2 LINEAR FUNCTIONS AND GRAPHS

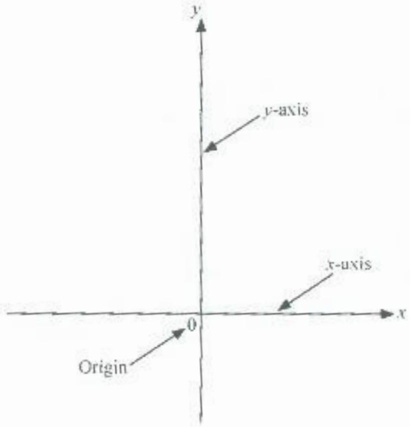
LEARNING OBJECTIVES

In this topic, we will learn to:

- state the coordinates of points on the Cartesian plane
- plot points and draw the graph of a linear function on graph paper
- recognise that the general equation of any straight line is $y = mx + c$
- find the gradient of a straight line

2.1 CARTESIAN COORDINATE SYSTEM

1. Graphs work on the coordinate system, where two major axes are used.

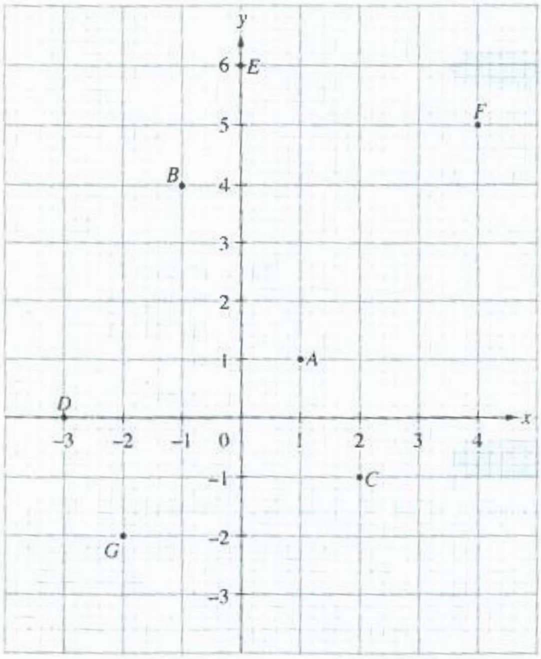


2. The position of any point on the Cartesian plane is represented as an ordered pair (a, b) of real numbers known as coordinates.

> *Note:*
> *a is known as the x-coordinate.*
> *b is known as the y-coordinate.*
> *The coordinates of the origin are (0,0).*

WORKED EXAMPLE 1

State the coordinates of each point shown in the figure.



Worked Solution:

Coordinates of A = **(1, 1)**

Coordinates of B = **(–1, 4)**

Coordinates of C = **(2, –1)**

Coordinates of D = **(–3, 0)**

Coordinates of E = **(0, 6)**

Coordinates of F = **(4, 5)**

Coordinates of G = **(–2, –2)**

Student's common mistake:

Students may incorrectly write the coordinates of B in the example as (4, –1). When writing the coordinates of a point, always begin with the x-coordinate.

2.2 LINEAR FUNCTIONS

1. A function is a relationship between two variables x and y such that each input x produces exactly one output y.

WORKED EXAMPLE 2

The equation of a function is $y = 3x + 2$. Find

(a) the value of y when $x = 1$,

(b) the value of x when $y = 8$,

Worked Solution:

(a) $y = 3x + 2$

When $x = 1$,

$y = 3(1) + 2$

$= \mathbf{5}$

(b) $y = 3x + 2$

When $y = 8$,

$8 = 3x + 2$

$3x = 6$

$x = \mathbf{2}$

WORKED EXAMPLE 3

The table shows the incomplete values of a graph of the function $y = x - 3$.

x	−2	−1	0	1	2
y	−5		−3		

(a) Complete the table.

(b) Using the completed table of values, plot the graph using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis.

(c) From the graph, find

(i) the value of x when $y = -1.5$,

(ii) the value of y when $x = -0.5$.

Worked Solution:

(a)

x	−2	−1	0	1	2
y	−5	**−4**	−3	**−2**	**−1**

When $x = -1$, $y = (-1) - 3$

$= -4$

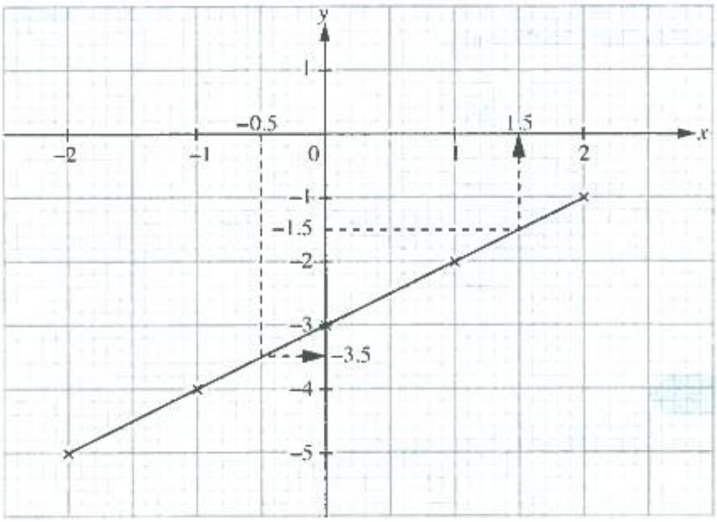
When $x = 1$, $y = (1) - 3$

$= -2$

When $x = 2$, $y = (2) - 3$

$= -1$

(b)



(c) From the graph,

(i) when $y = -1.5$, $x = \mathbf{1.5}$.

(ii) when $x = -0.5$, $y = \mathbf{-3.5}$.

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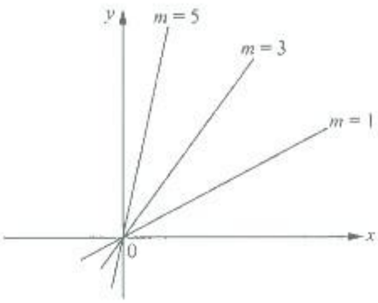
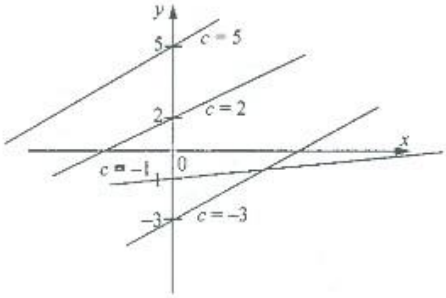
2. Any straight line in the coordinate system follows one general formula and that is $y = mx + c$.

3. The m in the equation represents the gradient of the line, which also means the steepness of the line.



> ***Note:***
> *In a positive gradient, the line rises from left to right. In a negative gradient, the line drops from left to right.*

4. The c in the equation represents the y-intercept of the line, which is the y-coordinate of the point of intersection of the line with the y-axis.



Worked Example 4

Find the gradient and y-intercept of each of the following lines.

(a) $y = 2x + 4$
(b) $y = 3x - 8$
(c) $y = -x + 2$
(d) $y = -4x - 7.5$

Worked Solution:

(a) $y = 2x + 4$
Gradient = **2**
y-intercept = **4**

(b) $y = 3x - 8$
Gradient = **3**
y-intercept = **−8**

(c) $y = -x + 2$
Gradient = **−1**
y-intercept = **2**

(d) $y = -4x - 7.5$
Gradient = **−4**
y-intercept = **−7.5**

Worked Example 5

State the equation of each of the following straight lines given the gradient and the y-intercept.

(a) gradient = 2, y-intercept = 3

(b) gradient = −1, y-intercept = $\frac{1}{2}$

(c) gradient = $\frac{1}{4}$, y-intercept = −3

Worked Solution:

(a) Equation of straight line: $y = mx + c$
$$\mathbf{y = 2x + 3}$$

(b) Equation of straight line: $y = mx + c$
$$\mathbf{y = -x + \frac{1}{2}}$$

(c) Equation of straight line: $y = mx + c$
$$\mathbf{y = \frac{1}{4}x - 3}$$

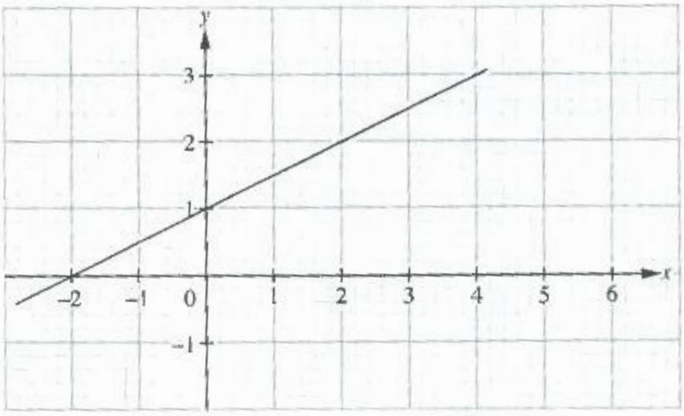
5. The gradient of a straight line is a measure of the slope or steepness of the line.

$$\text{Gradient of a line} = \frac{\text{vertical change}}{\text{horizontal change}} \text{ or } \frac{\text{rise}}{\text{run}}$$

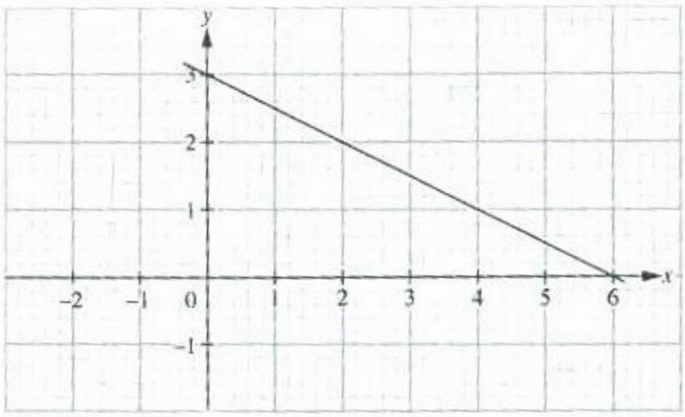
WORKED EXAMPLE 6

Find the gradient of each of the following lines.

(a)



(b)

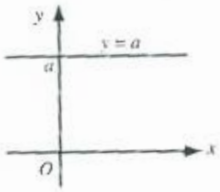


Worked Solution:

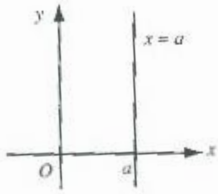
(a) $$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{3-0}{4-(-2)} = \frac{1}{2}$$

(b) $$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}} = -\frac{3-0}{6-0} = -\frac{1}{2}$$

6. The graph of $y = a$, where a is a constant, is a horizontal line. The gradient of a horizontal line is 0 since the vertical change is 0 for any horizontal change.



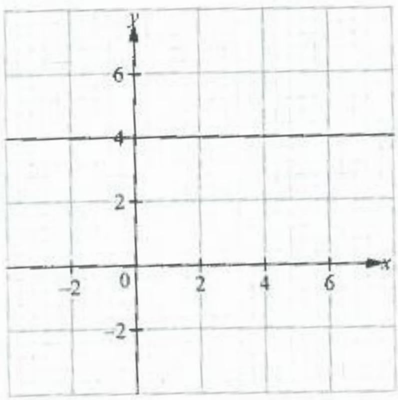
7. The graph of $x = a$, where a is a constant, is a vertical line. The gradient of a vertical line is undefined since the horizontal change is always 0.



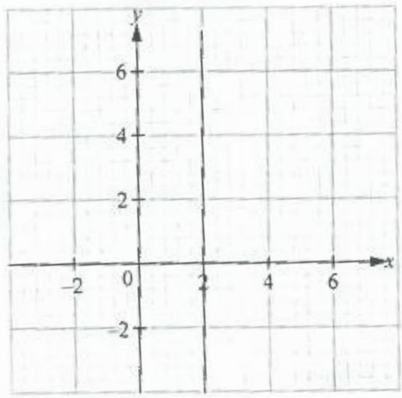
WORKED EXAMPLE 7

State the equation of each of the lines shown.

(a)



(b)



Worked Solution:

(a) $y = 4$

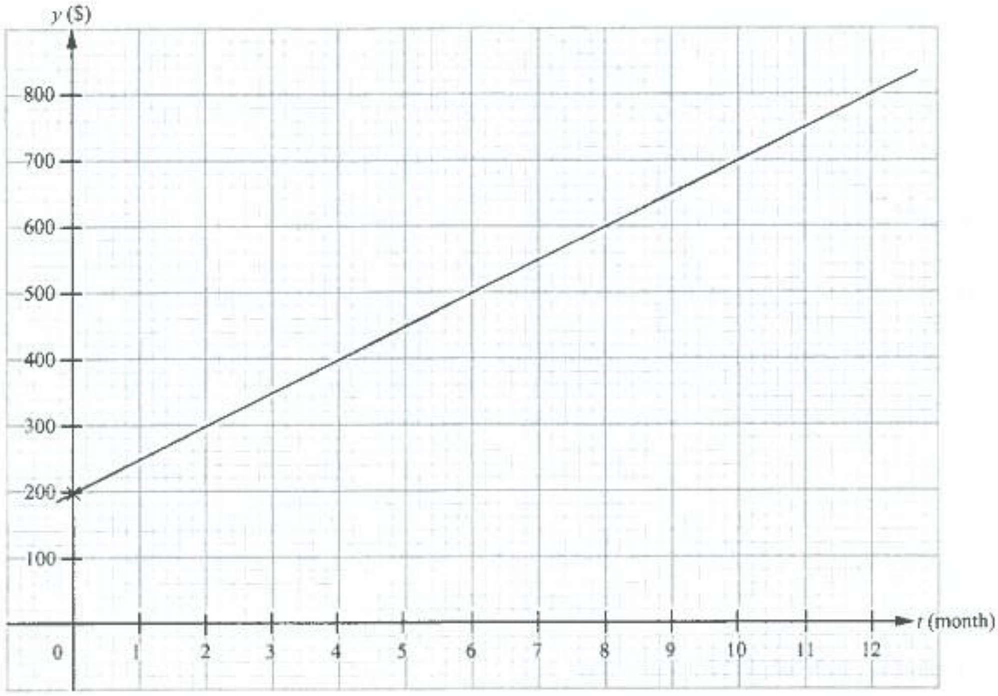
(b) $x = 2$

2.3 LINEAR GRAPHS IN REAL-WORLD CONTEXTS

1. Linear functions can be used to model some real-world situations.

WORKED EXAMPLE 8

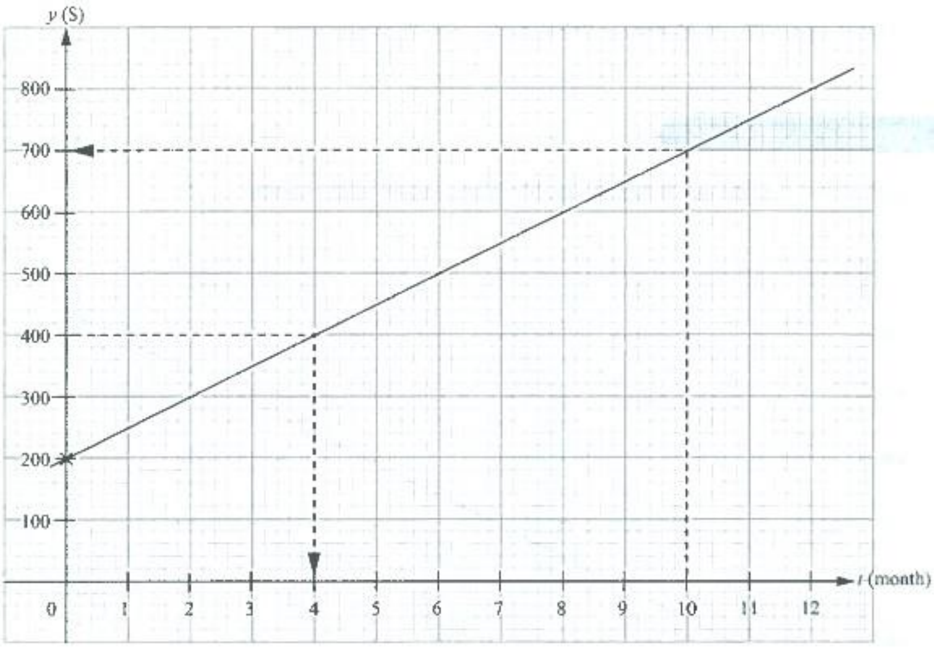
The graph shows the amount of money, $\$y$, Mr Wong earned after t months.



(a) How much did Mr Wong earn after 10 months?

(b) How many months will it take Mr Wong to earn $400?

Worked Solution:



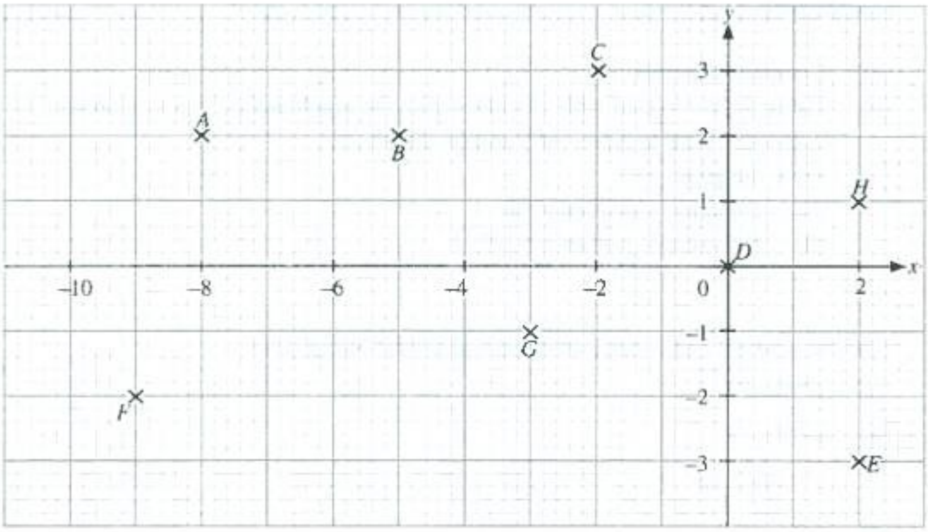
> ***Note:***
> *In this case, the variable used for the x-axis is t (not x).*

(a) From the graph, when $t = 10$, $y = 700$.
Mr Wong earned **$700** after 10 months.

(b) From the graph, when $y = 400$, $t = 4$.
Mr Wong will take **4 months** to earn $400.

PRACTICE QUESTIONS

1. State the coordinates of each point shown in the figure.



2. The equation of a function is $y = 2x + 3$. Find
   (a) the value of y when $x = 1$,
   (b) the value of x when $y = 7$.

3. The equation of a function is $y = \frac{1}{2}x - 1$. Find
   (a) the value of y when $x = 6$,
   (b) the value of x when $y = 8$.

4. The table shows the incomplete values of the graph $y = 2x + 3$.

x	–1	0	1	2	3
y		3		7	

   (a) Complete the table.
   (b) Using the completed table of values, plot the graph using 2 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y-axis.
   (c) From the graph, find
      (i) the value of x when $y = 2.5$,
      (ii) the value of y when $x = 1.5$.

5. The table shows the incomplete values of the graph $y = 7 - 2x$.

x	-3	-2	-1	0	1	2	3
y	13		9	7		3	1

(a) Complete the table.

(b) Using the completed table of values, plot the graph using 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis.

(c) From the graph, find

(i) the value of x when $y = 6$,

(ii) the value of y when $x = 1.5$.

6. Find the gradient and y-intercept of each of the following lines.

(a) $y = 3x + 4$

(b) $y = 4x - 6$

(c) $y = -2x + 12$

(d) $y = \frac{1}{2}x - 3$

7. State the equation of each of the following straight lines given the gradient and the y-intercept.

(a) gradient = 3, y-intercept = 5

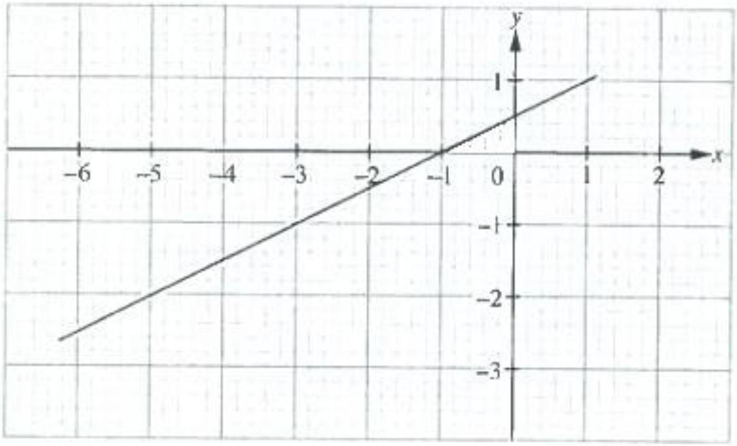
(b) gradient = -2, y-intercept = 12

(c) gradient = $-\frac{1}{3}$, y-intercept = -4

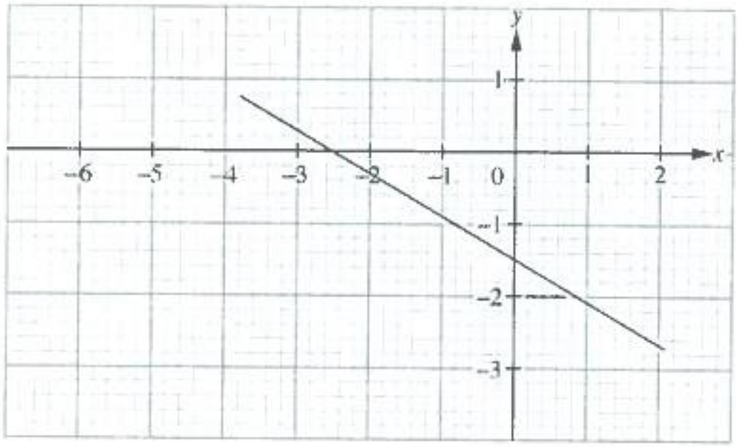
(d) gradient = $1\frac{1}{3}$, y-intercept = -2

8. Find the gradient and y-intercept of each of the following lines.

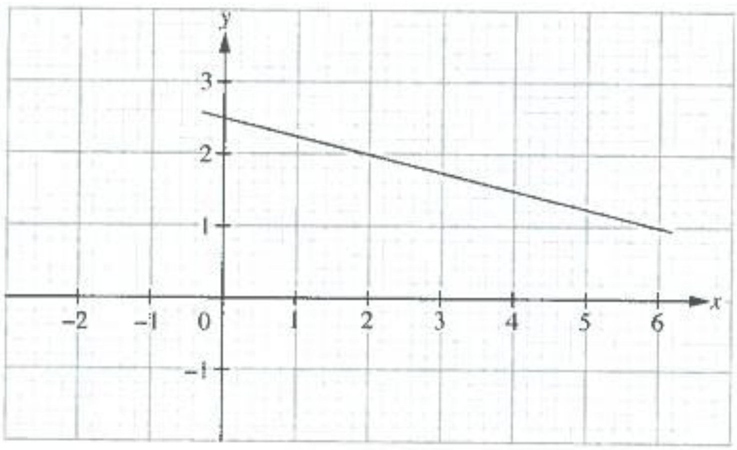
(a)



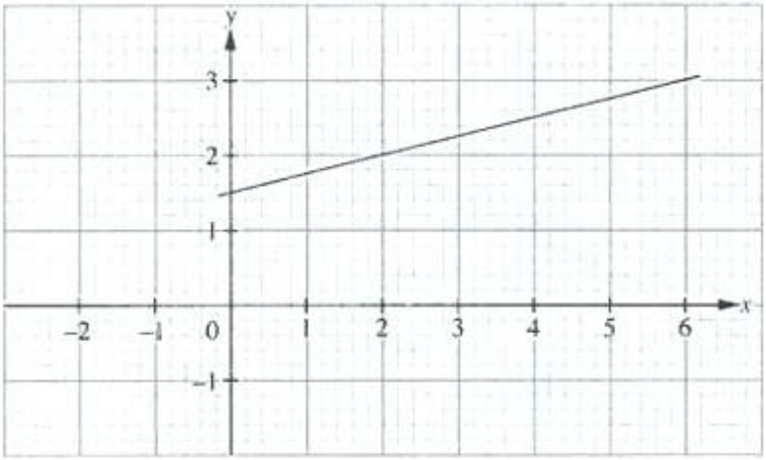
(b)



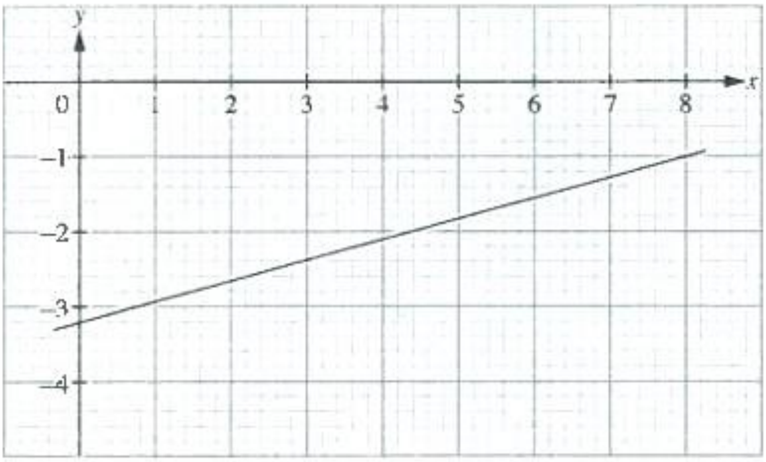
(c)



(d)



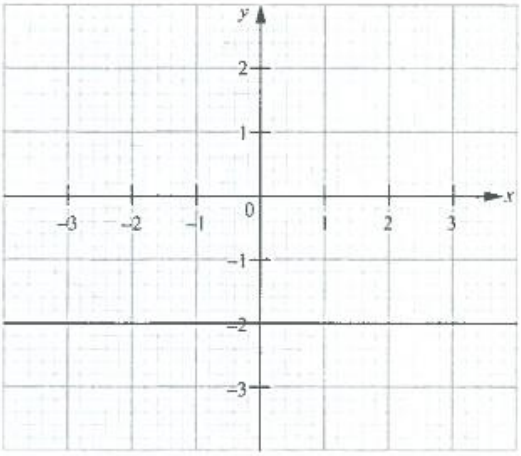
(e)



9. State the equation of each of the lines shown.

(a)

(b)



10. A motorcyclist travels 6 km every 8 minutes.

(a) Find the distance travelled by the motorcyclist in 32 minutes time.

(b) Based on the above information, draw a graph of distance travelled, D km, against time, t min, for $0 \leq t \leq 32$.

(c) From the graph, find

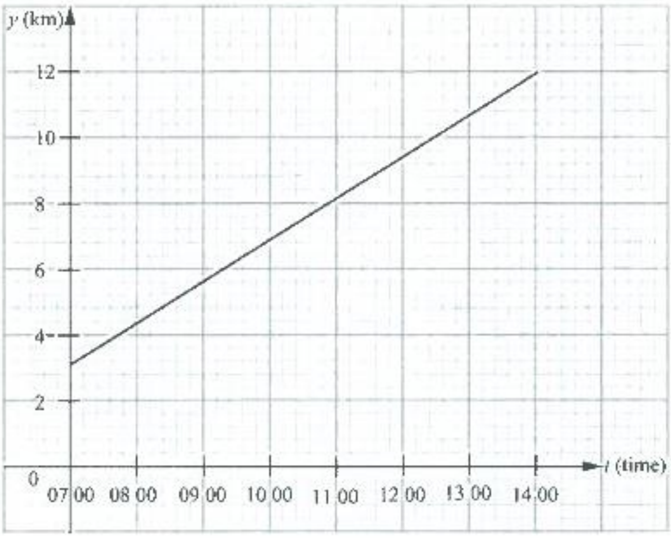
(i) the distance travelled by the motorcyclist in 22 minutes,

(ii) the time taken for the motorcyclist to travel 21 km.

(d) (i) Calculate the gradient of the graph.

(ii) Explain what the gradient represents.

11. The graph shows the distance, y km, travelled by a salesman from his home at time t on a particular day.



The salesman started to travel at 07 00.

(a) From the graph, find

(i) the time when he was 10 km away from his home,

(ii) the distance he was away from his home just before he started his journey.

(b) Assuming that the salesman was travelling at a constant speed, find his speed for the whole journey.

(c) Find the linear relationship between y and t in the form $y = mt + c$, where m is the gradient of the graph.