2 Kinematics

Study Station ▸▸

A Speed, Velocity and Acceleration

Learning Outcomes

- State the meaning of speed and velocity.
- Calculate average speed using the relationship *distance travelled / time taken*.
- Define acceleration and calculate uniform acceleration using the relationship *change in velocity / time taken*.
- Interpret given examples of non-uniform acceleration.

1. Kinematics is the physics topic that describes *objects in motion* using physical quantities including speed, velocity and acceleration.
2. When you are jogging, *speed describes how fast you move* while *velocity describes how fast you move and in which direction*.

3. **Speed** is defined as distance travelled per unit time. Its SI unit is **m/s**.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- If you *maintain a steady pace* while you jog, your *speed is constant*. If you cover 100 m in 20 s, your constant speed is $\frac{100\text{ m}}{20\text{ s}} = 5$ m/s.
- If you *do not maintain a steady pace* while you jog, your *speed is not constant*. Your changing speed at any moment of time is called **instantaneous speed**. If you also cover 100 m in 20 s, your **average speed** is similarly $\frac{100\text{ m}}{20\text{ s}} = 5$ m/s.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Tip

Note that speed is a scalar quantity. This physical quantity does not have direction even though the motion has direction.

Worked Example 2.1

The distance between town **A** and town **B** is 100 km. A car travels from town **A** to town **B** at a constant speed of 50 km/h. The same car travels back from town **B** to town **A** at a constant speed of 75 km/h. What is the average speed of the car for the whole journey?

Strategy

Use the formula:

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Therefore, we have to find the total distance travelled and total time taken first.

Solution

Total distance travelled = distance travelled from town **A** to town **B** + distance travelled from town **B** to town **A**

= (100 + 100) km

= 200 km

Total time taken = time taken to travel from town **A** to town **B** + time taken to travel from town **B** to town **A**

$$\text{Time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

$$\text{Time taken to travel from town A to town B} = \frac{100\text{ km}}{50\text{ km/h}} = 2\text{ h}$$

$$\text{Time taken to travel from town B to town A} = \frac{100\text{ km}}{75\text{ km/h}} = 1\tfrac{1}{3}\text{ h}$$

$$\text{Time taken for the whole journey} = (2 + 1\tfrac{1}{3})\text{ h} = 3\tfrac{1}{3}\text{ h}$$

$$\therefore \text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{200\text{ km}}{3\frac{1}{3}\text{ h}} = 60\text{ km/h}$$

Explanation

Note that the average speed is *not* calculated by finding the average of the two speeds, i.e. finding the average of 50 km/h and 75 km/h.

$$\text{The average of these two speeds} = \frac{50\text{ km/h} + 75\text{ km/h}}{2} = 62.5\text{ km/h} \neq \text{average speed for the whole journey}$$

4. *Speed with direction is known as velocity*. Formally, **velocity** is defined as the *rate of change of displacement*. Its unit is also **m/s**.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time taken}}$$

5. Velocity is a vector quantity because it has an associated direction. *Its magnitude is speed*.
   - If you jog at a speed of 5 m/s towards the north, your velocity is 5 m/s in the north direction.
   - If you are standing still (stationary), your speed is zero. Your velocity is also zero.

6. *The rate at which an object changes its velocity is known as acceleration*. Formally, **acceleration** is defined as the *rate of change of velocity*. Its unit is **m/s²**.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

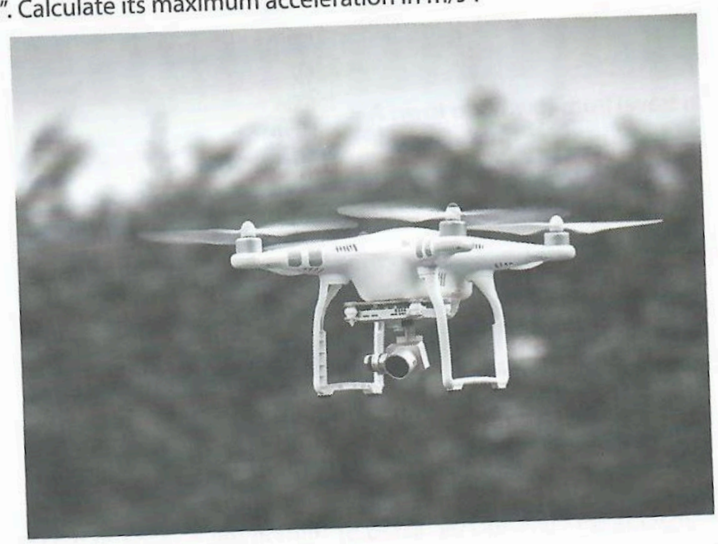
$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{\Delta t}$$

where a = acceleration
Δv = change in velocity
v = final velocity
u = initial velocity
Δt = change in time

7. Acceleration is also a vector quantity, like displacement and velocity.

Worked Example 2.2

The maximum acceleration of a drone, a small flying device, is given by its manufacturer as "0–90 kph in 2 s". Calculate its maximum acceleration in m/s².



Solution

$$\text{Final velocity} = 90 \text{ kph} = \frac{90 \text{ km}}{1 \text{ hour}}$$
$$= \frac{90\,000 \text{ m}}{60 \times 60 \text{ s}}$$
$$= 25 \text{ m/s}$$
$$\text{Acceleration} = \frac{25 \text{ m/s} - 0 \text{ m/s}}{2 \text{ s}}$$
$$= 12.5 \text{ m/s}^2$$

8. There are some objects that move in only one direction, such as:
   (a) An MRT train moving forward on the train track.
   (b) A coconut falling from a tree to the ground.
   (c) Sprinters running forward in a 100-m race.
9. For simple motions where the object moves in just one direction,
   - increasing speed means increasing velocity. Thus, acceleration is positive.
   - decreasing speed means decreasing velocity. Thus, acceleration is negative, and is also known as **deceleration**.
10. Other objects may move in two opposite directions (i.e. motions in a straight line). This type of motion is also called motions in one dimension.
    (a) An LRT train can move forward and backward on the track.
    (b) A ball thrown upwards in the air moves upwards then downwards.
11. For simple motions where the object moves in two opposite directions, we can *assign positive and negative symbols to the directions* to simplify the vector addition.
    - If you walk 10 m to the right and then 4 m to the left, you will end up 6 m to the right of your original position.
    - If we define the positive direction to be the right, the negative direction will be to the left. Thus, the total displacement = +10 m + (–4 m) = 6 m.

    - Consequently, a positive velocity (e.g. 5 m/s) would mean a movement to the right while a negative velocity (e.g. –5 m/s) would mean a movement to the left. Note that it is possible to have a positive velocity and negative displacement at the same time and vice versa.

Common Error

✗ In solving a kinematics question, we assign a positive symbol to displacement to the right and a positive symbol to velocity of motion to the left.

✓ In solving a kinematics question, we assign a positive symbol to displacement to the right but a negative symbol to velocity of motion to the left.

Explanation

In solving questions on kinematics, we define one direction (i.e. either to the right or to the left) as the positive direction. All vector quantities (including displacement, velocity and acceleration) in that direction will be positive. Vector quantities in the opposite direction will be negative.

Worked Example 2.3

A car is moving forward at a speed of 4 m/s. It then takes 10 seconds to stop and reverse at 1 m/s. Taking the forward motion as positive, find the following, giving all your answers in m/s:

(a) The change in speed

(b) The change in velocity

(c) The acceleration of the car during the 10 seconds when it stops and reverses

Strategy

(a) As speed is a scalar quantity, the change in speed can be found simply by using the following formula:

Change in speed = final speed – initial speed

(b) It is important to define positive and negative directions for vector calculations for motion in a straight line.

As velocity is a vector quantity, the change in velocity is given by the following formula:

Change in velocity = final velocity – initial velocity

(c) Acceleration can be calculated using the formula:

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

Solution

(a) Change in speed = final speed – initial speed
= (1 – 4) m/s
= –3 m/s

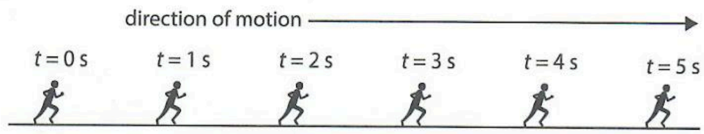
(b) Taking movement forward as positive,
Initial velocity = 4 m/s
Final velocity = –1 m/s
∴ Change in velocity = (–1 – 4) m/s
= –5 m/s

(c) $\text{Acceleration} = \frac{-5\text{ m/s}}{10\text{ s}}$
$= -0.5\text{ m/s}^2$

Checkpoint 2.1

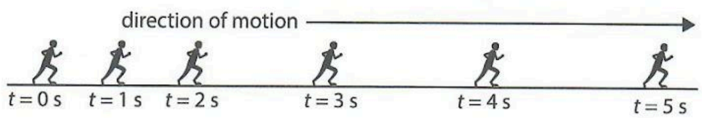
1. A train travels south at a velocity of 30 m/s along a straight, horizontal railway line. Its velocity changes and its acceleration is -1.0 m/s^2. Describe the train's movement after 10 s.
   A It is travelling south and slowing down.
   B It is travelling south and speeding up.
   C It is travelling north and slowing down.
   D It is travelling north and speeding up.

12. We can also use motion maps to visually represent the motion of an object. A **motion map** of a moving object shows the *different locations of the moving object at a regular time interval.*
13. When you jog at a constant speed, you will cover the same distance every second. For example, at the speed of 5 m/s, you will cover 5 m every second.



A motion map showing constant distance travelled and constant speed.

14. When you start sprinting, you will gain speed and cover a larger distance every second. The motion is undergoing **uniform acceleration** if the speed is increasing at a constant rate.



A motion map showing increasing distance travelled and increasing speed.

- The table below shows the differences between uniform and non-uniform acceleration.

Motion	Acceleration	Speed	Distance Travelled
Motion starting from stationary with a constant acceleration of 2 m/s^2.	Uniform acceleration	Speed increases at a constant rate. For every one-second interval, speed increases by 2 m/s (0 m/s, 2 m/s, 4 m/s, 6 m/s, 8 m/s).	Distance travelled increases at a constant rate. For every one-second interval, distance travelled during that one second increases by 2 m (1 m, 3 m, 5 m, 7 m, 9 m). Total distance travelled (measured from $t = 0$ s) increases exponentially (0 m, 1 m, 4 m, 9 m, 16 m, 25 m).
Motion starting from stationary with a non-constant acceleration.	Non-uniform acceleration	Speed increases at a changing rate. E.g., for every one-second interval, speed increases irregularly (0 m/s, 1 m/s, 4 m/s, 6 m/s, 7 m/s).	Distance travelled increases at a constant rate. E.g., at every one-second interval, distance travelled within that one second increases irregularly (1 m, 3 m, 2 m, 1 m). Total distance travelled (measured from $t = 0$ s) increases irregularly (0 m, 1 m, 4 m, 6 m, 7 m).

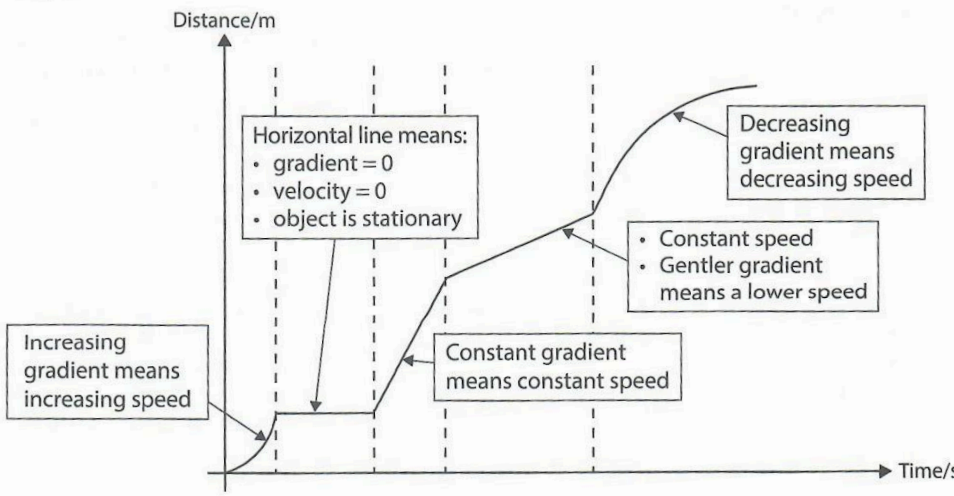
Link — Discover Physics (5th Edition) Textbook — Section 2.1

B Distance–Time and Speed–Time Graphs

Learning Outcomes

- Plot and interpret distance–time graphs and speed–time graphs for motion in one direction.
- Deduce from the shape of a distance–time graph if a body is at rest, moving with uniform speed or moving with non-uniform speed.
- Deduce from the shape of a speed–time graph if a body is at rest, moving with uniform velocity, moving with uniform acceleration or moving with non-uniform acceleration.
- Calculate the area under a speed–time graph to determine the distance travelled for motions in one direction with uniform speed or uniform acceleration.

1. We can use **distance–time graphs** and **speed–time graphs** to *represent changes in distance travelled and speed over time* for motions in one direction.
2. For motions in only one direction, the change in displacement is equal to the distance travelled. We can determine the distance travelled from the change in displacement.
   (a) If the displacements of a 100-m race runner are 20 m at $t = 2$ s and 30 m at $t = 3$ s, the distance travelled is 30 m – 20 m = 10 m.
   (b) If the displacement of a falling coconut is 20 m downwards at $t = 2$ s, the total distance travelled is 20 m – 0 m = 20 m.
3. The *gradient of the distance–time graph is equivalent to the rate of change of distance travelled, i.e., speed.*

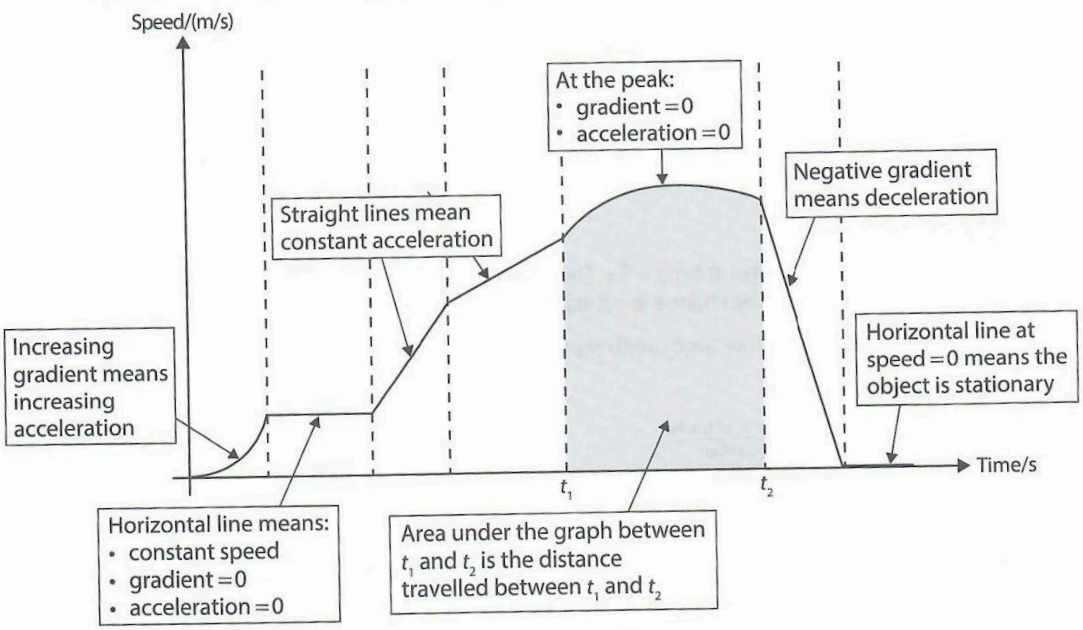


A distance–time graph

Tip

Note that distance travelled is always positive or zero, so the line in the graph is always above the horizontal axis.

4. For motions in only one direction, the change in velocity is equal to the change in speed. We can determine the change in speed from the change in velocity.
   (a) If the velocities of a 100-m race runner are 10 m/s at $t = 2$ s and 12 m/s at $t = 3$ s, the change in speed is 12 m/s – 10 m/s = 2 m/s.
   (b) If the velocity of a falling coconut is 20 m/s downwards at $t = 2$ s, the change in speed is 20 m/s – 0 m/s = 20 m/s.
5. The *gradient of the speed–time graph, which is the rate of change of speed, is also the rate of change of velocity, i.e., acceleration.*
6. The *area under the speed–time graph is equivalent to the distance travelled.*



A speed–time graph

Tip

Note that speed is always positive or zero, so the line in the graph is always above the horizontal axis.

Checkpoint 2.2

1. State how the acceleration of a moving object can be determined from a speed–time graph.

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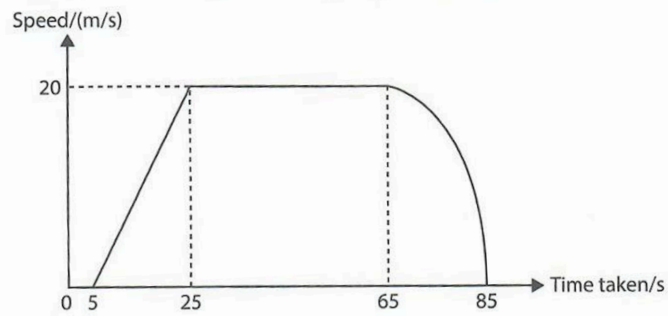
Tip

Questions on speed–time graphs have appeared in examinations, e.g., GCE 'O' Level Science Physics Oct/Nov 2021, Paper 1, Q2.

Worked Example 2.4

The graph below shows the motion of a train travelling in a straight line from station **A** to station **B**.



(a) Find the acceleration of the train as it leaves station **A**.

(b) Determine if the distance travelled in the first 25 seconds is greater than the distance travelled in the last 20 seconds.

Strategy

(a) The train starts moving at time = 5 s. Therefore, the time taken is (25 – 5) s = 20 s. The change in speed is equal to the change in velocity for motions in one direction.

(b) The area under a speed–time graph represents the distance travelled.

Solution

(a) $\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$

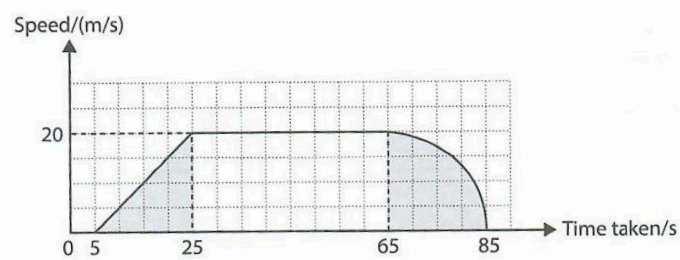
$= \frac{(20 - 0)\text{ m/s}}{(25 - 5)\text{ s}}$

$= 1\text{ m/s}^2$

(b) The train spends the same amount of time in motion between 0 s to 25 s and between 65 s to 85 s. The maximum speed reached in each interval is also the same. As the graph curves outwards between 65 s and 85 s, the area under the graph is larger. Since the area under the graph is equivalant to the distance travelled, the distance travelled in the last 20 seconds is greater.

Explanation

The area under the graph can also be estimated by counting the number of boxes or grids on the graph.



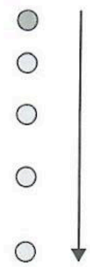
Link Discover Physics (5th Edition) Textbook — Section 2.2

C Free Fall Motion

Learning Outcome

- State that the acceleration of free fall for a body near the surface of the Earth is constant and is approximately 10 m/s^2.

1. Free fall motion is a special motion with constant acceleration *g*.
2. When you drop a coin, the coin will fall faster and faster until it hits the ground.



A motion map of a coin falling down.

3. If you measure the acceleration of the falling coin, you will get a value of approximately 10 m/s^2. This value is known as the **acceleration of free fall, *g***. It varies according to a few factors as summarised in the following table.

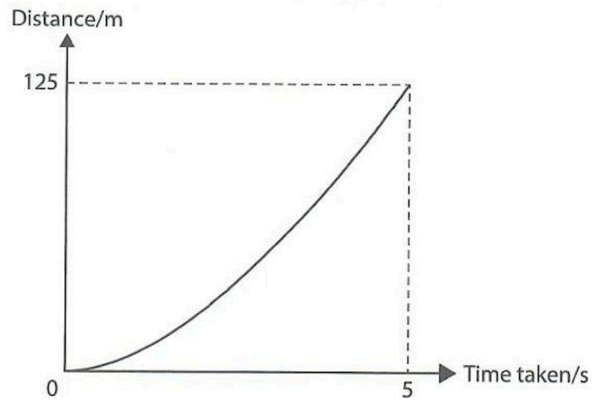
Example			Factor
g of a falling hammer on the moon (which is near-vacuum)	is the same as	*g* of a feather falling on the moon (which is near-vacuum)	*g* does not depend on the mass
g of falling motion in vacuum	is larger than	*g* of falling motion in air	*g* is affected by **air resistance** (see Chapter 3)
g far from the Equator (e.g. Europe)	is larger than	*g* near the Equator (e.g. Singapore)	*g* depends on the location on Earth
g at sea level	is larger than	*g* at mountain top	*g* decreases further away from the Earth

Tip!

For general falling motions in air, we usually assume that the effect of air resistance is negligible and *g* is a constant of 10 m/s^2 to simplify calculations.

Worked Example 2.5

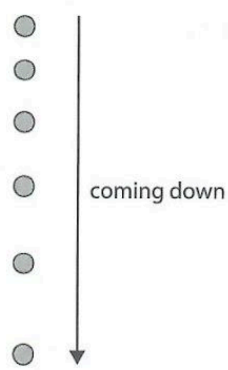
A flower pot falls from the balcony of a flat at a height of 125 m. The graph below shows the distance travelled by the flower pot before it hits the ground.



Assuming that the effect of air resistance is negligible, sketch the corresponding speed–time graph.

Strategy

We can draw a motion map to help us better understand the situation.



We note that this motion is in one direction. The speed is equal to the gradient. However, measuring directly from the graph is difficult and inaccurate. It is better to calculate the speeds using the knowledge that this is a free fall motion with constant acceleration $g = 10\ \text{m/s}^2$.

Solution

Acceleration $a = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$

$= \frac{v - u}{t}$

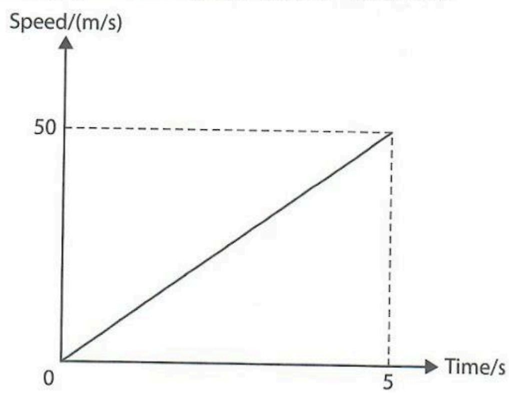
$at = v - u$

$u = v - at$

$= 0 + (10\ \text{m/s}^2)(5\ \text{s})$

Final velocity = final speed = 50 m/s

Therefore, the corresponding speed–time graph is as shown below.



Explanation

The speed–time graph must be a continuous straight line with a gradient of 10 m/s^2 as the object is in free fall. The acceleration is positive because we take positive direction as downwards.

Link Discover Physics (5th Edition) Textbook — Section 2.3

Checkpoint 2.3

1. An object falls through a vacuum. Describe how the speed and acceleration of the object are changing.

   A Speed is increasing, acceleration is decreasing

   B Speed is increasing, acceleration is constant

   C Speed is increasing, acceleration is increasing

   D Speed is constant, acceleration is zero

Test Station ▸▸

1. A car is moving at 15 m/s towards the traffic light. Its brakes are applied and the car slows down at a constant rate of 5.0 m/s^2. How much time does the car need before it comes to a stop?

   A 3.0 s

   B 10.0 s

   C 2.0 s

   D 7.5 s

2. The distance travelled by an object is shown in the graph in Figure 2.1.

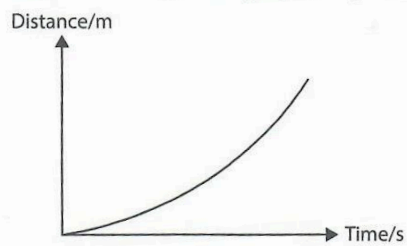


Figure 2.1

   Which of the following statements is **not** correct?

   A The distance travelled by the object is increasing.

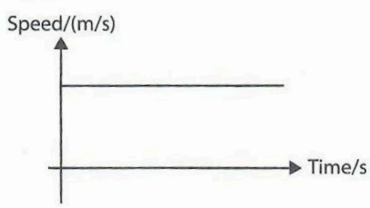
   B The gradient of the graph is increasing.

   C The area under the graph is the speed of the object.

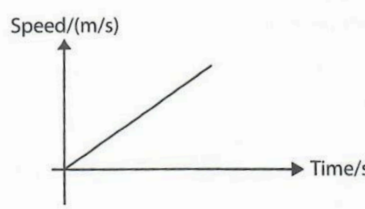
   D The object is continually moving faster.

3. Which of the following speed–time graphs **correctly** describes the motion of a stone falling in air?

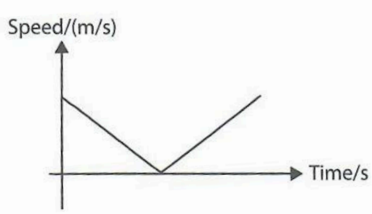
   A



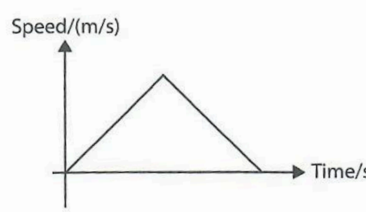
   B



   C



   D



4. The Changi Airport Skytrain (Figure 2.2) travels a distance of 200 m between Terminal 1 and Terminal 2 at a constant speed of 5 m/s.

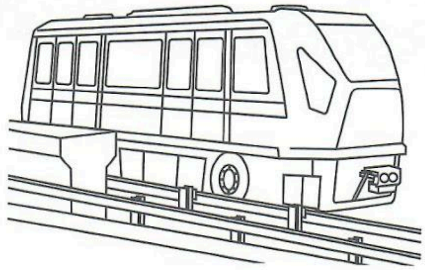


Figure 2.2

(a) How much time does the journey from Terminal 1 to Terminal 2 take? [2]

(b) Calculate its distance from Terminal 2 12 s after it leaves Terminal 2. [2]

(c) State why the assumption of constant speed is unrealistic. [1]

5. After robbing a bank, a robber escaped in a car. A policeman got into his car and chased after the robber 10 seconds later. The speed–time graphs of both cars are shown in the graph in Figure 2.3.

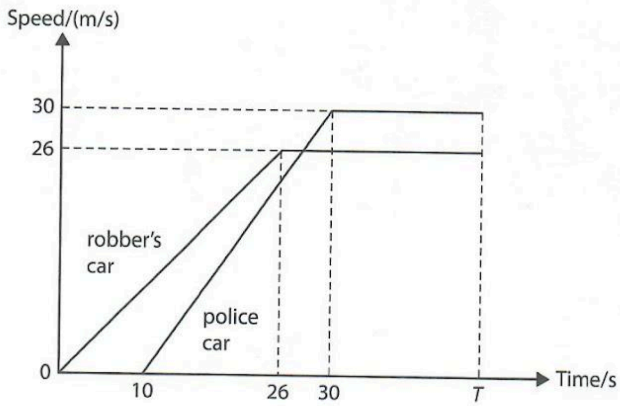


Figure 2.3

(a) Calculate the initial acceleration of the robber's car. [2]

(b) Calculate the initial acceleration of the police car. [2]

(c) Calculate the distance travelled by the robber's car after 30 seconds. [3]

(d) Calculate the distance travelled by the police car after 30 seconds. [2]

(e) The police car finally caught up with the robber's car at $t = T$. Determine the value of T. [3]