3$
Substitute $y = 3$ into (3):
$x = 2(3) + 2$
$= 6 + 2$
$= 8$
Length of 1 side of the square $= 2x + 4y$
$= 2(8) + 4(3)$
$= 16 + 12$
$= 28$ cm

$28 \times 4 = 112$ cm
The perimeter of the square is **112 cm**.

20. Let x be the first number.
Let y be the second number.
$2x + 5y = 69$ .... (1)
$y = \frac{3}{4}x$ .... (2)
Substitute (2) into (1):
$2x + 5\left(\frac{3}{4}x\right) = 69$
$2x + \frac{15}{4}x = 69$
$\frac{8}{4}x + \frac{15}{4}x = 69$
$\frac{23}{4}x = 69$
$23x = 276$
$x = 12$
Substitute $x = 12$ into (2):
$y = \frac{3}{4}(12)$
$= 9$
The first number is **12**.
The second number is **9**.

21. Let x be the first number.
Let y be the second number.
$2(x + y) = 7 + 3x$
$2x + 2y = 7 + 3x$
$2x + 2y - 3x = 7$
$2y - x = 7$ .... (1)
$x = y + 4$ .... (2)
Substitute (2) into (1):
$2y - (y + 4) = 7$
$2y - y - 4 = 7$
$y = 7 + 4$
$y = 11$
Substitute $y = 11$ into (2):
$x = 11 + 4$
$= 15$
The first number is **15**.
The second number is **11**.

22. Let x be the first number.
Let y be the second number.
$\frac{1}{2}x + \frac{1}{3}y = 5$ .... (1)
$x + y = 11$ .... (2)
From (2), make x the subject,
$x = 11 - y$ .... (3)
Substitute (3) into (1):
$\frac{1}{2}(11 - y) + \frac{1}{3}y = 5$
$\frac{3(11 - y)}{6} + \frac{2y}{6} = 5$
$\frac{3(11 - y) + 2y}{6} = 5$
$3(11 - y) + 2y = 30$
$33 - 3y + 2y = 30$
$-y = 30 - 33$
$-y = -3$
$y = 3$
Substitute $y = 3$ into (3):
$x = 11 - 3$
$= 8$
The first number is **8**.
The second number is **3**.

23. Let x be the first number.
Let y be the second number.
$x + y = 4x$ .... (1)
$\frac{1}{3}y + x = 8$ .... (2)
From (1), make y the subject,
$y = 4x - x$
$y = 3x$ .... (3)
Substitute (3) into (2):
$\frac{1}{3}(3x) + x = 8$
$x + x = 8$
$2x = 8$
$x = 4$
Substitute $x = 4$ into (3):
$y = 3(4)$
$= 12$
The first number is **4**.
The second number is **12**.

4 EXPANSION AND FACTORISATION OF ALGEBRAIC EXPRESSIONS

1. (a) $-5x^2 + (-x^2) + 3x - 7x = -6x^2 + (-4x)$
$= \mathbf{-6x^2 - 4x}$

(b) $-3x^2 + (-5x^2) + 2xy - 7x = \mathbf{-8x^2 + 2xy - 7x}$

(c) $6y^2 - yz - 3y^2 - (-8yz) = 6y^2 - 3y^2 - yz - (-8yz)$
$= 3y^2 - yz + 8yz$
$= \mathbf{3y^2 + 7yz}$

(d) $x^2 + (-6yx) - (-4x^2) - 3xy$
$= x^2 - (-4x^2) + (-6xy) - 3xy$
$= x^2 + 4x^2 + (-9xy)$
$= \mathbf{5x^2 - 9xy}$

2. (a) $-5(3x - y) = \mathbf{-15x + 5y}$

(b) $3 - x(2y + 3z) = \mathbf{3 - 2xy - 3xz}$

(c) $-2x(y + 3z) - 5x(2y - z)$
$= -2xy - 6xz - 10xy + 5xz$
$= \mathbf{-12xy - xz}$

(d) $2p(q - r) - 7q(r + 5p)$
$= 2pq - 2pr - 7qr - 35pq$
$= \mathbf{-33pq - 2pr - 7qr}$

3. (a) $(2c + d)(4x + 5y)$
$= (2c)(4x) + (2c)(5y) + (d)(4x) + (d)(5y)$
$= \mathbf{8cx + 10cy + 4dx + 5dy}$

(b) $(3p + 5q)(6r - s)$
$= (3p)(6r) + (3p)(-s) + (5q)(6r) + (5q)(-s)$
$= \mathbf{18pr - 3ps + 30qr - 5qs}$

(c) $(-3a - 2b)(-5c + 3d)$
$= (-3a)(-5c) + (-3a)(3d) + (-2b)(-5c)$
$+ (-2b)(3d)$
$= \mathbf{15ac - 9ad + 10bc - 6bd}$

(d) $(-4r - s)(3 - 2t - 4u)$
$= (-4r)(3) + (-4r)(-2t) + (-4r)(-4u) + (-s)(3)$
$+ (-s)(-2t) + (-s)(-4u)$
$= \mathbf{-12r + 8rt + 16ru - 3s + 2st + 4su}$

4. (a) $(4x + 3)(6x + 1)$
$= (4x)(6x) + (4x)(1) + (3)(6x) + (3)(1)$
$= 24x^2 + 4x + 18x + 3$
$= \mathbf{24x^2 + 22x + 3}$

(b) $(7x - 2)(x + 6)$
$= (7x)(x) + (7x)(6) + (-2)(x) + (-2)(6)$
$= 7x^2 + 42x - 2x - 12$
$= \mathbf{7x^2 + 40x - 12}$

(c) $(8x + 3)(4x - 7)$
$= (8x)(4x) + (8x)(-7) + (3)(4x) + (3)(-7)$
$= 32x^2 - 56x + 12x - 21$
$= \mathbf{32x^2 - 44x - 21}$

(d) $(4x - 9)(3 - 8x)$
$= (4x)(3) + (4x)(-8x) + (-9)(3) + (-9)(-8x)$
$= 12x - 32x^2 - 27 + 72x$
$= \mathbf{-32x^2 + 84x - 27}$

5. (a) $7x + 21 = \mathbf{7(x + 3)}$

(b) $-12x + 36 = 36 - 12x$
$= \mathbf{12(3 - x)}$

(c) $15x - 10xy = \mathbf{5x(3 - 2y)}$

(d) $-9x - 36xy + 18xz = 18xz - 36xy - 9x$
$= \mathbf{9x(2z - 4y - 1)}$

6. (a) $-9x - 27 = \mathbf{-9(x + 3)}$

(b) $-15ab - 25a = \mathbf{-5a(3b + 5)}$

(c) $-6c - 9ac - 15bc = \mathbf{-3c(2 + 3a + 5b)}$

(d) $-8 - 16x - 24xy = \mathbf{-8(1 + 2x + 3xy)}$

7. (a) $15y^2 + 10y = \mathbf{5y(3y + 2)}$

(b) $6x^2 - 12x = \mathbf{6x(x - 2)}$

(c) $16a - 24a^2 = \mathbf{-8a(3a - 2)}$

(d) $-70y - 14y^2 = \mathbf{-14y(y + 5)}$

8. (a) $x^2 + 8x + 12$

$\times$	x	$+2$
x	x^2	$+2x$
$+6$	$+6x$	$+12$

$x^2 + 8x + 12 = \mathbf{(x + 2)(x + 6)}$

(b) $x^2 - 7x + 12$

$\times$	x	-3
x	x^2	$-3x$
-4	$-4x$	$+12$

$x^2 - 7x + 12 = \mathbf{(x - 3)(x - 4)}$

(c) $x^2 + 2x - 3$

$\times$	x	-1
x	x^2	$-x$
$+3$	$+3x$	-3

$x^2 + 2x - 3 = \mathbf{(x - 1)(x + 3)}$

(d) $x^2 - 7x - 8$

×	x	$+1$
x	x^2	$+x$
-8	$-8x$	-8

$x^2 - 7x - 8 = \mathbf{(x + 1)(x - 8)}$

9. (a) $8x^2 + 26x + 15$

×	$4x$	$+3$
$2x$	$8x^2$	$+6x$
$+5$	$+20x$	$+15$

$8x^2 + 26x + 15 = \mathbf{(4x + 3)(2x + 5)}$

(b) $15x^2 + 2x - 24$

×	$5x$	-6
$3x$	$15x^2$	$-18x$
$+4$	$+20x$	-24

$15x^2 + 2x - 24 = \mathbf{(5x - 6)(3x + 4)}$

(c) $4x^2 - 17x + 15$

×	$4x$	-5
x	$4x^2$	$-5x$
-3	$-12x$	$+15$

$4x^2 - 17x + 15 = \mathbf{(4x - 5)(x - 3)}$

(d) $-3x^2 - 8x - 4 = -(3x^2 + 8x + 4)$

×	$3x$	$+2$
x	$3x^2$	$+2x$
$+2$	$+6x$	$+4$

$-3x^2 - 8x - 4 = \mathbf{-(3x + 2)(x + 2)}$

(e) $-4x^2 + 13x - 10 = -(4x^2 - 13x + 10)$

×	$4x$	-5
x	$4x^2$	$-5x$
-2	$-8x$	$+10$

$-4x^2 + 13x - 10 = \mathbf{-(4x - 5)(x - 2)}$

(f) $5 - 9x - 2x^2 = -(2x^2 + 9x - 5)$

×	$2x$	-1
x	$2x^2$	$-x$
$+5$	$+10x$	-5

$5 - 9x - 2x^2 = \mathbf{-(2x - 1)(x + 5)}$

10. (a) $x^2 + 2xy - 3y^2$

×	x	$-y$
x	x^2	$-xy$
$+3y$	$+3xy$	$-3y^2$

$x^2 + 2xy - 3y^2 = \mathbf{(x - y)(x + 3y)}$

(b) $3x^2 - 13xy - 10y^2$

×	$3x$	$+2y$
x	$3x^2$	$+2xy$
$-5y$	$-15xy$	$-10y^2$

$3x^2 - 13xy - 10y^2 = \mathbf{(3x + 2y)(x - 5y)}$

(c) $3x^2 - 12xy + 9y^2$
$= 3(x^2 - 4xy + 3y^2)$

×	x	$-y$
x	x^2	$-xy$
$-3y$	$-3xy$	$+3y^2$

$3(x^2 - 4xy + 3y^2) = \mathbf{3(x - y)(x - 3y)}$

(d) $-2x^2 + 8xy - 8y^2$
$= -2(x^2 - 4xy + 4y^2)$

×	x	$-2y$
x	x^2	$-2xy$
$-2y$	$-2xy$	$+4y^2$

$-2(x^2 - 4xy + 4y^2) = \mathbf{-2(x - 2y)^2}$

11. Betty is wrong. Since the coefficient of x is 3, we should obtain a quadratic expression where the coefficient of x^2 is more than 3 when expanding $(3x + 4)^2$ to find the area.

12. $2(8x^2 - 3) + 3(2x^2 + 5x - 5)$
$= 16x^2 - 6 + 6x^2 + 15x - 15$
$= 22x^2 + 15x - 21$
The total cost of 2 such pens and 3 such pencils is **$\$(22x^2 + 15x - 21)$**.

5 EXPANSION AND FACTORISATION USING SPECIAL ALGEBRAIC IDENTITIES

1. (a) $(x+5)^2 = x^2 + 2(x)(5) + 5^2$
$= \mathbf{x^2 + 10x + 25}$

(b) $(3x+1)^2 = (3x)^2 + 2(3x)(1) + 1^2$
$= \mathbf{9x^2 + 6x + 1}$

(c) $(2+3x)^2 = 2^2 + 2(2)(3x) + (3x)^2$
$= \mathbf{4 + 12x + 9x^2}$

(d) $(7x+9y)^2 = (7x)^2 + 2(7x)(9y) + (9y)^2$
$= \mathbf{49x^2 + 126xy + 81y^2}$

2. (a) $(x-4)^2 = x^2 - 2(x)(4) + 4^2$
$= \mathbf{x^2 - 8x + 16}$

(b) $(2x-5)^2 = (2x)^2 - 2(2x)(5) + 5^2$
$= \mathbf{4x^2 - 20x + 25}$

(c) $(6-x)^2 = 6^2 - 2(6)(x) + (x)^2$
$= \mathbf{36 - 12x + x^2}$

(d) $(x-3y)^2 = x^2 - 2(x)(3y) + (3y)^2$
$= \mathbf{x^2 - 6xy + 9y^2}$

3. (a) $(x+5)(x-5) = x^2 - 5^2$
$= \mathbf{x^2 - 25}$

(b) $(3x-5y)(3x+5y) = (3x)^2 - (5y)^2$
$= \mathbf{9x^2 - 25y^2}$

4. $(x+y)^2 = x^2 + 2xy + y^2 = x^2 + y^2 + 2xy$
$= 1000 + 2(56)$
$= \mathbf{1112}$

5. $m^2 - n^2 = (m+n)(m-n)$
$48 = (m+n)(5)$
$m + n = \frac{48}{5}$
$= 9.6$
$2(m+n)^2 = 2(9.6)^2$
$= \mathbf{184.32}$

6. (a) $102^2 = (100+2)^2 = 100^2 + 2(100)(2) + 2^2$
$= 10\,000 + 400 + 4$
$= \mathbf{10\,404}$

(b) $48^2 = (50-2)^2$
$= 50^2 - 2(50)(2) + 2^2$
$= 2500 - 200 + 4$
$= \mathbf{2304}$

(c) $196 \times 204 = (200-4)(200+4)$
$= 200^2 - 4^2$
$= 40\,000 - 16$
$= \mathbf{39\,984}$

7. (a) $x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$
$= \mathbf{(x+3)^2}$

(b) $9x^2 + 12x + 4 = (3x)^2 + 2(3x)(2) + 2^2$
$= \mathbf{(3x+2)^2}$

(c) $25x^2 + 30xy + 9y^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$
$= \mathbf{(5x+3y)^2}$

8. (a) $x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$
$= \mathbf{(x-3)^2}$

(b) $12x^2 - 12x + 3 = 3(4x^2 - 4x + 1)$
$= 3[(2x)^2 - 2(2x)(1) + 1^2]$
$= \mathbf{3(2x-1)^2}$

(c) $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$
$= \mathbf{(2x-3y)^2}$

9. (a) $4x^2 - 9 = (2x)^2 - 3^2$
$= \mathbf{(2x+3)(2x-3)}$

(b) $x^2 - 25y^2 = x^2 - (5y)^2$
$= \mathbf{(x+5y)(x-5y)}$

10. (a) $75^2 - 25^2 = (75+25)(75-25)$
$= 100 \times 50$
$= \mathbf{5000}$

(b) $105^2 - 25 = 105^2 - 5^2$
$= (105+5)(105-5)$
$= 110 \times 100$
$= \mathbf{11\,000}$

11. (a) Since $2x$ is divisible by 2, $2x$ is an even number.
When adding an even number and an odd number, we will always get an odd number.
Hence $2x + 3$ is an odd number.
Stella is correct.

(b) $2x + 3 - 2 = 2x + 1$
$(2x+1)^2 = (2x)^2 + 2(2x)(1) + 1^2$
$= \mathbf{4x^2 + 4x + 1}$

6 ALGEBRAIC FRACTIONS

1. (a) $\frac{2ab}{4a^2b^2} = \frac{2ab}{4a^2b^2}$
$= \frac{1}{2ab}$

(b) $\frac{3x^2y}{9xy^2} = \frac{3x^2y}{9xy^2}$
$= \frac{x}{3y}$

(c) $\frac{2ab^2}{8a^3b} = \frac{2ab^2}{8a^3b}$
$= \frac{b}{4a^2}$

(d) $\frac{9p^3q}{27pq^4} = \frac{9p^3q}{27pq^4}$
$= \frac{p^2}{3q^3}$

(e) $\frac{8a^2b^3}{(4ab)^2} = \frac{8a^2b^3}{16a^2b^2}$
$= \frac{8a^2b^3}{16a^2b^2}$
$= \frac{b}{2}$

(f) $\frac{(2xy)^2}{8x^2y} = \frac{4x^2y^2}{8x^2y}$
$= \frac{4x^2y^2}{8x^2y}$
$= \frac{y}{2}$

(g) $\frac{15f^2g}{6(fg)^2} = \frac{15f^2g}{6f^2g^2}$
$= \frac{5}{2g}$

(h) $\frac{(3abc)^3}{9a^4bc^2} = \frac{27a^3b^3c^3}{9a^4bc^2}$
$= \frac{27a^3b^3c^3}{9a^4bc^2}$
$= \frac{3b^2c}{a}$

(i) $\frac{8(mn)^3}{(4n)^2m} = \frac{8m^3n^3}{16n^2m}$
$= \frac{m^2n}{2}$

(j) $\frac{10y(x+2)^2}{15(x+2)} = \frac{10y(x+2)^2}{15(x+2)}$
$= \frac{2y(x+2)}{3}$

2. (a) $\frac{-10m^5}{15m^3} = \frac{-10m^5}{15m^3}$
$= -\frac{2m^2}{3}$

(b) $\frac{33y^2}{-11y} = \frac{33y^2}{-11y}$
$= \frac{3y}{-1}$
$= -3y$

(c) $\frac{(-2b)^3}{10b^2} = \frac{-8b^3}{10b^2}$
$= \frac{-8b^3}{10b^2}$
$= -\frac{4b}{5}$

(d) $\frac{(-4xy)^2}{20xy^3} = \frac{16x^2y^2}{20xy^3}$
$= \frac{4x}{5y}$

(e) $\frac{-3hk}{-12h^2k^2} = \frac{-3hk}{-12h^2k^2}$
$= \frac{1}{4hk}$

(f) $\frac{-4mn^2}{-12m^3n} = \frac{-4mn^2}{-12m^3n}$
$= \frac{n}{3m^2}$

(g) $\frac{(-2ab)^3}{-12a^4c} = \frac{-8a^3b^3}{-12a^4c}$
$= \frac{-8a^3b^3}{-12a^4c}$
$= \frac{2b^3}{3ac}$

(h) $\frac{s^2t}{(-2st)^2} = \frac{s^2t}{4s^2t^2}$
$= \frac{s^2t}{4s^2t^2}$
$= \frac{1}{4t}$

(i) $\frac{2m^3n^3}{8m(-n)^2} = \frac{2m^3n^3}{8mn^2}$
$= \frac{m^2n}{4}$

(j) $\frac{5^2xy}{50(-xy)^3} = \frac{25xy}{-50x^3y^3}$
$= -\frac{1}{2x^2y^2}$

5 EXPANSION AND FACTORISATION USING SPECIAL ALGEBRAIC IDENTITIES

1. (a) $(x + 5)^2 = x^2 + 2(x)(5) + 5^2$
$= \mathbf{x^2 + 10x + 25}$

(b) $(3x + 1)^2 = (3x)^2 + 2(3x)(1) + 1^2$
$= \mathbf{9x^2 + 6x + 1}$

(c) $(2 + 3x)^2 = 2^2 + 2(2)(3x) + (3x)^2$
$= \mathbf{4 + 12x + 9x^2}$

(d) $(7x + 9y)^2 = (7x)^2 + 2(7x)(9y) + (9y)^2$
$= \mathbf{49x^2 + 126xy + 81y^2}$

2. (a) $(x - 4)^2 = x^2 - 2(x)(4) + 4^2$
$= \mathbf{x^2 - 8x + 16}$

(b) $(2x - 5)^2 = (2x)^2 - 2(2x)(5) + 5^2$
$= \mathbf{4x^2 - 20x + 25}$

(c) $(6 - x)^2 = 6^2 - 2(6)(x) + (x)^2$
$= \mathbf{36 - 12x + x^2}$

(d) $(x - 3y)^2 = x^2 - 2(x)(3y) + (3y)^2$
$= \mathbf{x^2 - 6xy + 9y^2}$

3. (a) $(x + 5)(x - 5) = x^2 - 5^2$
$= \mathbf{x^2 - 25}$

(b) $(3x - 5y)(3x + 5y) = (3x)^2 - (5y)^2$
$= \mathbf{9x^2 - 25y^2}$

4. $(x + y)^2 = x^2 + 2xy + y^2 = x^2 + y^2 + 2xy$
$= 1000 + 2(56)$
$= \mathbf{1112}$

5. $m^2 - n^2 = (m + n)(m - n)$
$48 = (m + n)(5)$
$m + n = \frac{48}{5}$
$= 9.6$
$2(m + n)^2 = 2(9.6)^2$
$= \mathbf{184.32}$

6. (a) $102^2 = (100 + 2)^2 = 100^2 + 2(100)(2) + 2^2$
$= 10\ 000 + 400 + 4$
$= \mathbf{10\ 404}$

(b) $48^2 = (50 - 2)^2$
$= 50^2 - 2(50)(2) + 2^2$
$= 2500 - 200 + 4$
$= \mathbf{2304}$

(c) $196 \times 204 = (200 - 4)(200 + 4)$
$= 200^2 - 4^2$
$= 40\ 000 - 16$
$= \mathbf{39\ 984}$

7. (a) $x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$
$= \mathbf{(x + 3)^2}$

(b) $9x^2 + 12x + 4 = (3x)^2 + 2(3x)(2) + 2^2$
$= \mathbf{(3x + 2)^2}$

(c) $25x^2 + 30xy + 9y^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$
$= \mathbf{(5x + 3y)^2}$

8. (a) $x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$
$= \mathbf{(x - 3)^2}$

(b) $12x^2 - 12x + 3 = 3(4x^2 - 4x + 1)$
$= 3[(2x)^2 - 2(2x)(1) + 1^2]$
$= \mathbf{3(2x - 1)^2}$

(c) $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$
$= \mathbf{(2x - 3y)^2}$

9. (a) $4x^2 - 9 = (2x)^2 - 3^2$
$= \mathbf{(2x + 3)(2x - 3)}$

(b) $x^2 - 25y^2 = x^2 - (5y)^2$
$= \mathbf{(x + 5y)(x - 5y)}$

10. (a) $75^2 - 25^2 = (75 + 25)(75 - 25)$
$= 100 \times 50$
$= \mathbf{5000}$

(b) $105^2 - 25 = 105^2 - 5^2$
$= (105 + 5)(105 - 5)$
$= 110 \times 100$
$= \mathbf{11\ 000}$

11. (a) Since $2x$ is divisible by 2, $2x$ is an even number.
When adding an even number and an odd number, we will always get an odd number.
Hence $2x + 3$ is an odd number.
Stella is correct.

(b) $2x + 3 - 2 = 2x + 1$
$(2x + 1)^2 = (2x)^2 + 2(2x)(1) + 1^2$
$= \mathbf{4x^2 + 4x + 1}$

6 ALGEBRAIC FRACTIONS

1. (a) $\frac{2ab}{4a^2b^2} = \frac{^1\cancel{2}\cancel{a}b}{_2\cancel{4}a^{\cancel{2}}b^{\cancel{2}}}$
$= \frac{1}{2ab}$

(b) $\frac{3x^2y}{9xy^2} = \frac{^1\cancel{3}x^{\cancel{2}}\cancel{y}}{_3\cancel{9}\cancel{x}y^{\cancel{2}}}$
$= \frac{x}{3y}$

(c) $\frac{2ab^2}{8a^3b} = \frac{^1\cancel{2}\cancel{a}b^{\cancel{2}}}{_4\cancel{8}a^{\cancel{3}2}\cancel{b}}$
$= \frac{b}{4a^2}$

(d) $\frac{9p^3q}{27pq^4} = \frac{^1\cancel{9}p^{\cancel{3}2}\cancel{q}}{_3\cancel{27}\cancel{p}q^{\cancel{4}3}}$
$= \frac{p^2}{3q^3}$

(e) $\frac{8a^2b^3}{(4ab)^2} = \frac{8a^2b^3}{16a^2b^2}$
$= \frac{^1\cancel{8}\cancel{a^2}b^{\cancel{3}}}{_2\cancel{16}\cancel{a^2}\cancel{b^2}}$
$= \frac{b}{2}$

(f) $\frac{(2xy)^2}{8x^2y} = \frac{4x^2y^2}{8x^2y}$
$= \frac{^1\cancel{4}\cancel{x^2}y^{\cancel{2}}}{_2\cancel{8}\cancel{x^2}\cancel{y}}$
$= \frac{y}{2}$

(g) $\frac{15f^2g}{6(fg)^2} = \frac{^5\cancel{15}\cancel{f^2}\cancel{g}}{_2\cancel{6}\cancel{f^2}g^{\cancel{2}}}$
$= \frac{5}{2g}$

(h) $\frac{(3abc)^3}{9a^4bc^2} = \frac{27a^3b^3c^3}{9a^4bc^2}$
$= \frac{^3\cancel{27}\cancel{a^3}b^{\cancel{3}2}c^{\cancel{3}1}}{_1\cancel{9}a^{\cancel{4}}\cancel{b}\cancel{c^2}}$
$= \frac{3b^2c}{a}$

(i) $\frac{8(mn)^3}{(4n)^2m} = \frac{^1\cancel{8}m^{\cancel{3}2}n^{\cancel{3}}}{_2\cancel{16}\cancel{n^2}\cancel{m}}$
$= \frac{m^2n}{2}$

(j) $\frac{10y(x+2)^2}{15(x+2)} = \frac{^2\cancel{10}y(x+2)^{\cancel{2}}}{_3\cancel{15}\cancel{(x+2)}}$
$= \frac{2y(x+2)}{3}$

2. (a) $\frac{-10m^5}{15m^3} = \frac{-^2\cancel{10}m^{\cancel{5}2}}{_3\cancel{15}\cancel{m^3}}$
$= -\frac{2m^2}{3}$

(b) $\frac{33y^2}{-11y} = \frac{^3\cancel{33}y^{\cancel{2}}}{_{-1}\cancel{-11}\cancel{y}}$
$= \frac{3y}{-1}$
$= -3y$

(c) $\frac{(-2b)^3}{10b^2} = \frac{-8b^3}{10b^2}$
$= \frac{-^4\cancel{8}b^{\cancel{3}}}{_5\cancel{10}\cancel{b^2}}$
$= -\frac{4b}{5}$

(d) $\frac{(-4xy)^2}{20xy^3} = \frac{^4\cancel{16}x^{\cancel{2}}\cancel{y^2}}{_5\cancel{20}\cancel{x}y^{\cancel{3}1}}$
$= \frac{4x}{5y}$

(e) $\frac{-3hk}{-12h^2k^2} = \frac{^1\cancel{-3}\cancel{h}\cancel{k}}{_4\cancel{-12}h^{\cancel{2}}k^{\cancel{2}}}$
$= \frac{1}{4hk}$

(f) $\frac{-4mn^2}{-12m^3n} = \frac{^1\cancel{-4}\cancel{m}n^{\cancel{2}}}{_3\cancel{-12}m^{\cancel{3}2}\cancel{n}}$
$= \frac{n}{3m^2}$

(g) $\frac{(-2ab)^3}{-12a^4c} = \frac{-8a^3b^3}{-12a^4c}$
$= \frac{^2\cancel{-8}\cancel{a^3}b^3}{_3\cancel{-12}a^{\cancel{4}}c}$
$= \frac{2b^3}{3ac}$

(h) $\frac{s^2t}{(-2st)^2} = \frac{s^2t}{4s^2t^2}$
$= \frac{\cancel{s^2}\cancel{t}}{4\cancel{s^2}t^{\cancel{2}}}$
$= \frac{1}{4t}$

(i) $\frac{2m^3n^3}{8m(-n)^2} = \frac{^1\cancel{2}m^{\cancel{3}2}n^{\cancel{3}}}{_4\cancel{8}\cancel{m}\cancel{n^2}}$
$= \frac{m^2n}{4}$

(j) $\frac{5^2xy}{50(-xy)^3} = \frac{\cancel{25}\cancel{xy}}{-_2\cancel{50}x^{\cancel{3}2}y^{\cancel{3}2}}$
$= -\frac{1}{2x^2y^2}$

3. (a) $\frac{-7p-7q}{a(p+q)^2} = \frac{-7\cancel{(p+q)}}{a(p+q)^{\cancel{2}}}$
$= -\frac{\mathbf{7}}{\mathbf{a(p+q)}}$

(b) $\frac{m^3+m^2n}{2mn+2n^2} = \frac{m^2\cancel{(m+n)}}{2n\cancel{(m+n)}}$
$= \frac{\mathbf{m^2}}{\mathbf{2n}}$

(c) $\frac{5e-f}{2ef-10e^2} = \frac{5e-f}{2e(f-5e)}$
$= -\frac{\cancel{5e-f}}{2e\cancel{(5e-f)}}$
$= -\frac{\mathbf{1}}{\mathbf{2e}}$

(d) $\frac{(x+y)^2}{x^2-y^2} = \frac{(x+y)^{\cancel{2}}}{(x-y)\cancel{(x+y)}}$
$= \frac{\mathbf{x+y}}{\mathbf{x-y}}$

(e) $\frac{9n-18m}{16m^2-4n^2} = \frac{9(n-2m)}{4(4m^2-n^2)}$
$= \frac{9(n-2m)}{4(2m-n)(2m+n)}$
$= \frac{-9\cancel{(2m-n)}}{4\cancel{(2m-n)}(2m+n)}$
$= -\frac{\mathbf{9}}{\mathbf{4(2m+n)}}$

(f) $\frac{15x^2-2x-8}{25x^2-16} = \frac{\cancel{(5x-4)}(3x+2)}{\cancel{(5x-4)}(5x+4)}$
$= \frac{\mathbf{3x+2}}{\mathbf{5x+4}}$

4. (a) $\frac{2}{3b}\times\frac{b^2}{8} = \frac{\cancel{2}^1}{3\cancel{b}}\times\frac{b^{\cancel{2}}}{{}_4\cancel{8}}$
$= \frac{\mathbf{b}}{\mathbf{12}}$

(b) $\frac{n}{3m^2}\times\frac{6m}{n^3} = \frac{\cancel{n}}{\cancel{3}m^{\cancel{2}}}\times\frac{{}^2\cancel{6}\cancel{m}}{n^{\cancel{3}2}}$
$= \frac{\mathbf{2}}{\mathbf{mn^2}}$

(c) $\frac{5a}{3}\times\frac{12b^2}{15a^2} = \frac{{}^1\cancel{5}\cancel{a}}{\cancel{3}}\times\frac{{}^4\cancel{12}b^2}{{}_3\cancel{15}a^{\cancel{2}}}$
$= \frac{\mathbf{4b^2}}{\mathbf{3a}}$

(d) $\frac{(2y)^3}{7x}\times\frac{21x^2}{36y^2} = \frac{8y^3}{7x}\times\frac{21x^2}{36y^2}$
$= \frac{{}^2\cancel{8}y^{\cancel{3}}}{{}_1\cancel{7}\cancel{x}}\times\frac{{}^{1}\cancel{21}x^{\cancel{2}}}{{}_3\cancel{36}\cancel{y^2}}$
$= \frac{\mathbf{2xy}}{\mathbf{3}}$

(e) $\frac{x^2z}{15y}\times\frac{12y^2}{xz^2} = \frac{{}^1x^{\cancel{2}}\cancel{z}}{{}_5\cancel{15}\cancel{y}}\times\frac{{}^4\cancel{12}y^{\cancel{2}}}{{}_1\cancel{x}z^{\cancel{2}}}$
$= \frac{\mathbf{4xy}}{\mathbf{5z}}$

(f) $\frac{25b^3}{7a^4}\times\frac{21a^3}{10b^2} = \frac{{}^5\cancel{25}b^{\cancel{3}}}{\cancel{7}a^{\cancel{4}1}}\times\frac{{}^3\cancel{21}\cancel{a^3}}{{}_2\cancel{10}\cancel{b^2}}$
$= \frac{\mathbf{15b}}{\mathbf{2a}}$

(g) $\frac{(a+b)^3}{9}\times\frac{3}{a+b} = \frac{(a+b)^{\cancel{3}2}}{{}_3\cancel{9}}\times\frac{\cancel{3}^1}{\cancel{a+b}}$
$= \frac{\mathbf{(a+b)^2}}{\mathbf{3}}$

(h) $\frac{m^2}{(m+n)^2}\times\frac{2(m+n)}{m} = \frac{m^{\cancel{2}}}{(m+n)^{\cancel{2}}}\times\frac{2\cancel{(m+n)}}{\cancel{m}}$
$= \frac{\mathbf{2m}}{\mathbf{m+n}}$

(i) $\frac{2b}{5b^2}\times\frac{10b}{4a^2}\times\frac{a^2}{b^2} = \frac{{}^1\cancel{2}\cancel{b}}{{}_1\cancel{5}\cancel{b^2}}\times\frac{{}^1\cancel{10}\cancel{b}}{{}_2\cancel{4}\cancel{a^2}}\times\frac{\cancel{a^2}}{b^2}$
$= \frac{1}{1}\times\frac{1}{1}\times\frac{1}{b^2}$
$= \frac{\mathbf{1}}{\mathbf{b^2}}$

(j) $\frac{2m}{n^3}\times\frac{3n}{8n^2}\times\frac{4}{9m^5} = \frac{{}^1\cancel{2}\cancel{m}}{n^3}\times\frac{{}^1\cancel{3}\cancel{n}}{{}_4\cancel{8}n^{\cancel{2}}}\times\frac{{}^1\cancel{4}}{{}_3\cancel{9}m^{\cancel{5}4}}$
$= \frac{1}{n^3}\times\frac{1}{n}\times\frac{1}{3m^4}$
$= \frac{\mathbf{1}}{\mathbf{3m^4n^4}}$

(k) $\frac{xy}{5z}\times\frac{4}{3y^3}\times\frac{10z^2}{8x^3} = \frac{\cancel{xy}}{{}_1\cancel{5}\cancel{z}}\times\frac{{}^1\cancel{4}}{3y^{\cancel{3}2}}\times\frac{{}^1\cancel{10}z^{\cancel{2}}}{{}_1\cancel{8}x^{\cancel{3}2}}$
$= \frac{\mathbf{z}}{\mathbf{3x^2y^2}}$

(l) $\frac{x^2}{3y^2}\times\frac{9y}{5z^3}\times\frac{10z^2}{x} = \frac{x^{\cancel{2}}}{{}_1\cancel{3}y^{\cancel{2}}}\times\frac{{}^3\cancel{9}\cancel{y}}{\cancel{5}z^{\cancel{3}}}\times\frac{{}^2\cancel{10}\cancel{z^2}}{\cancel{x}}$
$= \frac{\mathbf{6x}}{\mathbf{yz}}$

(m) $\frac{a^2-b^2}{5c}\times\frac{10c^2}{a^2-ab} = \frac{(a+b)\cancel{(a-b)}}{\cancel{5c}}\times\frac{{}^2\cancel{10}c^{\cancel{2}}}{a\cancel{(a-b)}}$
$= \frac{a+b}{1}\times\frac{2c}{a}$
$= \frac{\mathbf{2c(a+b)}}{\mathbf{a}}$

(n) $\frac{a^2+2a+1}{ac+c}\times\frac{bc^2}{ab^2+b^2} = \frac{\cancel{(a+1)^2}}{\cancel{c}\cancel{(a+1)}}\times\frac{\cancel{b}c^{\cancel{2}}}{b^{\cancel{2}}\cancel{(a+1)}}$
$= \frac{\mathbf{c}}{\mathbf{b}}$

(o) $\frac{3u-2v}{7v^2}\times\frac{2u^3v}{9u-6v}=\frac{\cancel{3u-2v}}{7v^{\cancel{2}}}\times\frac{2u^3\cancel{v}}{3(\cancel{3u-2v})}$

$=\mathbf{\frac{2u^3}{21v}}$

(p) $\frac{x^2+6x+8}{16x-16}\times\frac{4x^2+4x}{2x+8}=\frac{\cancel{(x+4)}(x+2)}{_4\cancel{16}(x-1)}\times\frac{\cancel{4}x(x+1)}{2\cancel{(x+4)}}$

$=\mathbf{\frac{x(x+1)(x+2)}{8(x-1)}}$

5. (a) $\frac{b^2}{5}\div\frac{2}{5}=\frac{b^2}{_1\cancel{5}}\times\frac{\cancel{5}^1}{2}$

$=\mathbf{\frac{b^2}{2}}$

(b) $\frac{3}{4a}\div\frac{3}{2a^2}=\frac{^1\cancel{3}}{_2\cancel{4a}}\times\frac{\cancel{2}a^{\cancel{2}}}{_1\cancel{3}}$

$=\mathbf{\frac{a}{2}}$

(c) $\frac{a^2}{b}\div\frac{a}{b^3}=\frac{a^{\cancel{2}}}{\cancel{b}}\times\frac{b^{\cancel{3}2}}{\cancel{a}}$

$=\frac{ab^2}{1}$

$=\mathbf{ab^2}$

(d) $\frac{y}{4x^2}\div\frac{3y^2}{8x^3}=\frac{\cancel{y}}{_1\cancel{4x^2}}\times\frac{^2\cancel{8}x^{\cancel{3}}}{3y^{\cancel{2}}}$

$=\mathbf{\frac{2x}{3y}}$

(e) $\frac{15n}{3m^3}\div\frac{9n^2}{3m^2}=\frac{^5\cancel{15n}}{_1\cancel{3}m^{\cancel{3}}}\times\frac{^1\cancel{3m^2}}{_3\cancel{9}n^{\cancel{2}}}$

$=\mathbf{\frac{5}{3mn}}$

(f) $\frac{27}{4ab^2}\div\frac{9a}{8b}=\frac{^3\cancel{27}}{_1\cancel{4}ab^{\cancel{2}}}\times\frac{^2\cancel{8b}}{_1\cancel{9}a}$

$=\mathbf{\frac{6}{a^2b}}$

(g) $\frac{(2mn)^2}{5}\div\frac{m^3}{10n}=\frac{4m^2n^2}{5}\div\frac{m^3}{10n}$

$=\frac{4\cancel{m^2}n^2}{_1\cancel{5}}\times\frac{^2\cancel{10}n}{m^{\cancel{3}}}$

$=\mathbf{\frac{8n^3}{m}}$

(h) $\frac{p}{(2q)^3}\div\frac{p^4}{2q^2}=\frac{p}{8q^3}\div\frac{p^4}{2q^2}$

$=\frac{\cancel{p}}{_4\cancel{8}q^{\cancel{3}}}\times\frac{^1\cancel{2q^2}}{p^{\cancel{4}3}}$

$=\mathbf{\frac{1}{4p^3q}}$

(i) $\frac{a^2+6ab+9b^2}{a^2}\div\frac{a+3b}{a}$

$=\frac{a^2+6ab+9b^2}{a^2}\times\frac{a}{a+3b}$

$=\frac{(a+3b)^{\cancel{2}}}{a^{\cancel{2}}}\times\frac{\cancel{a}}{\cancel{a+3b}}$

$=\mathbf{\frac{a+3b}{a}}$

(j) $\frac{3+3x}{x^2}\div\frac{1+2x+x^2}{x^3}=\frac{3+3x}{x^2}\times\frac{x^3}{1+2x+x^2}$

$=\frac{3\cancel{(1+x)}}{\cancel{x^2}}\times\frac{x^{\cancel{3}}}{(x+1)^{\cancel{2}}}$

$=\frac{3}{1}\times\frac{x}{x+1}$

$=\mathbf{\frac{3x}{x+1}}$

(k) $\frac{9a-12b}{3ab}\div(3a-4b)^3$

$=\frac{\cancel{3}\cancel{(3a-4b)}}{\cancel{3}ab}\times\frac{1}{(3a-4b)^{\cancel{3}2}}$

$=\mathbf{\frac{1}{ab(3a-4b)^2}}$

(l) $\frac{5a+5b}{2p^2-18}\div\frac{a^2-b^2}{3p+9}$

$=\frac{5(a+b)}{2(p^2-9)}\times\frac{3p+9}{a^2-b^2}$

$=\frac{5\cancel{(a+b)}}{2(p-3)\cancel{(p+3)}}\times\frac{3\cancel{(p+3)}}{\cancel{(a+b)}(a-b)}$

$=\mathbf{\frac{15}{2(p-3)(a-b)}}$

7 DIRECT AND INVERSE PROPORTIONS

1. Let the number of passengers that 7 school buses can ferry be x.

$\frac{x}{7}=\frac{42}{1}$

$x=294$

7 school buses can ferry **294** passengers.

2. Let the number of houses that 120 men can paint be x.

$\frac{x}{120}=\frac{1}{20}$

$x=6$

120 men can paint **6** houses.

3. Let the number of tubes needed for 156 shuttlecocks be x.
$\frac{x}{156} = \frac{2}{24}$
$24x = 312$
$x = 13$
13 tubes are needed for 156 shuttlecocks.

4. Let the number of days Janice needs to read 12 novels be x.
$\frac{x}{12} = \frac{9}{3}$
$3x = 108$
$x = 36$
Janice needs **36** days to read 12 novels.

5. (a) $y = k(5x - 2)$, where k is a constant.
When $y = 6$, $x = 1$.
$6 = k(5 - 2)$
$3k = 6$
$k = 2$
$\therefore \mathbf{y = 2(5x - 2)}$

(b) When $y = 10$,
$10 = 2(5x - 2)$
$5x - 2 = 5$
$5x = 7$
$x = \mathbf{\frac{7}{5}}$

(c) When $x = -3$,
$y = 2[5(-3) - 2]$
$= 2[-15 - 2]$
$= \mathbf{-34}$

6. (a) $y = kx^2$, where k is a constant.
When $y = 10$, $x = 2$.
$10 = k(2)^2$
$10 = 4k$
$k = \frac{10}{4}$
$k = 2.5$
$\therefore \mathbf{y = 2.5x^2}$

(b) When $y = 22.5$,
$22.5 = 2.5x^2$
$x^2 = \frac{22.5}{2.5}$
$x^2 = 9$
$x = \mathbf{3}$ or $\mathbf{-3}$

(c) When $x = -5$,
$y = 2.5(-5)^2$
$= 2.5(25)$
$= \mathbf{62.5}$

7. (a) $a^2 = k(b + 1)$, where k is a constant.
When $b = 2$, $a = 3$.
$3^2 = k(2 + 1)$
$9 = k(3)$
$k = 3$
$\therefore \mathbf{a^2 = 3(b + 1)}$

(b) When $a = -5$,
$(-5)^2 = 3(b + 1)$
$25 = 3(b + 1)$
$b + 1 = \frac{25}{3}$
$b = \frac{25}{3} - 1$
$= \mathbf{7\frac{1}{3}}$

(c) When $b = 5\frac{3}{4}$,
$a^2 = 3\left(5\frac{3}{4} + 1\right)$
$a^2 = 20\frac{1}{4}$
$a^2 = \frac{81}{4}$
$a = \mathbf{\frac{9}{2}}$ or $\mathbf{-\frac{9}{2}}$

8. (a) $2a = k(3b - 1)$, where k is a constant.
When $b = 2$, $a = 5$.
$2 \times 5 = k(3 \times 2 - 1)$
$10 = k(6 - 1)$
$10 = k(5)$
$5k = 10$
$k = 2$
$\therefore \mathbf{2a = 2(3b - 1)}$

(b) When $a = -9$,
$2(-9) = 2(3b - 1)$
$-18 = 2(3b - 1)$
$3b - 1 = -9$
$3b = -8$
$b = \mathbf{-\frac{8}{3}}$

(c) When $b = \frac{2}{3}$,
$2a = 2\left[3\left(\frac{2}{3}\right) - 1\right]$
$2a = 2[2 - 1]$
$2a = 2$
$a = \mathbf{1}$

9. $\frac{4}{x} = ky$, where k is a constant.

When $y = 4$, $x = \frac{1}{2}$.

$$\frac{4}{\frac{1}{2}} = 4k$$
$$4 \div \frac{1}{2} = 4k$$
$$8 = 4k$$
$$k = 2$$
$$\therefore \frac{4}{x} = 2y$$

(a) When $y = \frac{1}{3}$,

$$\frac{4}{x} = 2\left(\frac{1}{3}\right)$$
$$\frac{4}{x} = \frac{2}{3}$$
$$2x = 12$$
$$x = \mathbf{6}$$

(b) When $x = -5$,

$$\frac{4}{-5} = 2y$$
$$2y = -\frac{4}{5}$$
$$y = \frac{-\frac{4}{5}}{2}$$
$$= -\mathbf{\frac{2}{5}}$$

10. Let the number of days 5 workers take to complete the project be x.

$$5x = 7 \times 10$$
$$= 70$$
$$x = 14$$

5 workers take **14 days** to complete the project.

11. Let the number of days the zoo can feed the lions if the zoo decides to bring in another 2 lions be x.

$$8x = 6 \times 4$$
$$= 24$$
$$x = 3$$

The zoo can feed the lions for **3 days**.

12. (a) $y = \frac{k}{2x}$, where k is a constant.

When $y = -1$, $x = 4$.

$$-1 = \frac{k}{2(4)}$$
$$-1 = \frac{k}{8}$$
$$k = -8$$
$$y = \frac{-8}{2x}$$
$$= \mathbf{\frac{-4}{x}}$$

(b) When $x = 5$,

$$y = -\mathbf{\frac{4}{5}}$$

(c) When $y = 6$,

$$6 = \frac{-4}{x}$$
$$6x = -4$$
$$x = \frac{-4}{6}$$
$$= -\mathbf{\frac{2}{3}}$$

13. (a) $y = \frac{k}{x^2 - 1}$, where k is a constant.

When $x = 2$, $y = 4$.

$$4 = \frac{k}{2^2 - 1}$$
$$4 = \frac{k}{4 - 1}$$
$$4 = \frac{k}{3}$$
$$k = 12$$
$$\therefore \mathbf{y = \frac{12}{x^2 - 1}}$$

(b) When $x = -3$,

$$y = \frac{12}{(-3)^2 - 1}$$
$$= \frac{12}{9 - 1}$$
$$= \frac{12}{8}$$
$$= \mathbf{\frac{3}{2}}$$

(c) When $y = \frac{1}{2}$,

$$\frac{1}{2} = \frac{12}{x^2 - 1}$$
$$x^2 - 1 = 24$$
$$x^2 = 25$$
$$x = \mathbf{\pm 5}$$

14. $\frac{1}{a} = \frac{k}{b + 3}$, where k is a constant.

When $b = 5$, $a = 2$.

$$\frac{1}{2} = \frac{k}{5 + 3}$$
$$\frac{1}{2} = \frac{k}{8}$$
$$2k = 8$$
$$k = 4$$
$$\therefore \frac{1}{a} = \frac{4}{b + 3}$$

(a) When $b = 9$,

$\frac{1}{a} = \frac{4}{9+3}$

$\frac{1}{a} = \frac{4}{12}$

$\frac{1}{a} = \frac{1}{3}$

$a = \mathbf{3}$

(b) When $a = -7$,

$\frac{1}{-7} = \frac{4}{b+3}$

$b + 3 = -28$

$b = \mathbf{-31}$

15. $\sqrt{m} = \frac{k}{5n+3}$, where k is a constant.

When $n = 1$, $m = 9$.

$\sqrt{9} = \frac{k}{5(1)+3}$

$3 = \frac{k}{5+3}$

$3 = \frac{k}{8}$

$k = 24$

$\therefore \sqrt{m} = \frac{24}{5n+3}$

(a) When $n = 3$,

$\sqrt{m} = \frac{24}{5(3)+3}$

$\sqrt{m} = \frac{24}{15+3}$

$\sqrt{m} = \frac{24}{18}$

$\sqrt{m} = \frac{4}{3}$

$m = \left(\frac{4}{3}\right)^2$

$m = \mathbf{\frac{16}{9}}$

(b) When $m = 4$,

$\sqrt{4} = \frac{24}{5n+3}$

$2 = \frac{24}{5n+3}$

$2(5n+3) = 24$

$5n + 3 = 12$

$5n = 9$

$n = \mathbf{\frac{9}{5}}$

16. (a) $2y\left(\frac{3}{6}\right) = 2(3)\left(\frac{3}{9}\right)$

$y = \mathbf{2}$

(b) $2y = \frac{kx}{3}$

When $x = 9$, $y = 3$.

$2(3) = \frac{9k}{3}$

$3k = 6$

$k = 2$

$\therefore 2y = \frac{2x}{3}$

$y = \mathbf{\frac{x}{3}}$

(c) When $y = -2$,

$-2 = \frac{x}{3}$

$x = \mathbf{-6}$

17. (a) Since f is directly proportional to $\sqrt{G}$,

$\boldsymbol{f = k\sqrt{G}}$, where k is a constant.

(b) Given that $f = 24.4$ and $G = 4.5$,

$24.4 = k\sqrt{4.5}$

$k = 11.502$ (5 sig. fig.)

When $f = 56$,

$56 = 11.502\sqrt{G}$

$G = 23.7$ (3 sig. fig.)

The growth of the tail is **23.7 mm**.

(c) Total amount of food fed during the 14 days

$= 14 \times 5$

$= 70$ g

$70 = 11.502\sqrt{G}$

$G = 37.0$ (3 sig. fig.)

The tail will grow **37.0 mm** by the end of the experiment.

18. (a)

No. of watches (w)	17	30	48	63
Amount of money earned (\$$S$)	425	750	1200	1575
$\frac{S}{w}$	25	25	25	25

Since the value of $\frac{S}{w}$ is 25 and is a constant, S and w are directly proportional.

(b) $\frac{S}{w} = 25$

$\boldsymbol{S = 25w}$

(c) When $w = 37$,
$S = 25 \times 37$
$= 925$
The amount of money earned when he sold 37 watches was **$925**.

19. (a)

Lap	1	2	3	4	5
Time taken (t s)	10.6	10.85	11.1	11.35	11.6
Speed (v m/s)	4.717	4.608	4.505	4.405	4.310

(b)

Lap	1	2	3	4	5
Time taken (t s)	10.6	10.85	11.1	11.35	11.6
Speed (v m/s)	4.717	4.608	4.505	4.405	4.31
vt	50	50	50	50	50

The distance of the lap is 50 m and is a constant for the five laps. Hence v and t are inversely proportional.

(c) $vt = 50$
$t = \frac{\mathbf{50}}{v}$

8 POLYGONS AND GEOMETRICAL CONSTRUCTIONS

1. (a) $\frac{180° - 90°}{2} = 45°$ (base ∠s of isos. △)
$x° = 90° - 45°$
$= 45°$
$x = \mathbf{45}$
$y° = 180° - 95° - 45°$ (∠ sum of △)
$= 40°$
$y = \mathbf{40}$

(b) $2x° = 70°$ (opp. ∠s of parallelogram are equal)
$x° = 35°$
$x = \mathbf{35}$
$10z° = 180° - 70°$ (int. ∠s, // lines)
$= 110°$
$z° = 11°$
$z = \mathbf{11}$
$5y° = 180° - 70°$ (int. ∠s, // lines)
$= 110°$
$y° = 22°$
$y = \mathbf{22}$

(c) $\frac{180° - 105°}{2} = 37.5°$ (base ∠s of isos. △)
$180° - 105° = 75°$ (int. ∠s, // lines)
$x° = 75° - 37.5°$
$= 37.5°$
$x = \mathbf{37.5}$
$y° = 180° - x° - 2x°$ (∠ sum of △)
$= 67.5°$
$y = \mathbf{67.5}$

(d) $32° + 28° = 60°$
$3y° = 180° - 60°$ (int. ∠s, // lines)
$= 120°$
$y° = 40°$
$y = \mathbf{40}$
$x° = 180° - 3y$ (int. ∠s, // lines)
$x° = 180° - 120°$
$= 60°$
$x = \mathbf{60}$

(e) $2y° = 40°$
$y° = 20°$
$y = \mathbf{20}$
$x = \mathbf{25}$
$z° = 180° - 40° - 25°$ (∠ sum of △)
$= 115°$
$z = \mathbf{115}$

(f) $x° = \frac{90°}{2}$ (diagonal bisects int. ∠)
$x° = 45°$
$x = \mathbf{45}$
$y° = 180° - 45°$ (∠ on a str. line)
$= 135°$
$y = \mathbf{135}$

2. (a) Refer to Appendix 13 (page 198).

(b) The shortest side is KM.
Length of KM = **6.3 cm**

3. (a) Refer to Appendix 14 (page 199).

(b) $\angle ACB = \mathbf{90°}$

(c) Right-angled triangle

4. (a) Refer to Appendix 15 (page 199).

(b) (i) 6.5 cm

(ii) $\angle PRQ = \mathbf{33°}$

5. (a) Refer to Appendix 16 (page 200).

(b) 90°

(c) Rectangle

6. (a) Refer to Appendix 17 (page 200).

(b) Rhombus

7. (a) Pentagon = 5 sides
Sum of interior angles
$= 180^\circ \times 5 - 360^\circ$
$= \mathbf{540^\circ}$
Sum of exterior angles
$= \mathbf{360^\circ}$
One interior angle
$= \frac{540^\circ}{5}$
$= \mathbf{108^\circ}$
One exterior angle
$= \frac{360^\circ}{5}$
$= \mathbf{72^\circ}$

(b) Octagon = 8 sides
Sum of interior angles
$= 180^\circ \times 8 - 360^\circ$
$= \mathbf{1080^\circ}$
Sum of exterior angles
$= \mathbf{360^\circ}$
One interior angle
$= \frac{1080^\circ}{8}$
$= \mathbf{135^\circ}$
One exterior angle
$= \frac{360^\circ}{8}$
$= \mathbf{45^\circ}$

(c) Decagon = 10 sides
Sum of interior angles
$= 180^\circ \times 10 - 360^\circ$
$= \mathbf{1440^\circ}$
Sum of exterior angles
$= \mathbf{360^\circ}$
One interior angle
$= \frac{1440^\circ}{10}$
$= \mathbf{144^\circ}$
One exterior angle
$= \frac{360^\circ}{10}$
$= \mathbf{36^\circ}$

8. Let x° be the exterior angle.
Interior angle $= 9x^\circ$
$x + 9x = 180$
$10x = 180$
$x = 18$
Number of sides $= \frac{360^\circ}{18^\circ}$
$= 20$
$\therefore n = \mathbf{20}$
Sum of interior angles
$= 180^\circ \times 20 - 360^\circ$
$= \mathbf{3240^\circ}$

9. Let x° be the exterior angle.
Interior angle $= 13x^\circ$
$x + 13x = 180$
$14x = 180$
$x = 12\frac{6}{7}$
Number of sides $= \frac{360^\circ}{12\frac{6}{7}^\circ}$
$= 28$
$\therefore n = \mathbf{28}$
Sum of interior angles
$= 180^\circ \times 28 - 360^\circ$
$= \mathbf{4680^\circ}$

10. Sum of interior angles
$= 100^\circ + 110^\circ + 114^\circ + (12 - 3) \times 2x^\circ$
$= 324^\circ + 18x^\circ$
Sum of interior angles
$= 180^\circ \times 12 - 360^\circ$
$= 1800^\circ$
$324^\circ + 18x^\circ = 1800^\circ$
$18x^\circ = 1476^\circ$
$x^\circ = 82^\circ$
$x = \mathbf{82}$

11. Sum of interior angles
$= 2x^\circ + (2x^\circ + 30^\circ) + (x^\circ + 16^\circ) + (3x^\circ - 20^\circ)$
$+ (10 - 4) \times 105^\circ$
$= 8x^\circ + 656^\circ$
Sum of interior angles
$= 180^\circ \times 10 - 360^\circ$
$= 1440^\circ$
$8x^\circ + 656^\circ = 1440^\circ$
$8x^\circ = 784^\circ$
$x^\circ = 98^\circ$
$x = \mathbf{98}$

12. $(n-3)\times 108° + 396° = 180° \times n - 360°$
$108n - 324 + 396 = 180n - 360$
$-72n = -432$
$n = \mathbf{6}$

13. $(n-1)\times 129° + 126° = 180° \times n - 360°$
$129n - 129 + 126 = 180n - 360$
$-51n = -357$
$n = \mathbf{7}$

14. (a) $\angle HGF = \frac{180° \times 5 - 360°}{5}$
$= \mathbf{108°}$

(b) $\angle GFH = \frac{180° - 108°}{2}$ (base $\angle$s of isos. $\triangle$)
$= \mathbf{36°}$

(c) Obtuse $\angle ABC = 360° - 108° - 108°$ ($\angle$ at a pt.)
$= \mathbf{144°}$

15. (a) $\angle CDE = \frac{180° \times 6 - 360°}{6}$
$= \mathbf{120°}$

(b) $\angle KJL = \frac{180° - 120°}{2}$ (base $\angle$s of isos. $\triangle$)
$= \mathbf{30°}$

(c) Reflex $\angle JOF = 360° - 120°$
$= \mathbf{240°}$

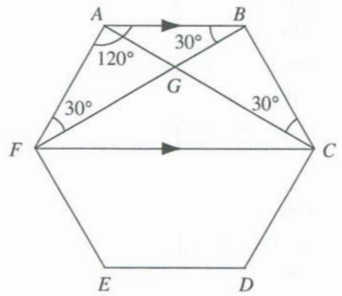
16. (a) (i) $\angle UQR + \angle PQR = 180°$ (adj. $\angle$s on a st. line)
$18° + \angle PQR = 180°$
$\angle PQR = 180° - 18°$
$= 162°$
$\angle QPR = \frac{180° - 162°}{2}$ (base $\angle$ of isos. $\triangle$)
$= \mathbf{9°}$

(ii) $\angle QRS = \angle RST = \angle PQR = 162°$
$\angle RQT + 162° = 180°$ (int. $\angle$s, $RS // QT$)
$\angle RQT = 180° - 162°$
$= \mathbf{18°}$

(iii) $\angle PQW = \angle PQR - \angle RQT$
$= 162° - 18°$
$= 144°$
$\angle QWP + 144° + 9° = 180°$ ($\angle$ sum of $\triangle$)
$\angle QWP + 153° = 180°$
$\angle QWP = 180° - 153°$
$= 27°$
$\angle RWT = \angle QWP$ (vert. opp. $\angle$s)
$= \mathbf{27°}$

(b) Size of each exterior angle $= 18°$
Number of sides of the polygon
$= \frac{360°}{18°}$
$= \mathbf{20}$

17. $\angle FAB = \frac{180° \times 6 - 360°}{6}$
$= 120°$
$\angle AFB = \frac{180° - 120°}{2}$ (base $\angle$ of isos. $\triangle$)
$= 30°$



$\angle ABC + \angle BCF = 180°$ (int. $\angle$s, $AB // FC$)
$120° + \angle BCF = 180°$
$\angle BCF = 180° - 120°$
$= 60°$
$\angle ACF = 60° - 30°$
$= 30°$
$\angle BAC = \angle ACF$ (alt. $\angle$s, $AB // FC$)
$= 30°$
$\angle AGB + 30° + 30° = 180°$ ($\angle$ sum of $\triangle$)
$\angle AGB = 180° - 30° - 30°$
$= 120°$
$\angle FGC = \angle AGB$ (vert. opp. $\angle$s)
$= \mathbf{120°\ (proven)}$

9 CONGRUENCE AND SIMILARITY

1. (a) $\angle CAB = 180° - 60° - 70°$ ($\angle$ sum of $\triangle$)
$= 50°$
$\angle DFE = 180° - 60° - 50°$ ($\angle$ sum of $\triangle$)
$= 70°$
$\angle CAB = \angle FDE = 50°$
$\angle ABC = \angle DEF = 60°$
$\angle ACB = \angle DFE = 70°$
$AB = DE = 7.4$ cm
$AC = DF = 6.8$ cm
$BC = EF = 6$ cm
Hence $\triangle ABC$ is **congruent** to $\triangle DEF$.

(b) $\angle QPR = 180° - 70° - 40°$ ($\angle$ sum of $\triangle$)
$= 70°$
$\angle TUS = 180° - 60° - 70°$ ($\angle$ sum of $\triangle$)
$= 50°$
Since not all the corresponding angles are equal, $\triangle PQR$ is **not congruent** to $\triangle STU$.

2. (a) $\angle PQM = \angle FEH$
$s° = 112°$
$s = \mathbf{112}$
$\angle NMQ = \angle GHE$
$t° = 89°$
$t = \mathbf{89}$
$PN = FG$
$w = \mathbf{6}$
$QP = EF$
$x = \mathbf{3}$
$MQ = HE$
$y = \mathbf{4}$
$NM = GH$
$z = \mathbf{7}$

3. (a) $AC = PR$
$x = \mathbf{10}$
$AB = PQ$
$y = \mathbf{7.5}$

$\angle ACB = \angle PRQ$
$= 180° - 95° - 37°$ ($\angle$ sum of $\triangle$)
$= 48°$
$z = \mathbf{48}$

(b) $\angle CAB = \angle RPQ$
$= 90°$
$\angle ACB = \angle PRQ$
$= 180° - 90° - 26°$ ($\angle$ sum of $\triangle$)
$= 64°$
$x = \mathbf{64}$

$\angle CBA = \angle RQP$
$= 26°$
$y = \mathbf{26}$

$BC = QR$
$z = \mathbf{15}$

4. (a) $\angle BAC = 180° - 100° - 45°$ ($\angle$ sum of $\triangle$)
$= 35°$
$\angle DEF = 180° - 35° - 45°$ ($\angle$ sum of $\triangle$)
$= 100°$
$\angle BAC = \angle EDF = 35°$
$\angle ABC = \angle DEF = 100°$
$\angle ACB = \angle DFE = 45°$

$\frac{AB}{DE} = \frac{9.9}{19.8} = \frac{1}{2}$
$\frac{AC}{DF} = \frac{13.7}{27.4} = \frac{1}{2}$
$\frac{BC}{EF} = \frac{8}{16} = \frac{1}{2}$
Hence $\triangle ABC$ is **similar** to $\triangle DEF$.

(b) $\angle QRP = 180° - 90° - 51°$ ($\angle$ sum of $\triangle$)
$= 39°$
$\angle TUS = 180° - 90° - 38°$ ($\angle$ sum of $\triangle$)
$= 52°$
Since not all the corresponding angles are equal, $\triangle PQR$ is **not similar** to $\triangle STU$.

5. (a) $x = \mathbf{67.4}$
$y = \mathbf{22.6}$
$\frac{z}{5} = \frac{6}{12}$
$z = \mathbf{2.5}$

(b) $x° = 180° - 105° - 53.6°$ ($\angle$ sum of $\triangle$)
$= 21.4°$
$x = \mathbf{21.4}$
$y = \mathbf{53.6}$
$\frac{z}{7.5} = \frac{6}{9}$
$z = \mathbf{5}$

(c) $x° = 180° - 75° - 24°$ ($\angle$ sum of $\triangle$)
$= 81°$
$x = \mathbf{81}$
$y = \mathbf{75}$
$\frac{z}{12.8} = \frac{1.85}{7.4}$
$z = \mathbf{3.2}$

(d) $x° = 180° - 104° - 31°$ ($\angle$ sum of $\triangle$)
$= 45°$
$x = \mathbf{45}$
$y = \mathbf{31}$
$\frac{z}{8.48} = \frac{5.4}{4.5}$
$z = \mathbf{10.176}$

(e) $x = \mathbf{63}$
$y° = 180° - 90° - 63°$ ($\angle$ sum of $\triangle$)
$= 27°$
$y = \mathbf{27}$
$\frac{z}{5} = \frac{4.5}{3}$
$z = \mathbf{7.5}$

(f) $x = \mathbf{62}$
$y° = 180° - 58° - 62°$ (∠ sum of △)
$= 60°$
$y = \mathbf{60}$
$\frac{z}{6.73} = \frac{2.75}{6.6}$
$z = \mathbf{2.80}$

6. (a) $x = \mathbf{110}$
$y = \mathbf{70}$
$\frac{z}{6.2} = \frac{8}{4}$
$z = \mathbf{12.4}$

(b) $x = \mathbf{49}$
$y = \mathbf{131}$
$\frac{z}{11} = \frac{4.4}{9.68}$
$z = \mathbf{5}$

(c) $x = \mathbf{50}$
$y° = 180° - 50°$ (int. ∠s)
$= 130°$
$y = \mathbf{130}$
$\frac{z}{30} = \frac{8}{12}$
$z = \mathbf{20}$

(d) $x = \mathbf{118}$
$y = \mathbf{78}$
$\frac{z}{15} = \frac{8}{12}$
$z = \mathbf{10}$

7. (a) $\triangle ABC \equiv \triangle EDC$
Length AC = Length EC
$AC = \mathbf{2.9\ cm}$

(b) $\angle ABC = 180° - 90° - 60°$ (∠ sum of △)
$= \mathbf{30°}$

(c) Line AB is parallel to line DE.

8. $\triangle ABC \equiv \triangle EDF$
$\angle ABC = \angle EDF$
$= 120°$
$x = \mathbf{120}$

Since $ED = DF$, $\triangle EDF$ is isosceles.
$\angle DEF = \frac{180° - 120°}{2}$ (base ∠s of isos. △)
$= \frac{60°}{2}$
$= 30°$
$y = \mathbf{30}$

$\angle CAB = \angle FED$
$= 30°$
$z = \mathbf{30}$

9. $\triangle ABC \equiv \triangle ADE$
$\angle BAE = \angle EAC = \angle DAC = \frac{150°}{3} = 50°$

$\angle ACB = \angle AED$
$x = 12°$
$x = \mathbf{12}$

$\angle ADE = 180° - 50° - 50° - 12°$ (∠ sum of △)
$= 68°$
$y = \mathbf{68}$

10. $\triangle BAC \equiv \triangle ABD$
$\angle ABD = \angle BAC$
$= 130°$
$\angle CAD = 130° - 20°$
$= 110°$
$x = \mathbf{110}$

$\angle ADB = 180° - 20° - 130°$ (∠ sum of △)
$= 30°$

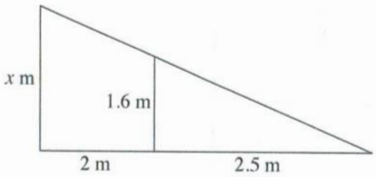
$\angle BCA = \angle ADB$
$= 30°$
$y = \mathbf{30}$

11. $\triangle ABC \equiv \triangle EBD$
$\angle BAD = 180° - 90° - 50°$ (∠ sum of △)
$= 40°$
$y = \mathbf{40}$

$\angle BCA = \angle BDE$
$x = \mathbf{50}$

12. $x = \mathbf{89}$
$\frac{y}{9} = \frac{16}{20}$
$y = \mathbf{7.2}$

13. Using similar triangles, let the height of lamppost be x metres.



$\frac{x}{1.6} = \frac{2+2.5}{2.5}$

$x = 2.88$

The height of the lamppost is **2.88 m**.

14. 1 : 250 000
1 cm : 250 000 cm
1 cm : 2.5 km
2.5 km $\longrightarrow$ 1 cm
1 km $\longrightarrow \frac{1}{2.5}$ cm
145 km $\longrightarrow \frac{1}{2.5} \times 145$
$= 58$ cm
The map distance that he has covered is **58 cm**.

15. 1 : 50 000
1 cm : 50 000 cm
1 cm : 0.5 km
1 cm $\longrightarrow$ 0.5 km
3 cm $\longrightarrow 0.5 \times 3$
$= 1.5$ km
The actual distance between the two cliffs is **1.5 km**.

16. 1 : 150 000
1 cm : 150 000 cm
1 cm : 1.5 km
1 cm $\longrightarrow$ 1.5 km
5.5 cm $\longrightarrow 1.5 \times 5.5$
$= 8.25$ km
The actual distance she covered was **8.25 km**.

17. 3 cm : 45 km
3 cm : 45 000 m
3 cm : 4 500 000 cm
3 : 4 500 000
1 : 1 500 000
The scale of the map is **1 : 1 500 000**.

18. 1 : 250 000
1 cm : 250 000 cm
1 cm : 2.5 km
$(1 \text{ cm})^2 : (2.5 \text{ km})^2$
$1 \text{ cm}^2 : 6.25 \text{ km}^2$
$2.3 \text{ cm}^2 \longrightarrow 6.25 \times 2.3$
$= 14.375 \text{ km}^2$
The ground area of the nature reserve is **14.375 km²**.

19. 1 : 60 000
1 cm : 60 000 cm
1 cm : 0.6 km
$(1 \text{ cm})^2 : (0.6 \text{ km})^2$
$1 \text{ cm}^2 : 0.36 \text{ km}^2$
$4.4 \text{ cm}^2 \longrightarrow 0.36 \times 4.4$
$= 1.584 \text{ km}^2$
The actual area of the school is **1.584 km²**.

20. $12 \text{ cm}^2 : 432 \text{ km}^2$
$1 \text{ cm}^2 : 36 \text{ km}^2$
$\sqrt{1 \text{ cm}^2} : \sqrt{36 \text{ km}^2}$
1 cm : 6 km
1 cm : 6000 m
1 cm : 600 000 cm
1 : 600 000
The value of n is **600 000**.

21. (a) 1 : 750 000
1 cm : 750 000 cm
1 cm : 7.5 km
7.5 km $\longrightarrow$ 1 cm
1 km $\longrightarrow \frac{1}{7.5}$ cm
510 km $\longrightarrow \frac{1}{7.5} \times 510$
$= 68$ cm
The distance between the two cities on the map is **68 cm**.

(b) 1 cm : 7.5 km
$(1 \text{ cm})^2 : (7.5 \text{ km})^2$
$1 \text{ cm}^2 : 56.25 \text{ km}^2$
$56.25 \text{ km}^2 \longrightarrow 1 \text{ cm}^2$
$1 \text{ km}^2 \longrightarrow \frac{1}{56.25} \text{ cm}^2$
$2.25 \text{ km}^2 \longrightarrow \frac{1}{56.25} \times 2.25$
$= 0.04 \text{ cm}^2$
The area of the reservoir on the map is **0.04 cm²**.

(c) 40.8 cm : 510 km
40.8 cm : 510 000 m
40.8 cm : 51 000 000 cm
1 cm : 1 250 000 cm
1 : 1 250 000
The value of n is **1 250 000**.

(d) 1 : 1 250 000
1 cm : 12.5 km
1 cm² : 156.25 km²
156.25 km² ⟶ 1 cm²
1 km² ⟶ $\frac{1}{156.25}$ cm²
2.25 km² ⟶ $\frac{1}{156.25} \times 2.25$
$= \mathbf{0.0144\ cm^2}$
The area of the reservoir on the new map is **0.0144 cm²**.

10 PYTHAGORAS' THEOREM

1. (a) $x^2 = 4^2 + 3^2$
$= 25$
$x = \sqrt{25}$
$= \mathbf{5}$

(b) $x^2 = 24^2 + 7^2$
$= 625$
$x = \sqrt{625}$
$= \mathbf{25}$

(c) $x^2 + 6^2 = 10^2$
$x^2 = 10^2 - 6^2$
$= 64$
$x = \sqrt{64}$
$= \mathbf{8}$

(d) $x^2 + 9^2 = 17^2$
$x^2 = 17^2 - 9^2$
$= 208$
$x = \sqrt{208}$
$= \mathbf{14.42}$ (2 d.p.)

(e) $16^2 + x^2 = 25^2$
$x^2 = 25^2 - 16^2$
$= 369$
$x = \sqrt{369}$
$= \mathbf{19.21}$ (2 d.p.)

(f) $x^2 = 30^2 + 5^2$
$= 925$
$x = \sqrt{925}$
$= \mathbf{30.41}$ (2 d.p.)

(g) $19^2 + x^2 = 34^2$
$x^2 = 34^2 - 19^2$
$= 795$
$x = \sqrt{795}$
$= \mathbf{28.20}$ (2 d.p.)

(h) $x^2 = 17^2 + 17^2$
$= 578$
$x = \sqrt{578}$
$= \mathbf{24.04}$ (2 d.p.)

(i) $10^2 + x^2 = 32^2$
$x^2 = 32^2 - 10^2$
$= 924$
$x = \sqrt{924}$
$= \mathbf{30.40}$ (2 d.p.)

(j) $19^2 + x^2 = 29^2$
$x^2 = 29^2 - 19^2$
$= 480$
$x = \sqrt{480}$
$= \mathbf{21.91}$ (2 d.p.)

2. (a) $AC^2 = 38^2$
$= 1444$
$AB^2 + BC^2 = 18^2 + 20^2$
$= 724$
Since $AC^2 \neq AB^2 + BC^2$, the triangle is **not a right-angled triangle**.

(b) $AC^2 = 40^2$
$= 1600$
$AB^2 + BC^2 = 32^2 + 24^2$
$= 1600$
Since $AC^2 = AB^2 + BC^2$, the triangle is **a right-angled triangle**.

(c) $AC^2 = 37.5^2$
$= 1406.25$
$BC^2 + AB^2 = 36^2 + 10.5^2$
$= 1406.25$
Since $AC^2 = BC^2 + AB^2$, the triangle is **a right-angled triangle**.

(d) $AC^2 = 27^2$
$= 729$
$AB^2 + BC^2 = 9^2 + 9^2$
$= 162$
Since $AC^2 \neq AB^2 + BC^2$, the triangle is **not a right-angled triangle**.

(e) $BC^2 = 25^2$
$= 625$
$AB^2 + AC^2 = 19^2 + 18^2$
$= 685$
Since $BC^2 \neq AB^2 + AC^2$, the triangle is **not a right-angled triangle**.

(f) $BC^2 = 12^2$
$= 144$
$AB^2 + AC^2 = 9.6^2 + 7.2^2$
$= 144$
Since $BC^2 = AB^2 + AC^2$, the triangle is **a right-angled triangle**.

(g) $BC^2 = 40^2$
$= 1600$
$AB^2 + AC^2 = 27^2 + 27^2$
$= 1458$
Since $BC^2 \neq AB^2 + AC^2$, the triangle is **not a right-angled triangle**.

(h) $BC^2 = 32.5^2$
$= 1056.25$
$AC^2 + AB^2 = 30^2 + 12.5^2$
$= 1056.25$
Since $BC^2 = AC^2 + AB^2$, the triangle is **a right-angled triangle**.

3. (a) $y^2 = 8^2 + 6^2$
$= 100$
$y = \sqrt{100}$
$= \mathbf{10}$

$y^2 + y^2 = x^2$
$10^2 + 10^2 = x^2$
$x^2 = 100 + 100$
$= 200$
$x = \sqrt{200}$
$= \mathbf{14.14}$ (2 d.p.)

(b) $x^2 = 7^2 + 5^2$
$= 74$
$x = \sqrt{74}$
$= \mathbf{8.60}$ (2 d.p.)

$y^2 = 7^2 + 10^2$
$= 149$
$y = \sqrt{149}$
$= \mathbf{12.21}$ (2 d.p.)

(c) $x^2 + 4^2 = 15^2$
$x^2 = 15^2 - 4^2$
$= 209$
$x = \sqrt{209}$
$= \mathbf{14.46}$ (2 d.p.)

$x^2 + y^2 = 22^2$
$209 + y^2 = 484$
$y^2 = 484 - 209$
$= 275$
$y = \sqrt{275}$
$= \mathbf{16.58}$ (2 d.p.)

(d) $x^2 = 17^2 + 15^2$
$= 514$
$x = \sqrt{514}$
$= \mathbf{22.67}$ (2 d.p.)
$x^2 + 20^2 = y^2$
$514 + 400 = y^2$
$y^2 = 914$
$y = \sqrt{914}$
$= \mathbf{30.23}$ (2 d.p.)

(e) $x^2 = 10^2 + 15^2$
$= 325$
$x = \sqrt{325}$
$= \mathbf{18.03}$ (2 d.p.)

$y^2 + y^2 = x^2$
$2y^2 = 325$
$= 162.5$
$y = \sqrt{162.5}$
$= \mathbf{12.75}$ (2 d.p.)

(f) $(2y)^2 = 24^2 + 7^2$
$= 625$
$2y = \sqrt{625}$
$= 25$
$y = \frac{25}{2}$
$= \mathbf{12.5}$

$$y^2 + x^2 = 36^2$$
$$12.5^2 + x^2 = 36^2$$
$$x^2 = 1296 - 156.25$$
$$= 1139.75$$
$$x = \sqrt{1139.75}$$
$$= \mathbf{33.76} \text{ (2 d.p.)}$$

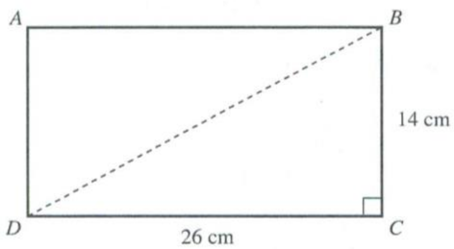
(g) $x^2 = 4^2 + 3^2$
$= 25$
$x = \sqrt{25}$
$= \mathbf{5}$

$x^2 + 12^2 = y^2$
$5^2 + 12^2 = y^2$
$y^2 = 25 + 144$
$= 169$
$y = \sqrt{169}$
$= \mathbf{13}$

(h) $x^2 = 12^2 + 10^2$
$= 244$
$x = \sqrt{244}$
$= \mathbf{15.62}$ (2 d.p.)

$y^2 = (12 + 3)^2 + 10^2$
$= 225 + 100$
$= 325$
$y = \sqrt{325}$
$= \mathbf{18.03}$ (2 d.p.)

4. Let the rectangle be $ABCD$.



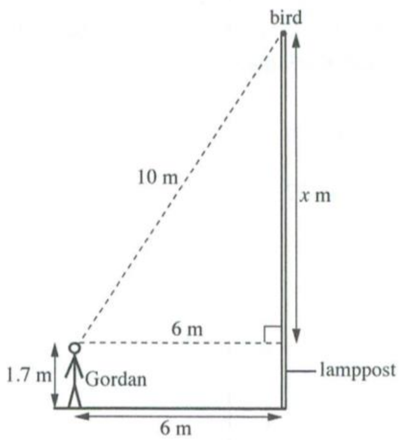
$BD^2 = BC^2 + CD^2$
$= 14^2 + 26^2$
$= 196 + 676$
$= 872$
$BD = \sqrt{872}$
$= 29.5$ cm (3 sig. fig.)
The length of the diagonal is **29.5 cm**.

5. (a) $KM^2 = 32.5^2$
$= 1056.25$
$KN^2 + NM^2 = 30^2 + 12.5^2$
$= 900 + 156.25$
$= 1056.25$
$= KM^2$
Since $KM^2 = KN^2 + NM^2$, $\triangle KMN$ is a right-angled triangle. Hence $\angle \mathbf{KNM = 90°}$. (shown)

(b) (i) Since $\angle KNM = 90°$,
$KN^2 + NL^2 = KL^2$
$30^2 + NL^2 = 36.34^2$
$NL^2 = 1320.5956 - 900$
$= 420.5956$
$NL = \sqrt{420.5956}$
≈ 20.5084 cm
Area of shaded region
= Area of $\triangle KNL$ – area of $\triangle KNM$
$= \left(\frac{1}{2} \times 30 \times 20.5084\right) - \left(\frac{1}{2} \times 30 \times 12.5\right)$
$= 307.626 - 187.5$
$= 120.126$
$= \mathbf{120.13}$ **cm²** (2 d.p.)

(ii) $ML = NL - NM$
$= 20.5084 - 12.5$
$= 8.0084$ cm
Perimeter of the shaded region
$= 32.5 + 36.34 + 8.0084$
$= 76.8484$
$= \mathbf{76.85}$ **cm** (2 d.p.)

6.



$x^2 + 6^2 = 10^2$
$x^2 + 36 = 100$
$x^2 = 100 - 36$
$= 64$
$x = \sqrt{64}$
$= 8$
Height of lamppost $= 8 + 1.7$
$=$ **9.7 m**

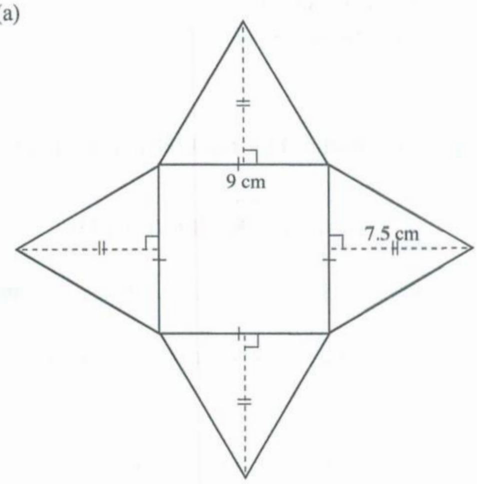
7. Let the height of the pole be h metres.
$h^2 = 0.6^2 + (h - 0.1)^2$
$= 0.36 + h^2 - 0.2h + 0.01$
$0.2h = 0.36 + 0.01$
$= 0.37$
$h = \frac{0.37}{0.2}$
$= 1.85$
The height of the pole is **1.85 m**.

11 VOLUME AND SURFACE AREA OF PYRAMIDS, CONES AND SPHERES

1. Volume $= \frac{1}{3} \times 108 \times 28$
$=$ **1008 cm³**

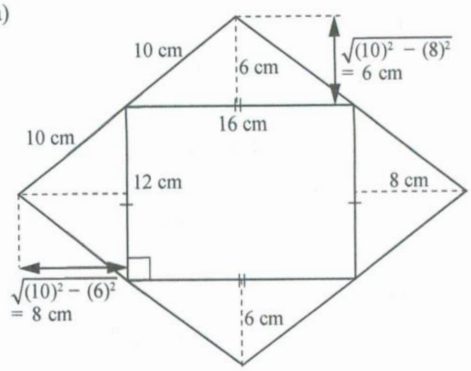
2. Volume $= \frac{1}{3} \times 12 \times 8 \times 10$
$=$ **320 cm³**

3. (a)



(b) Total surface area
$= (9 \times 9) + 4(\frac{1}{2} \times 9 \times 7.5)$
$=$ **216 cm²**

4. (a)



(b) Total surface area
$= (12 \times 16) + 2(\frac{1}{2} \times 16 \times 6) + 2(\frac{1}{2} \times 12 \times 8)$
$=$ **384 cm²**

5. Let the length of the square base be x cm.
$\frac{1}{3} \times x^2 \times 8 = 170\frac{2}{3}$
$x^2 = 64$
$x = 8$
The length of the square base is **8 cm**.

6. (a) Volume $= \frac{1}{3} \times \pi \times 6^2 \times 8$
$=$ **302 cm³** (3 sig. fig.)

(b) Volume $= \frac{1}{3} \times \pi \times 24^2 \times 7$
$=$ **4220 cm³** (3 sig. fig.)

7. (a) Surface area $= \pi \times 5^2 + \pi \times 5 \times 13$
$=$ **283 cm²** (3 sig. fig.)

(b) Surface area $= \pi \times 22^2 + \pi \times 22 \times 43.9$
$=$ **4550 cm²** (3 sig. fig.)

8. $\frac{1}{3} \times \pi \times r^2 \times 27 = 3118.5$
$r = 10.5$ (3 sig. fig.)
The base radius is **10.5 cm**.

9. $\pi \times r \times 18 = 435.6$
$r = 7.70$ (3 sig. fig.)
The base radius is **7.70 cm**.

10. (a) Volume $= \frac{4}{3} \times \pi \times 8^3$
$=$ **2140 cm³** (3 sig. fig.)
Surface area $= 4 \times \pi \times 8^2$
$=$ **804 cm²** (3 sig. fig.)

(b) Radius $= \frac{25}{2}$
$= 12.5$ cm
Volume $= \frac{4}{3} \times \pi \times 12.5^3$
$=$ **8180 cm³** (3 sig. fig.)
Surface area $= 4 \times \pi \times 12.5^2$
$=$ **1960 cm²** (3 sig. fig.)

11. (a) Volume $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3$
$=$ **718 cm³** (3 sig. fig.)
Surface area $= \pi \times 7^2 + \frac{1}{2} \times 4 \times \pi \times 7^2$
$=$ **462 cm²** (3 sig. fig.)

(b) Radius $= \frac{20}{2}$
$= 10$ cm
Volume $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 10^3$
$=$ **2090 cm³** (3 sig. fig.)
Surface area $= \pi \times 10^2 + \frac{1}{2} \times 4 \times \pi \times 10^2$
$=$ **942 cm²** (3 sig. fig.)

12. $\frac{4}{3} \times \pi \times r^3 = 36\pi$
$r^3 = 27$
$r = 3$
The radius of the sphere is **3 cm**.

13. $4 \times \pi \times r^2 = 200$
$r = 3.99$ (3 sig. fig.)
The radius of the sphere is **3.99 cm**.

14. (a) Volume of solid
= Volume of cylinder + Volume of hemisphere
$= \pi \times 7^2 \times 15 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3$
$=$ **3030 cm³** (3 sig. fig.)

(b) Surface area of solid
= Curved surface area of cylinder + Area of circle + Curved surface area of hemisphere
$= 2 \times \pi \times 7 \times 15 + \pi \times 7^2 + \frac{1}{2} \times 4 \times \pi \times 7^2$
$=$ **1120 cm²** (3 sig. fig.)

15. (a) Volume of solid
= Volume of cuboid + Volume of pyramid
$= 12 \times 8 \times 9 + \frac{1}{3} \times 12 \times 8 \times 6$
$=$ **1056 cm³**

(b) Surface area of solid
= Area of 5 sides of cuboid + Area of 4 triangular faces
$= 2 \times (12 \times 9) + 2 \times (8 \times 9) + (12 \times 8) + 2 \times (\frac{1}{2} \times 12 \times 7.2) + 2 \times (\frac{1}{2} \times 8 \times 10)$
$=$ **622.4 cm²** (3 sig. fig.)

16. Volume of ice cream in cone A
= Volume of cone + Volume of hemisphere
$= \frac{1}{3} \times \pi \times \left(\frac{6}{2}\right)^2 \times (14 - 3) + \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$
$= 160$ cm³ (3 sig. fig.)
Volume of ice cream in cone B
= Volume of cone + Volume of hemisphere
$= \frac{1}{3} \times \pi \times \left(\frac{4}{2}\right)^2 \times (18 - 2) + \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{4}{2}\right)^3$
$= 83.8$ cm³ (3 sig. fig.)
There is more ice cream in cone A than cone B. Hence **cone A** is a better buy if each cone costs the same price.

17. Volume of cube $= 30 \times 30 \times 30$
$= 27\ 000$ cm³
Volume of the bowl
= Volume of big hemisphere – Volume of small hemisphere
$= \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{30}{2}\right)^3 - \frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{28}{2}\right)^3$
$= 1321.6$ cm³ (5 sig. fig.)
Volume of wooden block that was not used
$= 27\ 000 - 1321.6$
$=$ **25 700 cm³** (3 sig. fig.)

12 PROBABILITY OF SINGLE EVENTS

1. (a) Sample space, S
= **{4, 5, 6, 7, 8, 9, 10, 11, 12, 13}**

(b) Total number of possible outcomes = **10**

2. (a) Total number of possible outcomes
$= 20 + 15$
$= 35$
P(the button is white) $= \frac{15}{35}$
$= \mathbf{\frac{3}{7}}$

(b) P(the button is red) $= 1 - \frac{3}{7}$
$= \mathbf{\frac{4}{7}}$

3. (a) Total number of possible outcomes
$= 27 + 15$
$= 42$
P(the ribbon is blue) $= \frac{27}{42}$
$= \mathbf{\frac{9}{14}}$

(b) P(the ribbon is red) = 0

(c) P(the ribbon is either black or blue) = 1

4. (a) $S = \{1, 2, 3, 4, 5, 6\}$
Total number of possible outcomes = 6
2, 4 and 6 are even numbers.
P(an even number is obtained) $= \frac{3}{6}$
$= \mathbf{\frac{1}{2}}$

(b) 1, 3 and 5 are odd numbers.
P(an odd number is obtained) $= \frac{3}{6}$
$= \mathbf{\frac{1}{2}}$

(c) 2, 3 and 5 are prime numbers.
P(a prime number is obtained) $= \frac{3}{6}$
$= \mathbf{\frac{1}{2}}$

(d) 1 and 2 are numbers below 3.
P(a number that is below 3 is obtained) $= \frac{2}{6}$
$= \mathbf{\frac{1}{3}}$

5. (a) Total number of possible outcomes
$= 12 + 15 + 13$
$= 40$
P(the pen is red) $= \frac{12}{40}$
$= \mathbf{\frac{3}{10}}$

(b) P(the pen is green) $= \frac{15}{40}$
$= \mathbf{\frac{3}{8}}$

(c) P(the pen is purple) $= \mathbf{\frac{13}{40}}$

(d) P(the pen is either green or blue) $= \frac{15 + 13}{40}$
$= \mathbf{\frac{7}{10}}$

6. (a) Total number of possible outcomes
$= 20 + 12 + 18$
$= 50$
P(a peanut is picked) $= \frac{20}{50}$
$= \mathbf{\frac{2}{5}}$

(b) P(a cashew nut is picked) $= \frac{12}{50}$
$= \mathbf{\frac{6}{25}}$

(c) P(either a peanut or a macadamia nut is picked) $= \frac{20 + 18}{50}$
$= \mathbf{\frac{19}{25}}$

(d) P(a nut that is not a macadamia nut is picked)
= 1 – P(a macadamia nut is picked)
$= 1 - \frac{18}{50}$
$= \mathbf{\frac{16}{25}}$

7. (a) P(the next in line to see the doctor is a female)
$= \frac{12}{8 + 12}$
$= \mathbf{\frac{3}{5}}$

(b) $\frac{12}{20 + x} = \frac{6}{11}$
$132 = 6(20 + x)$
$132 = 120 + 6x$
$6x = 12$
$x = \mathbf{2}$

8. (a) P(the fruit picked was a mango) $= \frac{4}{12 + 4 + 8}$
$= \mathbf{\frac{1}{6}}$

(b) $\frac{8 + x}{24 + x} = \frac{7}{15}$
$15(8 + x) = 7(24 + x)$
$120 + 15x = 168 + 7x$
$8x = 48$
$x = \mathbf{6}$

9. (a) $P(\text{it is a can of juice}) = \frac{12}{12+8+15}$

$= \mathbf{\frac{12}{35}}$

(b) $\frac{8}{12+8+15-y} = \frac{2}{7}$

$\frac{8}{35-y} = \frac{2}{7}$

$56 = 2(35-y)$

$56 = 70 - 2y$

$2y = 14$

$y = \mathbf{7}$

10. (a) $\frac{12}{12+12+x} = \frac{2}{5}$

$\frac{12}{24+x} = \frac{2}{5}$

$2(24+x) = 60$

$48 + 2x = 60$

$2x = 12$

$x = \mathbf{6}$

(b) $\frac{12+x}{24+x} = \frac{5}{8}$

$8(12+x) = 5(24+x)$

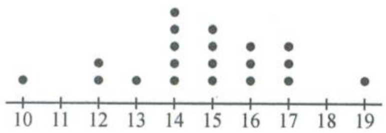
$96 + 8x = 120 + 5x$

$3x = 24$

$x = \mathbf{8}$

13 STATISTICAL DIAGRAMS

1. (a)



(b) Most common number of letters received in a month = **14**

(c) Percentage of adults who received at least 17 letters

$= \frac{4}{20} \times 100\%$

$= \mathbf{20\%}$

(d) The numbers of letters received for the adults range from 10 to 19. The numbers of letters received cluster around 12 to 17, with two extreme values of 10 and 19. The distribution is not symmetrical.

2. (a) Most adults took **78 minutes** to complete the task.

(b) Longest time taken for the adults to complete the task = **80 minutes**

(c) Fraction of adults who took at most 75 minutes to complete the task

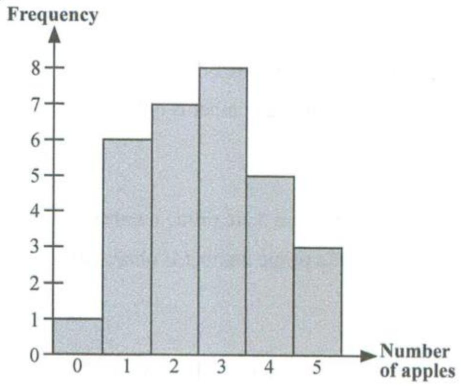
$= \frac{14}{30}$

$= \mathbf{\frac{7}{15}}$

3. (a)

Number of apples	0	1	2	3	4	5
Frequency	1	6	7	8	5	3

(b)



(c) Most common number of apples ate in a week = **3**

4. (a)

15	9 9
16	8 8 9 9
17	0 0 1 4 4 5 5 5 6 7 8 9
18	0 1 1 1 1 2 3 4 6
19	0 1 2

Key: 15 | 9 means 159 cm

(b) Height of tallest beauty contestant = **192 cm**

(c) Most common height = **181 cm**

(d) Percentage of beauty contestants that are shorter than 172 cm

$= \frac{9}{30} \times 100\%$

$= \mathbf{30\%}$

5. (a) Class interval with the most data value
= **325 to 329**

(b) Percentage of adults that saved less than $340 in a month
$= \frac{20}{40} \times 100\%$
= **50%**

(c) Number of adults that saved at least $x a month
$= \frac{3}{20} \times 40$
= 6
x = **360**

6. (a)

Group B						Stem	Group A					
				8	4	4	0	0	3	4	5	9
9	9	7	4	3	0	5	0	2	3	5	9	
		7	5	2	0	6	1					

Key (Group B)
4 | 4 means 44 kg

Key (Group A)
4 | 0 means 40 kg

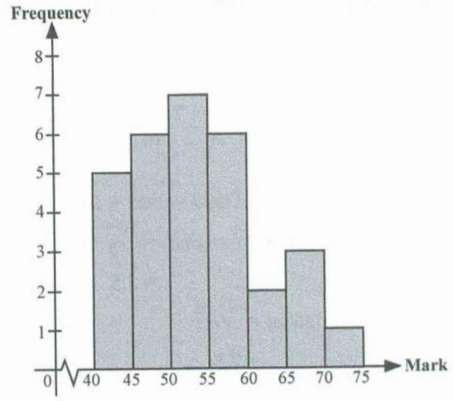
(b) For Group A, class interval that has the fewest students = **60 to 69**
For Group B, class interval that has the fewest students = **40 to 49**

(c) Group B is heavier. There are more students with greater masses in Group B than in Group A.

7. (a)

Mark (x)	Tally	Frequency
$40 < x \le 45$	~~////~~	5
$45 < x \le 50$	~~////~~ /	6
$50 < x \le 55$	~~////~~ //	7
$55 < x \le 60$	~~////~~ /	6
$60 < x \le 65$	//	2
$65 < x \le 70$	///	3
$70 < x \le 75$	/	1

(b)



(c) Percentage of students who obtained more than 60 marks
$= \frac{6}{30} \times 100\%$
= **20%**

8. (a)

Time (x hours)	Frequency
$0 \le x < 2$	**0**
$2 \le x < 4$	**20**
$4 \le x < 6$	**40**
$6 \le x < 8$	**50**
$8 \le x < 10$	**10**

(b) Class interval containing the most students
$= \mathbf{6 \le x < 8}$

(c) Total number of students
= 20 + 40 + 50 + 10
= 120
P(the student took at least 4 hours but less than 8 hours to complete the project)
$= \frac{90}{120}$
$= \mathbf{\frac{3}{4}}$

14 AVERAGES OF STATISTICAL DATA

1. (a) 1, 3, 3, 3, 7, 8, 9, 9, 10, 12

Mean

$= \frac{1+3+3+3+7+8+9+9+10+12}{10}$

$= \mathbf{6.5}$

Number of data values = 10

Position of median $= \frac{10+1}{2}$

$= 5.5$

Median $= \frac{7+8}{2}$

$= \mathbf{7.5}$

Mode = **3**

(b) 5.9, 6.1, 6.2, 6.2, 6.2, 6.3, 6.3, 6.3, 6.3

Mean

$= \frac{5.9+6.1+6.2+6.2+6.2+6.3+6.3+6.3+6.3}{9}$

$= \mathbf{6.2}$

Number of data values = 9

Position of median $= \frac{9+1}{2}$

$= 5$

Median = **6.2**

Mode = **6.3**

(c) 4, 5, 5, 5, 11, 14, 14, 14, 22, 30, 30

Mean

$= \frac{4+5+5+5+11+14+14+14+22+30+30}{11}$

$= \mathbf{14}$

Number of data values = 11

Position of median $= \frac{11+1}{2}$

$= 6$

Median = **14**

Mode = **5** and **14**

2. Total amount earned in 5 days

= \$85 × 5

= \$425

Minimum amount of money that she must earn on the fifth day

= \$425 – (\$92 + \$70 + \$86 + \$89)

= \$425 – \$337

= **\$88**

3. Total age of the 5 children

= 2.3 × 5

= 11.5 years

Total age of the 17 children

= 4.5 × 17

= 76.5 years

Mean age of all the children

$= \frac{11.5+76.5}{5+17}$

= **4 years**

4. Since the mode of the numbers is 126, one of the numbers must be 126.

114, 115, 115, 116, x, 124, 126, 126, y, 130

$\frac{x+124}{2} = 121$

$x = \mathbf{118}$

$y = \mathbf{126}$

5. (a) Modal amount spent = **\$3.50**

(b) Mean amount spent

= (2 × \$2.50 + \$2.80 + \$2.90 + \$ 3.20 + 5 × \$3.50 + \$3.70 + 2 × \$3.80 + \$3.90 + 2 × \$4.00 + 2 × \$4.20 + \$4.60 + \$4.90 + 2 × \$5.20 + \$5.30 + \$5.50 + \$5.80 + 2 × \$6.00 + \$6.50 + 2 × \$7.20) ÷ 30

= **\$4.41** (nearest cent)

(c) Number of data values = 30

Position of median $= \frac{30+1}{2}$

$= 15.5$

Median amount spent $= \frac{\$4.00+\$4.00}{2}$

= **\$4.00**

6. (a) Modal hourly wage = **\$13**

(b) Mean hourly wage

$= \frac{8\times10+10\times11+14\times12+17\times13+9\times14+12\times15}{8+10+14+17+9+12}$

= **\$12.64** (nearest cent)

(c) Number of data values = 70

Position of median $= \frac{70+1}{2}$

$= 35.5$

Median hourly wage $= \frac{\$13+\$13}{2}$

= **\$13**

7. (a) (i) Modal number of siblings = **1** and **2**

(ii) Number of data values
$= 4 + 6 + 6 + 5 + 3 + 1$
$= 25$
Position of median $= \frac{25+1}{2}$
$= 13$
Median = **2**

(iii) Mean number of siblings
$= \frac{4 \times 0 + 6 \times 1 + 6 \times 2 + 5 \times 3 + 3 \times 4 + 1 \times 5}{25}$
$= \mathbf{2}$

(b) Since there is no extreme value in the data, the **mean** best describes the data. The calculation of the mean also involves all the data.

8. (a) Modal score = **6**

(b) Mean score
$= \frac{3 \times 4 + 5 \times 5 + 7 \times 6 + 3 \times 7 + 4 \times 8 + 2 \times 9 + 1 \times 10}{25}$
$= \mathbf{6.4}$

(c) Number of data values = 25
Position of median $= \frac{25+1}{2}$
$= 13$
Median score = **6**

9. (a) Number of students who obtained a score of 6 or 7
$= 22 - 5 - 4 - 2 - 1$
$= 10$
Since the mode is 7, the minimum value of y is 0 and the maximum value of x is **10**.

(b) (i) Mean score
$= \frac{5 \times 5 + 0 \times 6 + 10 \times 7 + 4 \times 8 + 2 \times 9 + 1 \times 10}{22}$
$=$ **7.05** (3 sig. fig.)

(ii) Number of data values = 22
Position of median $= \frac{22+1}{2}$
$= 11.5$
Median score $= \frac{7+7}{2}$
$= \mathbf{7}$

10. (a) $3 + p + 7 + 5 + 2 + q + 5 = 30$
$\mathbf{p + q = 8}$

(b) $\frac{3 \times 0 + p + 7 \times 2 + 5 \times 3 + 2 \times 4 + 5q + 5 \times 6}{30} = 2.5$
$\frac{67 + p + 5q}{22 + p + q} = 2.5$
$67 + p + 5q = 55 + 2.5p + 2.5q$
$1.5p - 2.5q = 12$
$\mathbf{3p - 5q = 24}$

(c) $p + q = 8$
$p = 8 - q$ .... (1)
$3p - 5q = 24$ .... (2)
Substitute (1) into (2):
$3(8 - q) - 5q = 24$
$24 - 3q - 5q = 24$
$-8q = 0$
$q = 0$
Substitute $q = 0$ into (1):
$p = 8 - 0$
$= 8$
Hence $p = \mathbf{8}$ and $q = \mathbf{0}$.

(d) (i) Modal number of children = **1**

(ii) Number of data values = 30
Position of median $= \frac{30+1}{2}$
$= 15.5$
Median number of children $= \frac{2+2}{2}$
$= \mathbf{2}$

11. (a) Greatest possible value of x = **3**

(b) $\frac{0 \times 2 + x + 3 \times 2 + 4 \times 3 + 2 \times 4}{2 + x + 3 + 4 + 2} = 2$
$\frac{x + 26}{x + 11} = 2$
$2x + 22 = x + 26$
$x = \mathbf{4}$

(c) Position of median $= \frac{x + 11 + 1}{2}$
$= \frac{x + 12}{2}$
Position of median for largest value of x
$= 2 + x + 1$
$= x + 3$
$\frac{x + 12}{2} = x + 3$
$2x + 6 = x + 12$
$x = 6$

Largest possible value of x = **6**
Position of median for smallest value of x
$= 2 + x + 3$
$= x + 5$

$\frac{x+12}{2} = x + 5$

$2x + 10 = x + 12$

$x = 2$

Smallest possible value of x = **2**

12. (a) Modal time taken = **54 min**

(b) Number of data values = 40

Position of median $= \frac{40+1}{2}$

$= 20.5$

Median time taken $= \frac{45+46}{2}$

$=$ **45.5 min**

(c)

Time (min)	Frequency
$24 < x \leqslant 29$	3
$29 < x \leqslant 34$	4
$34 < x \leqslant 39$	5
$39 < x \leqslant 44$	6
$44 < x \leqslant 49$	7
$49 < x \leqslant 54$	7
$54 < x \leqslant 59$	8

(d)

Time (min)	Class Mark (x)	Frequency (f)	fx
$24 < x \leqslant 29$	$\frac{24+29}{2} = 26.5$	3	$26.5(3) = 79.5$
$29 < x \leqslant 34$	$\frac{29+34}{2} = 31.5$	4	$31.5(4) = 126$
$34 < x \leqslant 39$	$\frac{34+39}{2} = 36.5$	5	$36.5(5) = 182.5$
$39 < x \leqslant 44$	$\frac{39+44}{2} = 41.5$	6	$41.5(6) = 249$
$44 < x \leqslant 49$	$\frac{44+49}{2} = 46.5$	7	$46.5(7) = 325.5$
$49 < x \leqslant 54$	$\frac{49+54}{2} = 51.5$	7	$51.5(7) = 360.5$
$54 < x \leqslant 59$	$\frac{54+59}{2} = 56.5$	8	$56.5(8) = 452$
	Total	40	1775

Estimated mean time taken

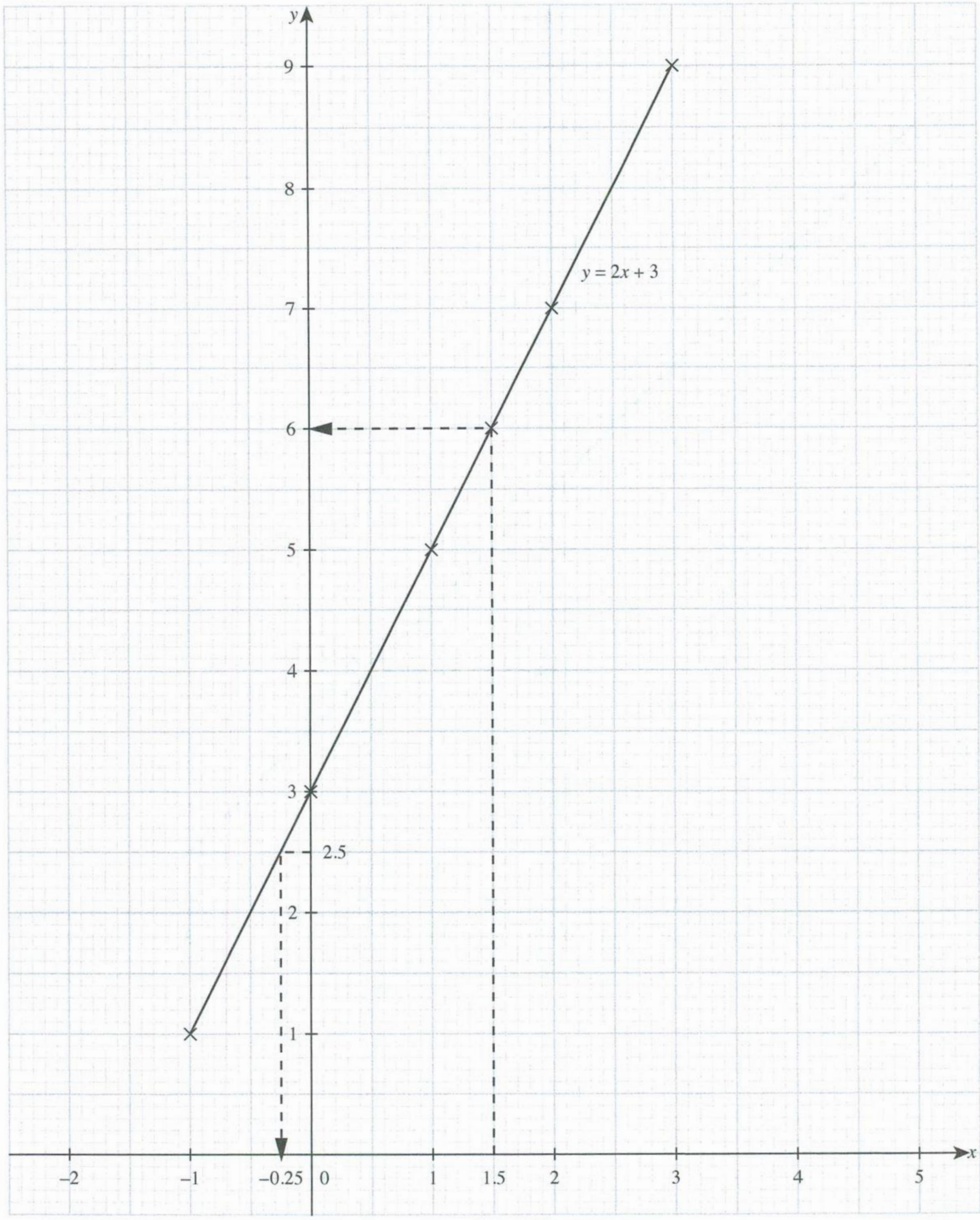
$= \frac{1775}{40}$

$=$ **44.4 min** (3 sig. fig.)

Appendix 1

Chapter 2

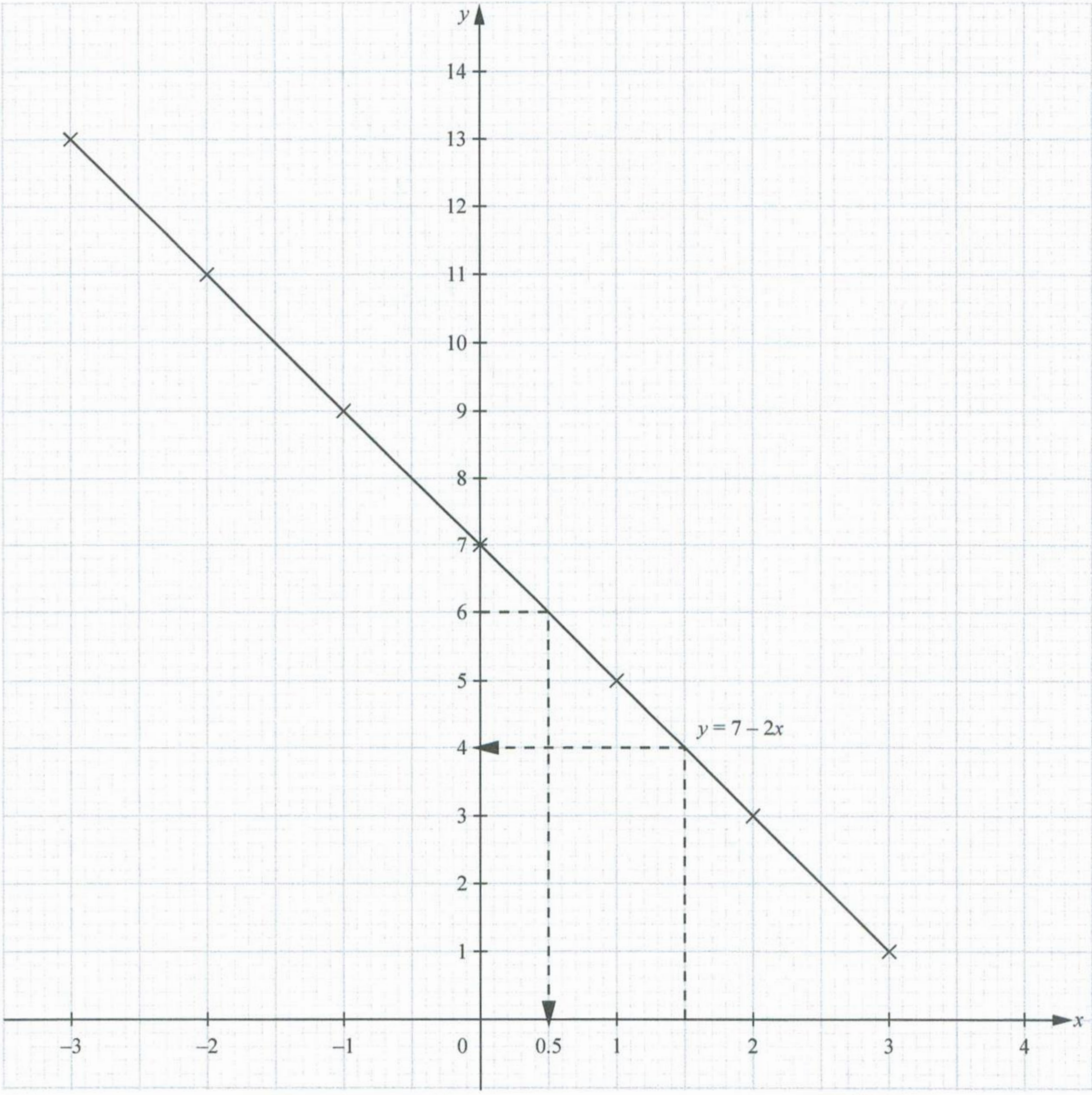
4. (b)



Appendix 2

Chapter 2

5. (b)

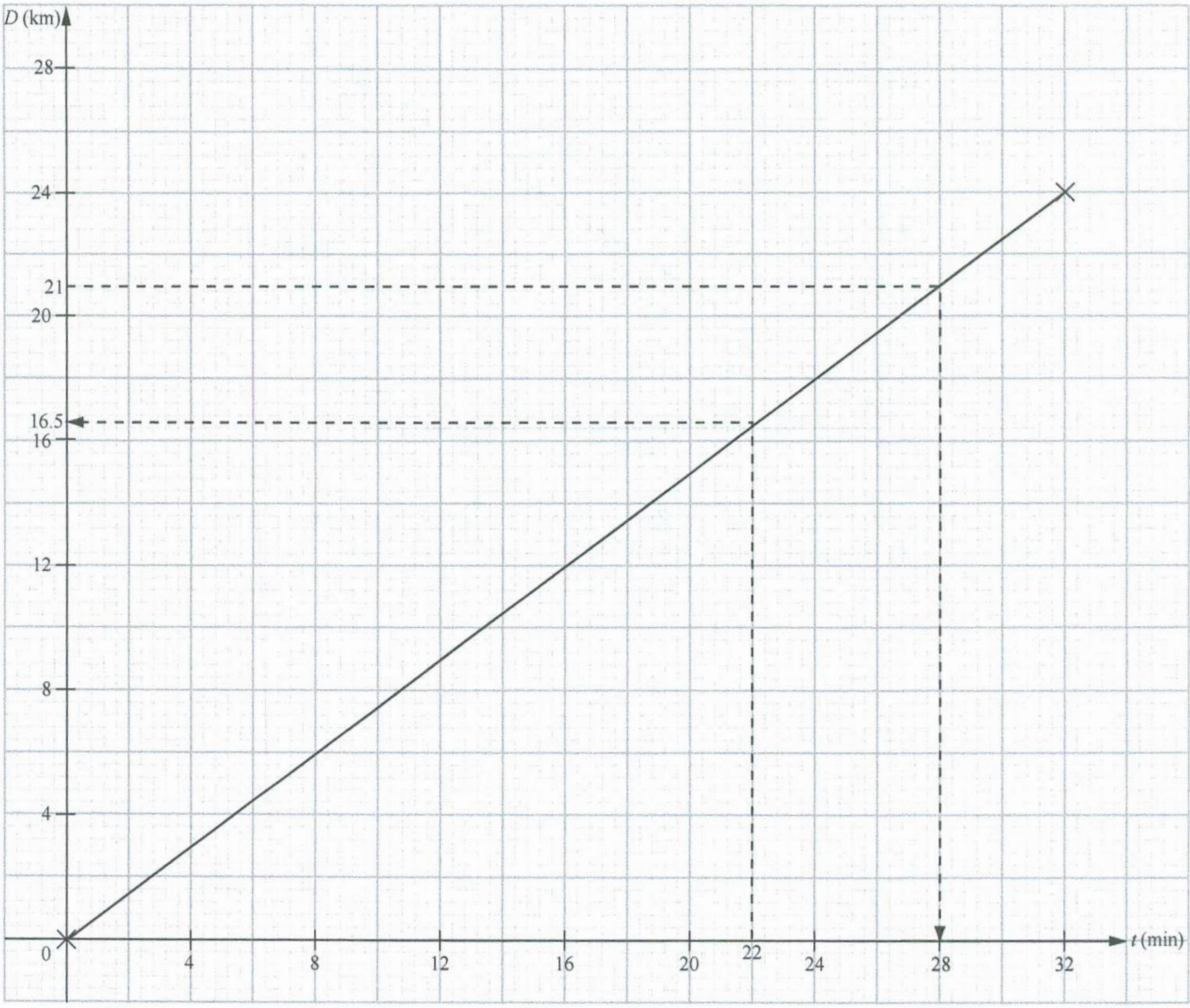


Appendix 3

Chapter 2

10. (b)

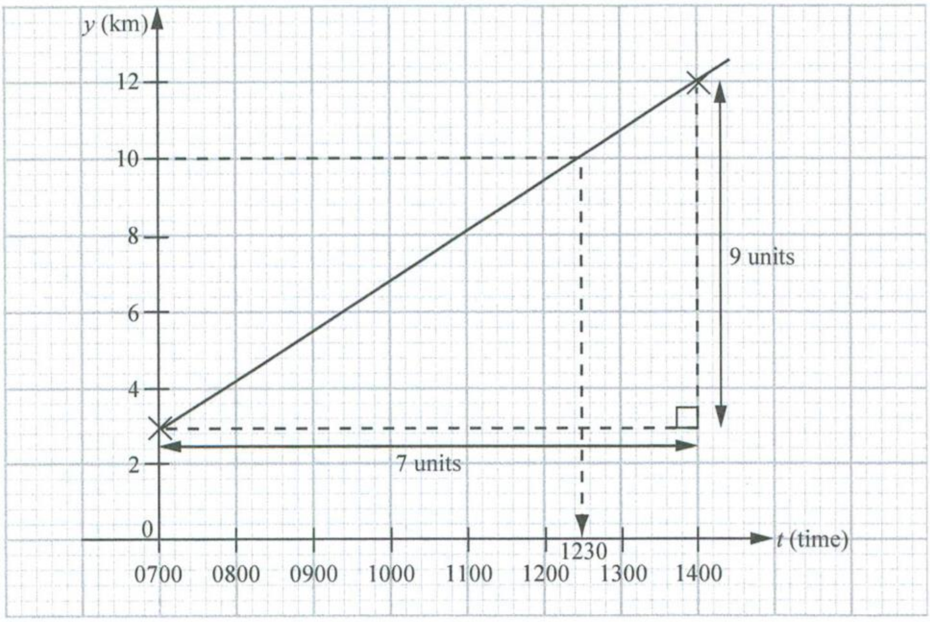
t	0	32
D	0	24



Appendix 4

Chapter 2

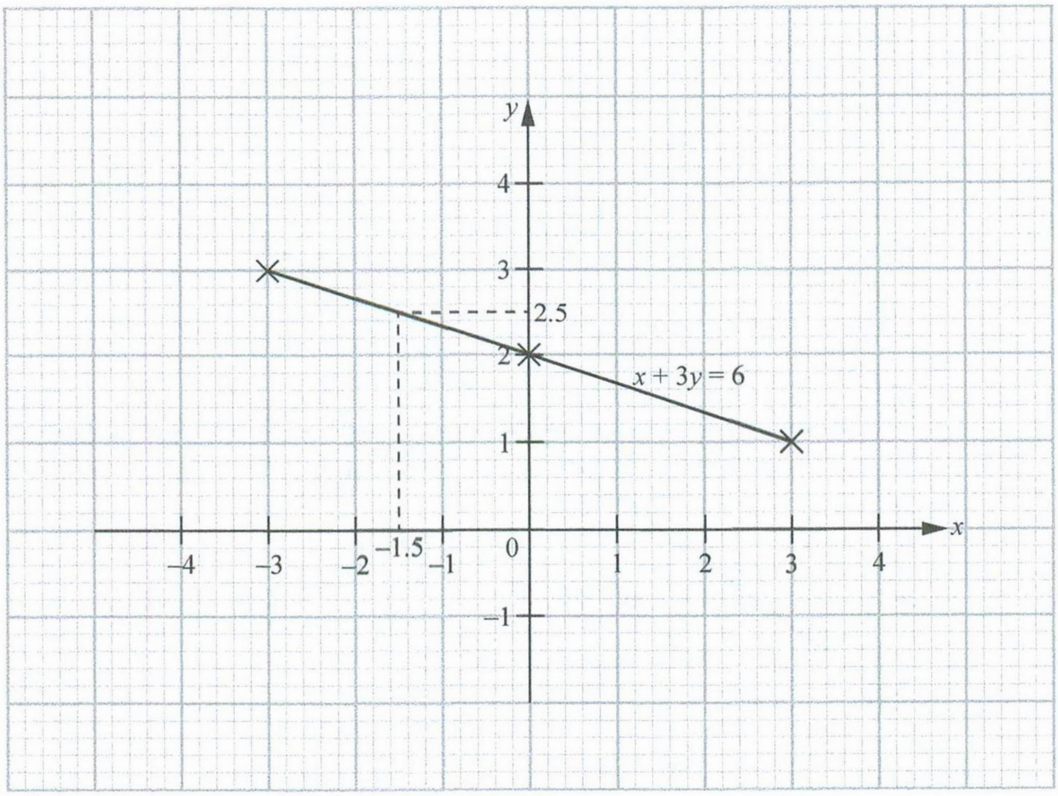
11.



Appendix 5

Chapter 3

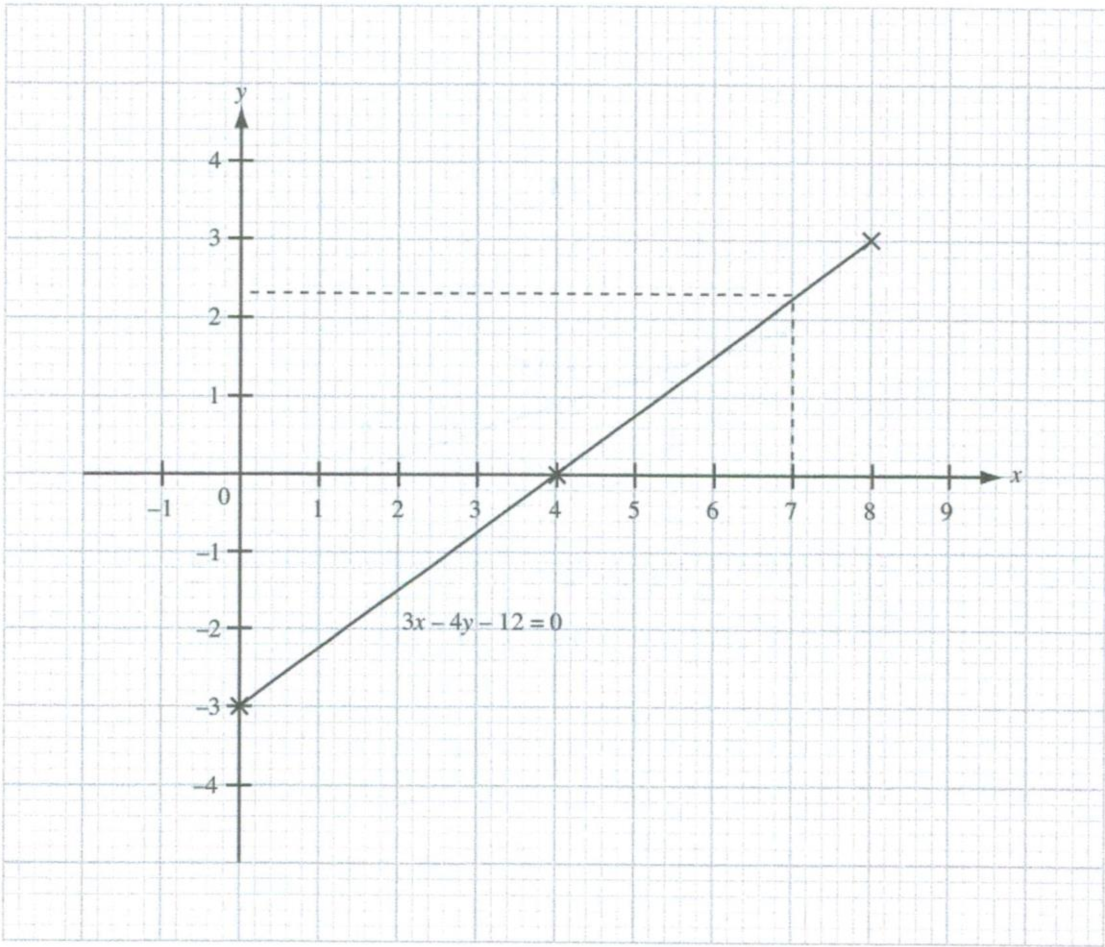
1. (b)



Appendix 6

Chapter 3

2. (b)



Appendix 7

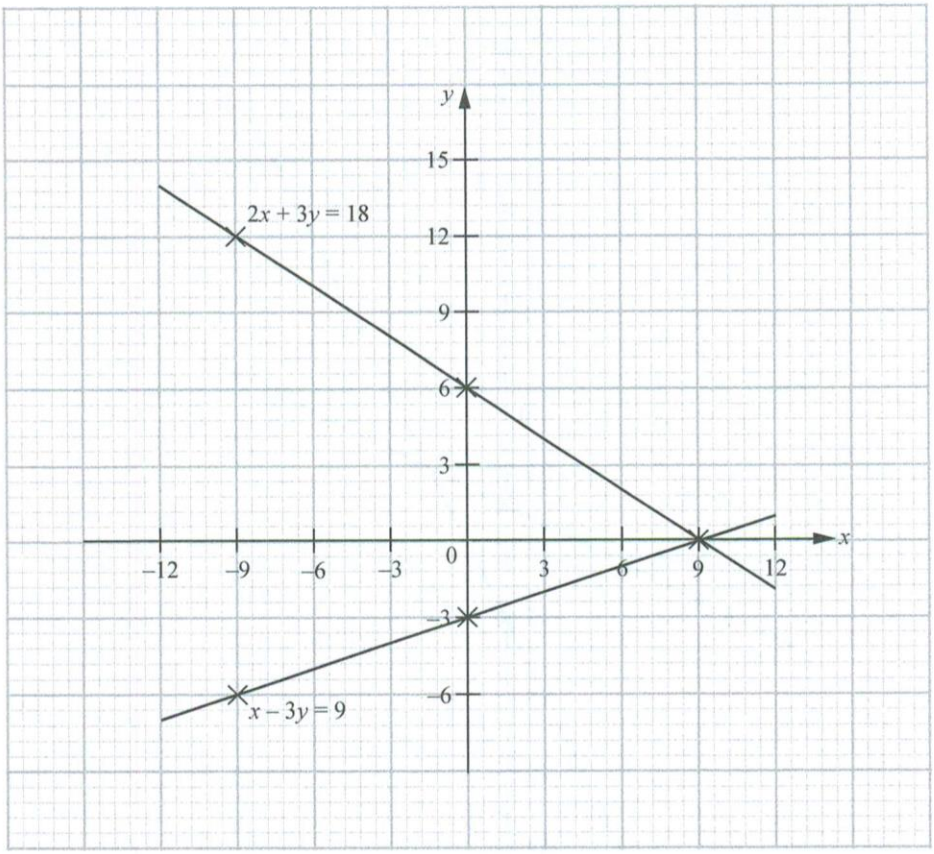
Chapter 3

3. (a) $2x + 3y = 18$

x	–9	0	9
y	12	6	0

$x - 3y = 9$

x	–9	0	9
y	–6	–3	0



Appendix 8

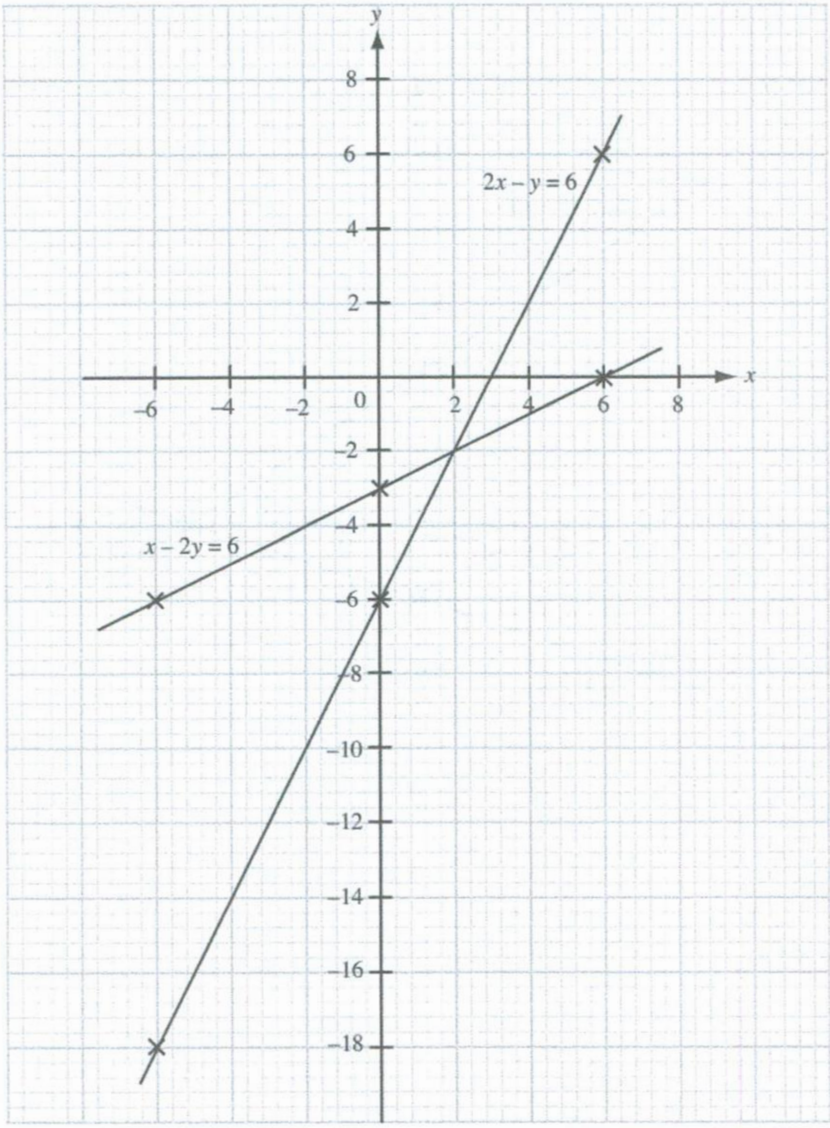
Chapter 3

3. (b) $2x - y = 6$

x	–6	0	6
y	–18	–6	6

$x - 2y = 6$

x	–6	0	6
y	–6	–3	0



Appendix 9

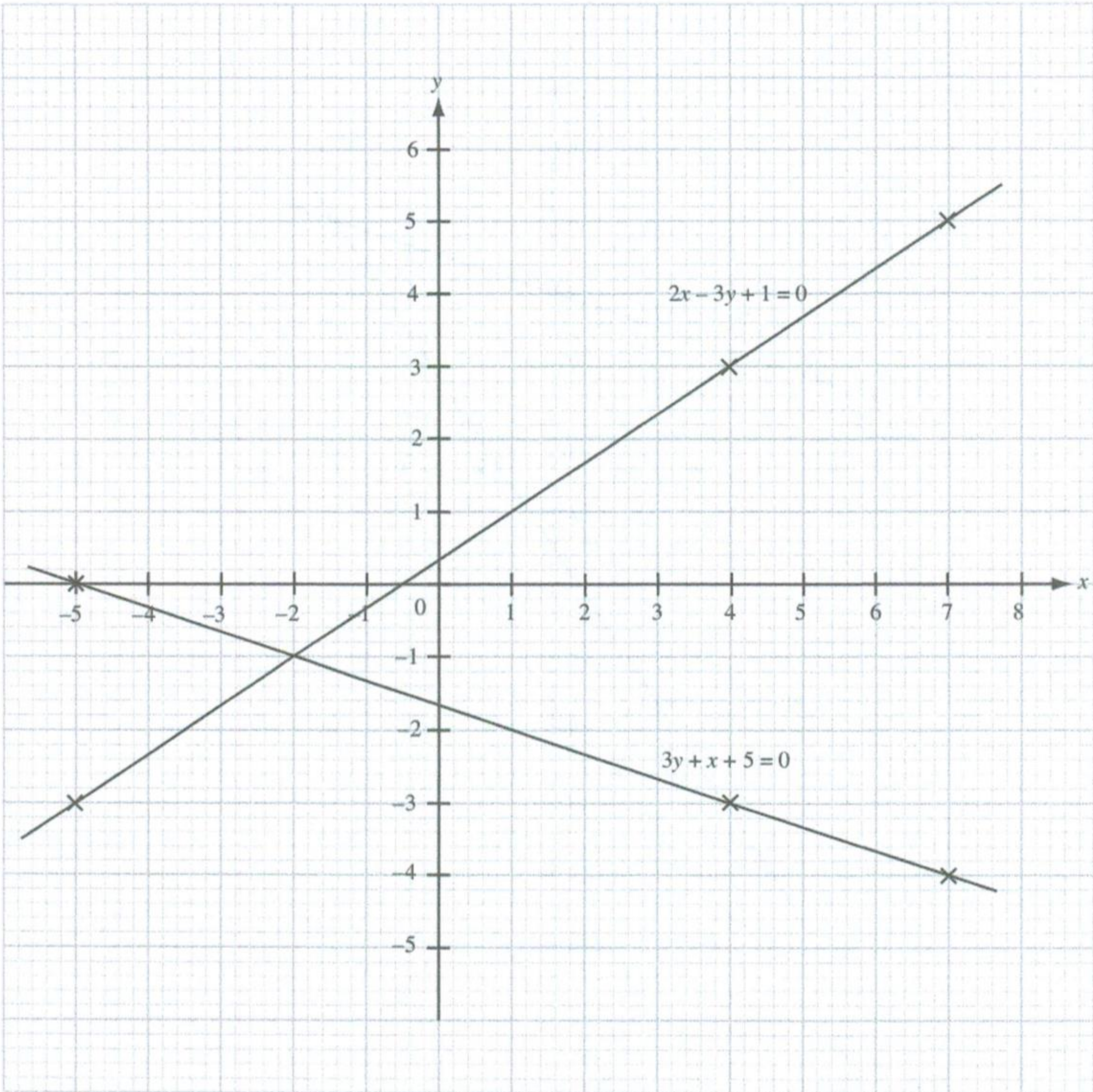
Chapter 3

3. (c) $3y + x + 5 = 0$

x	–5	4	7
y	0	–3	–4

$2x - 3y + 1 = 0$

x	–5	4	7
y	–3	3	5



Appendix 10

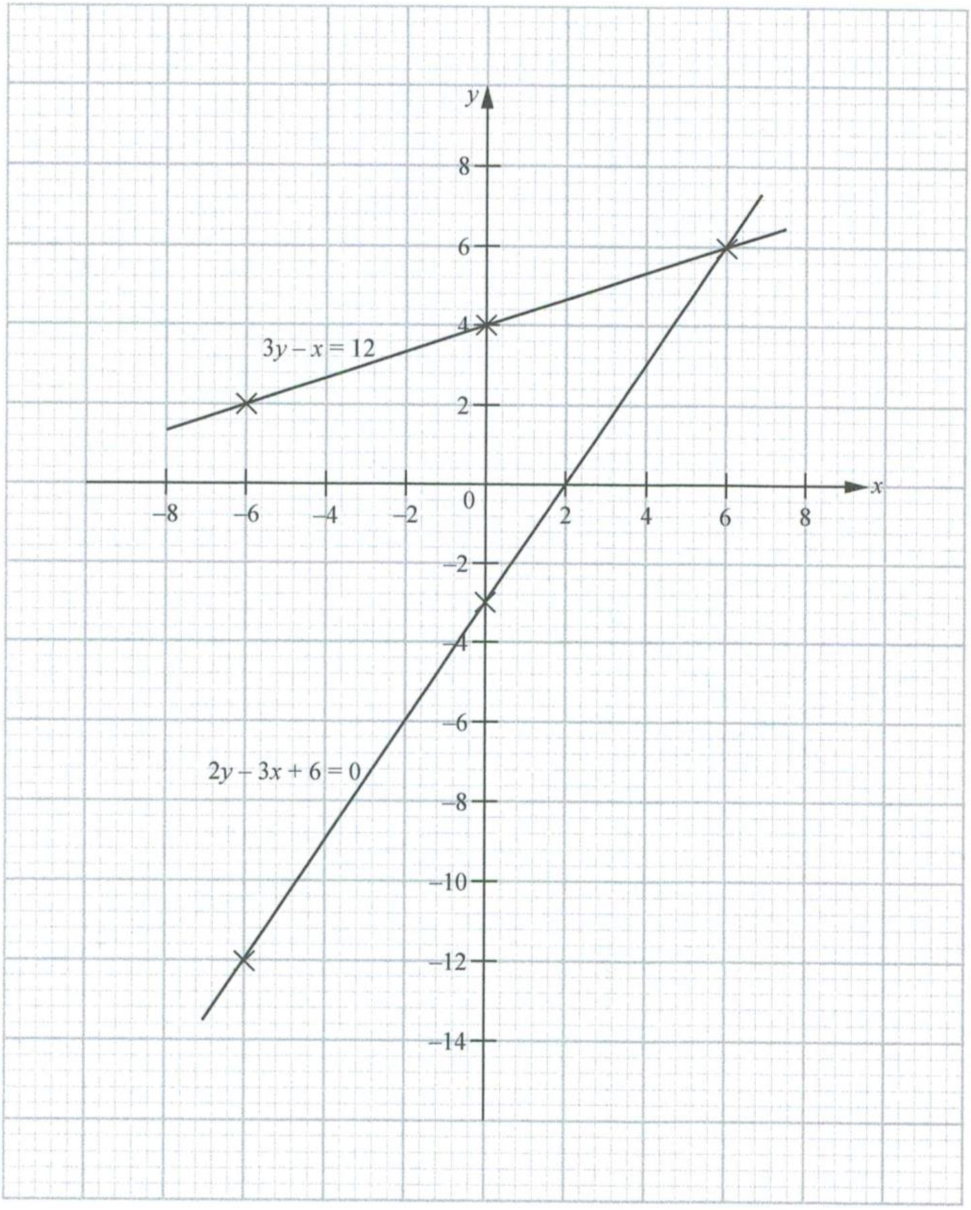
Chapter 3

3. (d) $3y - x = 12$

x	–6	0	6
y	2	4	6

$2y - 3x + 6 = 0$

x	–6	0	6
y	–12	–3	6



Appendix 11

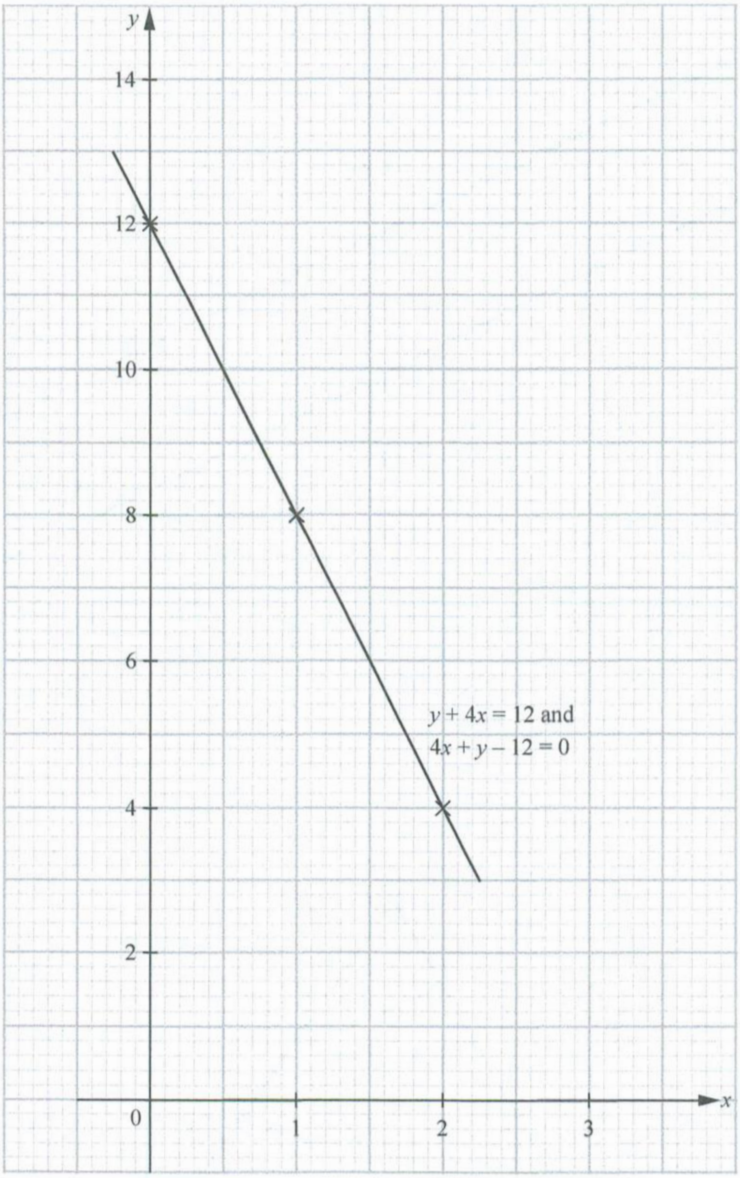
Chapter 3

4. (a) $y + 4x = 12$

x	0	1	2
y	12	8	4

$4x + y - 12 = 0$

x	0	1	2
y	12	8	4



Appendix 12

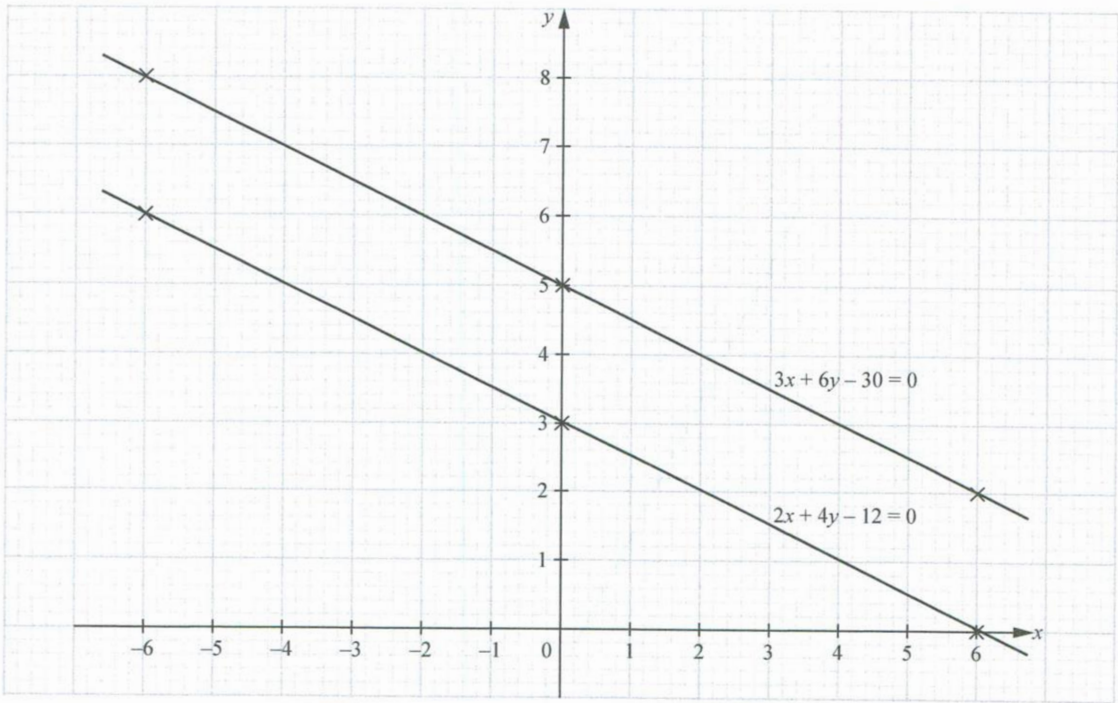
Chapter 3

4. (b) $2x + 4y - 12 = 0$

x	−6	0	6
y	6	3	0

$3x + 6y - 30 = 0$

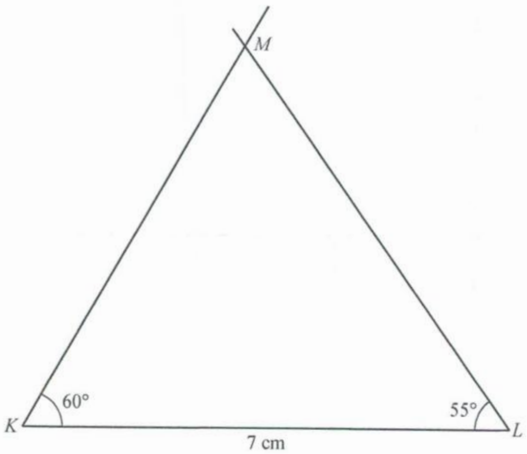
x	−6	0	6
y	8	5	2



Appendix 13

Chapter 8

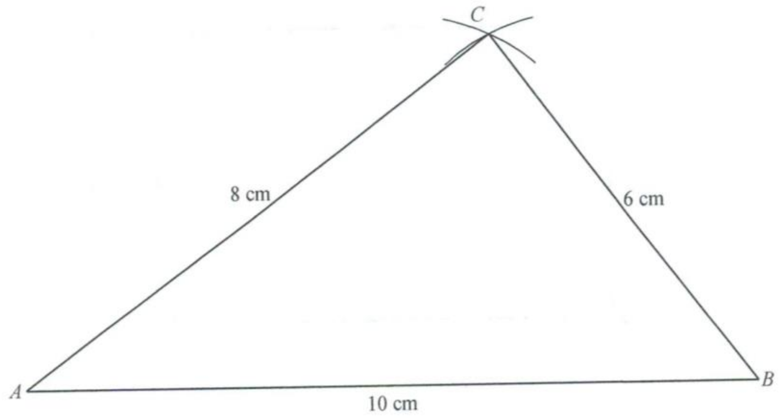
2. (a)



Appendix 14

Chapter 8

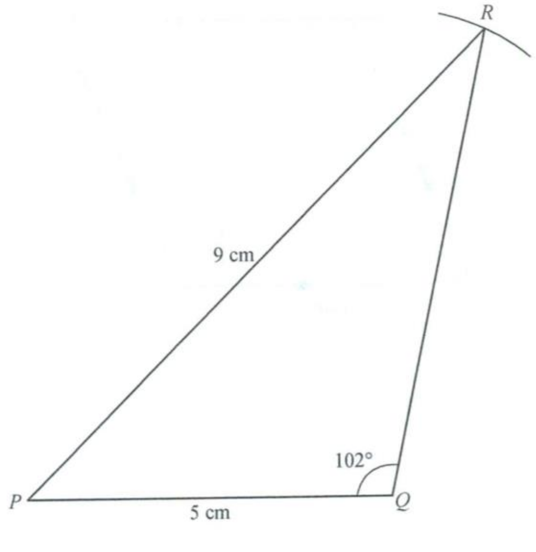
3. (a)



Appendix 15

Chapter 8

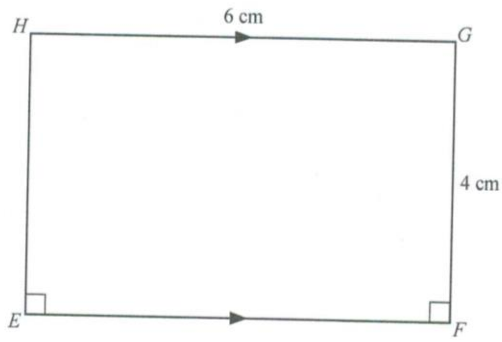
4. (a)



Appendix 16

Chapter 8

5. (a)



Appendix 17

Chapter 8

6. (a)