# 11 VOLUME AND SURFACE AREA OF PYRAMIDS, CONES AND SPHERES

**LEARNING OBJECTIVES**

In this topic, we will learn to:

- find the volume and surface area of pyramids, cones and spheres
- solve problems involving the volume and surface area of composite solids

## 11.1 PYRAMID

1. A pyramid is a solid that consists of a polygon-shaped base and a number of triangular faces which are known as lateral faces.

2. The vertices of a pyramid are the corner points.

3. The apex of a pyramid is the vertex where all the slanted faces meet.

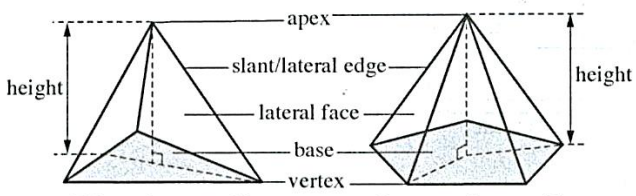
4. The height of a pyramid is the perpendicular distance from the apex to the base of the pyramid.

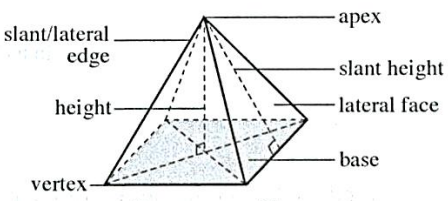
5. A slant height of a pyramid is the height of a lateral face.

6. The slant/lateral edge is the common edge between two adjacent lateral faces.

7. A right pyramid is a pyramid where the apex is vertically above the centre of the base. In this book, the term 'pyramid' is used to refer to a right pyramid.

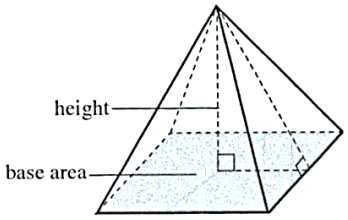
8. A right pyramid that has a base of a regular polygon is known as a regular pyramid.




> ***Note:***
> *A pyramid is named according to the shape of its base.*

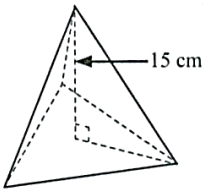
9. Volume of pyramid = $\frac{1}{3}$ × Base area × Height




10. Total surface area of pyramid = Base area + Area of all lateral faces

**WORKED EXAMPLE 1**

The diagram shows a pyramid with base area 35 cm$^2$ and height 15 cm. Find the volume of the pyramid.




***Worked Solution:***

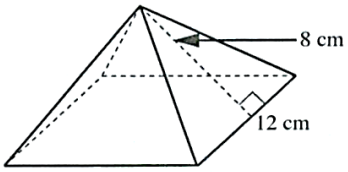
Volume = $\frac{1}{3}$ × Base area × Height

= $\frac{1}{3}$ × 35 × 15

= **175 cm$^3$**

**WORKED EXAMPLE 2**

The diagram shows a regular pyramid.

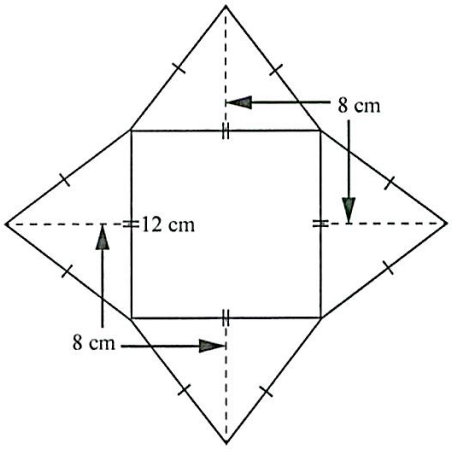



(a) Draw a net of the pyramid.

(b) Hence find the total surface area of the pyramid.

***Worked Solution:***

(a)




(b) Total surface area = Base area + Area of all lateral faces

$= (12 \times 12) + 4(\frac{1}{2} \times 8 \times 12)$

$= \mathbf{336\ cm^2}$

## 11.2 CONE

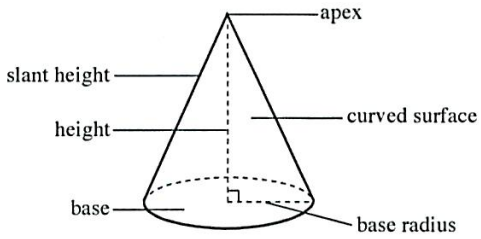
1. A cone is a solid that consists of a base bounded by a simple closed curve and where the curve surface tapers into a point known as the apex that is opposite the base.

2. When the apex of a cone is vertically above the centre of cone's circular base, the cone is a right circular cone. In this book, the term 'cone' is used to refer to a right circular cone.

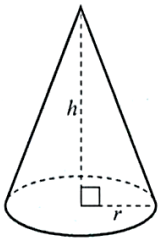
3. The height of a cone is the perpendicular distance from the apex to the base of the cone.

4. The slant height of a right circular cone is the distance from the apex to the circumference of the base of the cone.

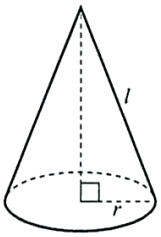
5. The base radius is the radius of the circular base of a cone.



6. Volume of cone = $\frac{1}{3}\pi r^2 h$, where $r$ is the base radius and $h$ is the height of the cone.

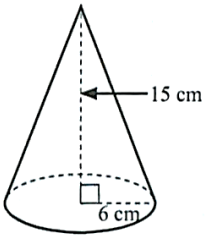



7. Curved surface area of cone = $\pi rl$, where $r$ is the base radius and $l$ is the slant height of the cone.




## Worked Example 3

The diagram shows a cone. It has a circular base of radius 6 cm and a height of 15 cm. Find the volume of the cone.




***Worked Solution:***

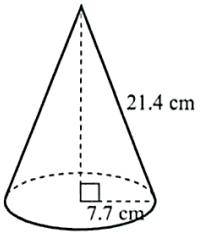
Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 15$

$=$ **565 cm³** (3 sig. fig.)

## Worked Example 4

The diagram shows a cone. It has a circular base of radius 7.7 cm and a slant height of 21.4 cm. Find the surface area of the cone.




***Worked Solution:***

Surface area = $\pi r^2 + \pi rl$

$= \pi \times 7.7^2 + \pi \times 7.7 \times 21.4$

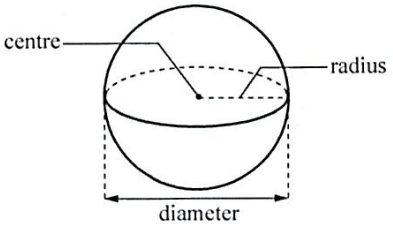
$=$ **704 cm²** (3 sig. fig.)

## 11.3 SPHERE

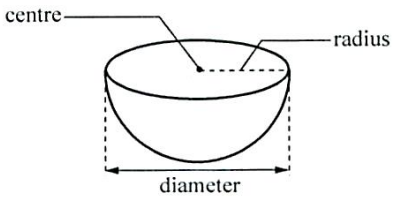
1. A sphere is a solid where all the points on its surface are at a uniform distance from its centre.

2. The uniform distance from the centre to the surface of a sphere is the radius of the sphere.

3. A line segment joining two points on a sphere and passing through the centre is a diameter of the sphere.




4. A hemisphere is half of a sphere.

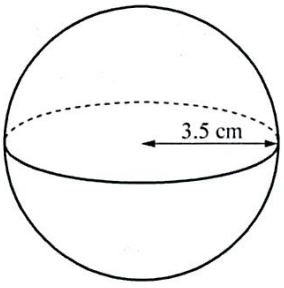



5. Volume of sphere $= \frac{4}{3}\pi r^3$, where $r$ is the radius of the sphere.

6. Surface area of sphere $= 4\pi r^2$, where $r$ is the radius of the sphere.

**WORKED EXAMPLE 5**

The diagram shows a sphere. It has a radius of 3.5 cm.
Find the volume and surface area of the sphere.




***Worked Solution:***

Volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3.5^3$

$=$ **180 cm³** (3 sig. fig.)

Surface area $= 4\pi r^2$

$= 4 \times \pi \times 3.5^2$

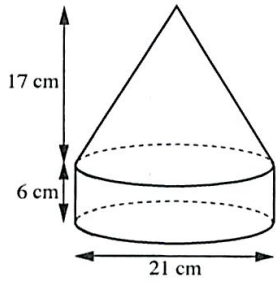
$=$ **154 cm²** (3 sig. fig.)

## 11.4 Composite Solid

1. In this section, we solve problems involving the volume and surface area of composite solids.

**WORKED EXAMPLE 6**

The diagram shows a solid made from a cone and a cylinder. The cone and the cylinder both have a diameter of 21 cm. The cone has a height of 17 cm. The cylinder has a height of 6 cm.



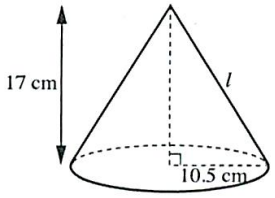

Calculate
(a) the volume of the solid,
(b) the surface area of the solid.

***Worked Solution:***

(a) Base radius $= \frac{21}{2}$
$= 10.5$ cm

Volume of solid = Volume of cylinder + Volume of cone
$= \pi \times 10.5^2 \times 6 + \frac{1}{3} \times \pi \times 10.5^2 \times 17$
$= \mathbf{4040\ cm^3}$ (3 sig. fig.)

(b) $l^2 = 17^2 + 10.5^2$
$= 289 + 110.25$
$= 399.25$
$l = \sqrt{399.25}$
$= 19.981$ cm (5 sig. fig.)



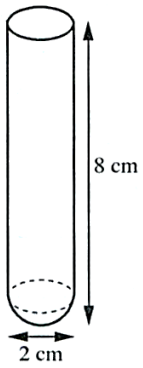

Surface area of solid = Base area of cylinder + Curved surface area of cylinder
+ Curved surface area of cone
$= \pi \times 10.5^2 + 2 \times \pi \times 10.5 \times 6 + \pi \times 10.5 \times 19.981$
$= \mathbf{1400\ cm^2}$ (3 sig. fig.)

**WORKED EXAMPLE 7**

The diagram shows a test tube made from a cylinder and a hemisphere.
The cylinder and the hemisphere both have diameter 2 cm.
The test tube has a height of 8 cm. Find

(a) the volume of the test tube,

(b) the external surface area of the test tube.




***Worked Solution:***

(a) Base radius $= \frac{2}{2}$

$= 1$ cm

Height of cylinder $= 8 - 1$

$= 7$ cm

Volume of test tube

= Volume of cylinder + Volume of hemisphere

$= \pi \times 1^2 \times 7 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 1^3$

$=$ **24.1 cm³** (3 sig. fig.)

(b) External surface area of test tube

= Curved surface area of the hemisphere + curved surface area of the cylinder

$= \frac{1}{2} \times 4 \times \pi \times 1^2 + 2 \times \pi \times 1 \times 7$

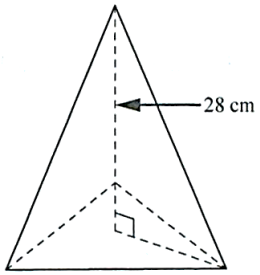
$=$ **50.3 cm²** (3 sig. fig.)

***Student's common mistake:***

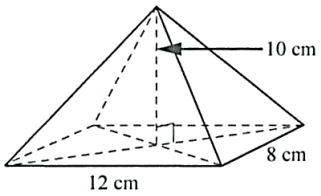
It is a common mistake to use the formula $\frac{4}{3}\pi r^3$ when calculating the volume of a hemisphere and the formula $4\pi r^2$ when calculating the surface area of a hemisphere. Since a hemisphere is half of a sphere, the volume of a hemisphere is $\frac{1}{2} \times \frac{4}{3}\pi r^3$ and the curved surface area of a hemisphere is $\frac{1}{2} \times 4\pi r^2$.

## PRACTICE QUESTIONS

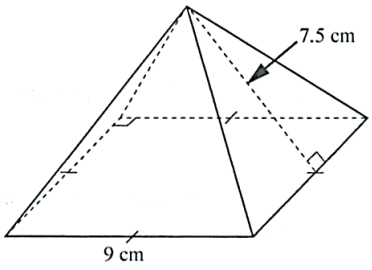
1. The diagram shows a pyramid with base area 108 cm² and height 28 cm. Find the volume of the pyramid.



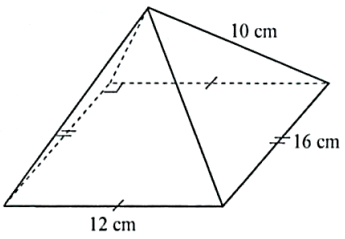

2. The diagram shows a pyramid. It has a rectangular base of sides 12 cm by 8 cm. The height of the pyramid is 10 cm. Find the volume of the pyramid.




3. The diagram shows a pyramid with a square base.
   (a) Draw a net of the pyramid.
   (b) Hence find the total surface area of the pyramid.



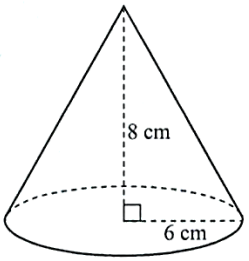

4. The diagram shows a pyramid with a rectangular base.
   (a) Draw a net of the pyramid.
   (b) Hence find the total surface area of the pyramid.



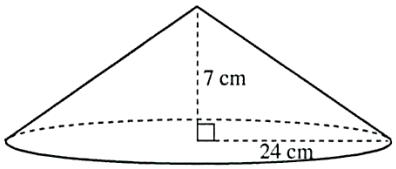
5. A square pyramid has height 8 cm. Its volume is $170\frac{2}{3}$ cm³. Find the length of its square base.

6. Find the volume of each of the following cones.

(a)

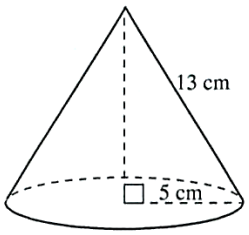



(b)

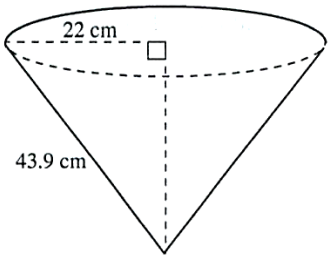



7. Find the surface area of each of the following cones.

(a)




(b)

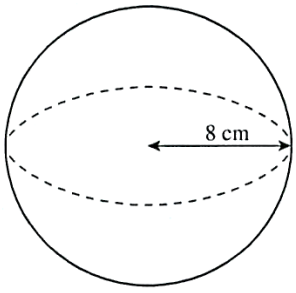



8. A cone has volume 3118.5 cm³ and height 27 cm. Find the base radius.

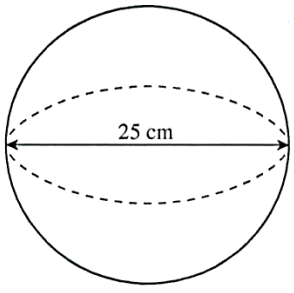
9. The curved surface area of a cone is 435.6 cm². The cone has a slant height of 18 cm. Find the base radius of the cone.

10. Find the volume and surface area of each of the following spheres.

(a)

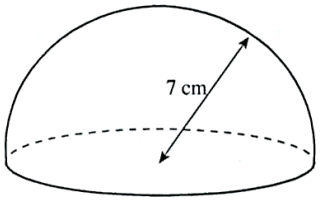



(b)

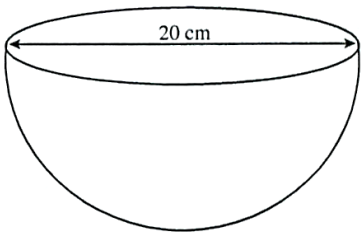


11. Find the volume and surface area of each of the following solid hemispheres.

(a)




(b)



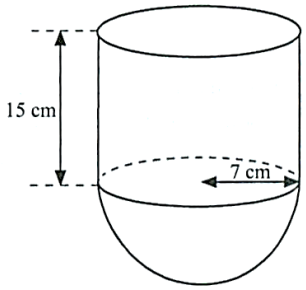

12. A sphere has volume $36\pi$ cm$^3$. Find the radius of the sphere.

13. The surface area of a sphere is 200 cm$^2$. Find the radius of the sphere.

14. The diagram shows a solid made from a cylinder and a hemisphere. The cylinder and the hemisphere both have radius 7 cm. The height of the cylinder is 15 cm. Find

(a) the volume of the solid,

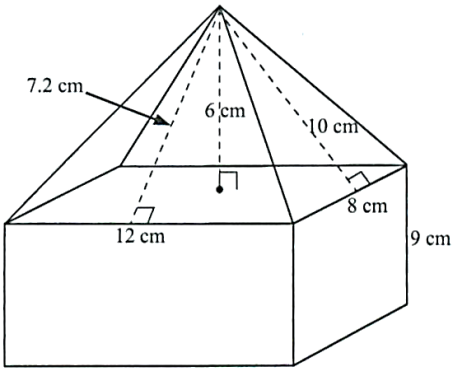
(b) the surface area of the solid.



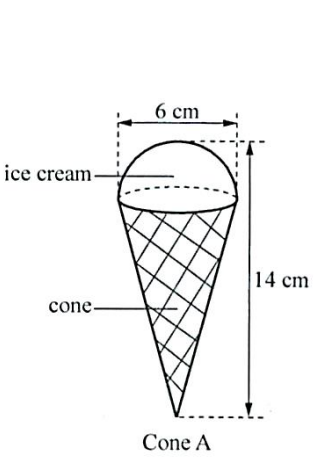

15. The diagram shows a solid made from a cuboid, 12 cm by 8 cm by 9 cm, attached to a pyramid. The pyramid has a rectangular base of sides 12 cm by 8 cm. The pyramid has height 6 cm and slant heights 7.2 cm and 10 cm. Find

(a) the volume of the solid,

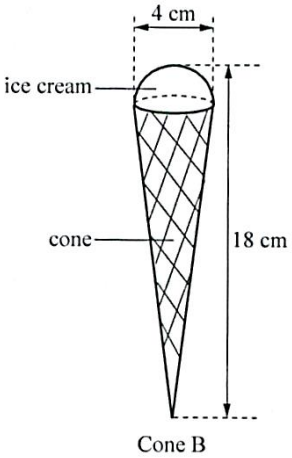
(b) the surface area of the solid.



16. The diagram shows two cones of ice cream. Each of the cones is in the shape of a circular cone. Ice cream is filled into each cone to its brim and a shape of a hemisphere is formed at the top of the cone. If each cone of ice cream costs the same price, which is a better buy? Explain your answer.



Cone A



Cone B

17. Peter used a wooden lathe to produce a bowl from a wooden block as shown in the diagram. The wooden block is a cube of side 30 cm. The bowl is a hollow hemisphere with an internal diameter of 28 cm and an external diameter of 30 cm. Find the volume of the wooden block that was not used.