9 CONGRUENCE AND SIMILARITY

LEARNING OBJECTIVES

In this topic, we will learn to:

- state the properties of congruent and similar figures
- solve problems involving congruence and similarity
- interpret scales on maps

9.1 CONGRUENCE

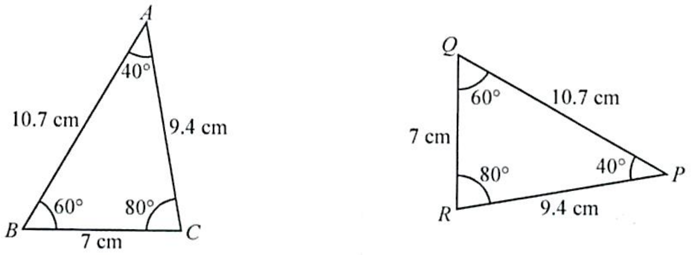
1. Two figures are congruent if they have the same shape and size. In other words, if two figures are congruent,
   (a) their corresponding sides are equal, and
   (b) their corresponding angles are equal.

2. The symbol '≡' is used to denote two figures are congruent.

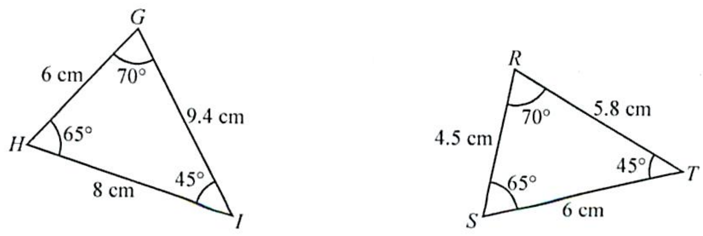
WORKED EXAMPLE 1

Are the triangles congruent? Explain your answer.

(a)



(b)



Worked Solution:

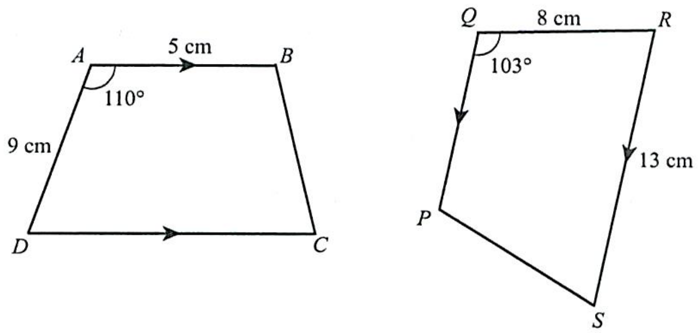
(a) $\angle BAC = \angle QPR = 40°$
$\angle ABC = \angle PQR = 60°$
$\angle ACB = \angle PRQ = 80°$
$AB = PQ = 10.7$ cm
$BC = QR = 7$ cm
$AC = PR = 9.4$ cm
Hence $\triangle ABC$ is **congruent** to $\triangle PQR$.

> ***Note:***
> *When naming congruent figures, ensure that the vertices of a figure correspond to the vertices of the other figure.*

(b) Since not all the corresponding sides are equal, $\triangle GHI$ is **not congruent** to $\triangle RST$.

WORKED EXAMPLE 2

Quadrilateral $ABCD$ is congruent to quadrilateral $PQRS$.



Find
(a) the length of PQ,
(b) $\angle BCD$.

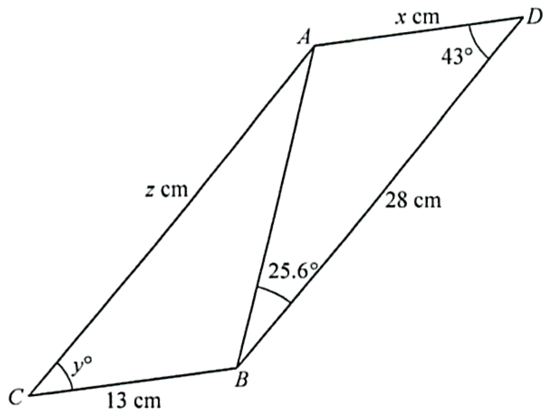
Worked Solution:

(a) Length of PQ = Length of AB
= **5 cm**

(b) $\angle ABC = \angle PQR$
$= 103°$
$\angle BCD = 180° - 103°$ (int. $\angle$s, AB // DC)
$= 77°$

WORKED EXAMPLE 3

It is given that $\triangle ACB$ is congruent to $\triangle BDA$. Find the values of x, y, and z.



Worked Solution:

$AD = BC$
$x = \mathbf{13}$

$\angle ACB = \angle BDA$
$y = \mathbf{43}$

$AC = BD$
$z = \mathbf{28}$

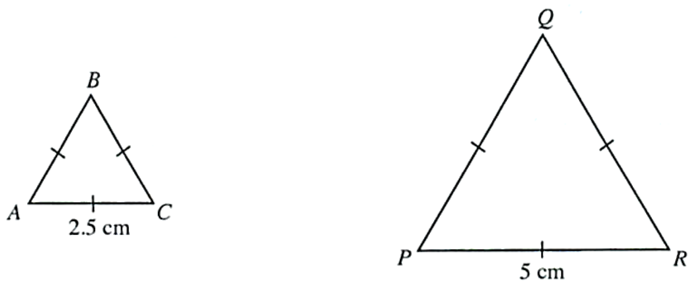
9.2 SIMILARITY

1. Two figures are similar if they have the same shape. In other words, if two figures are similar,
   (a) all their corresponding angles are equal, and
   (b) the ratios of their corresponding sides are equal.

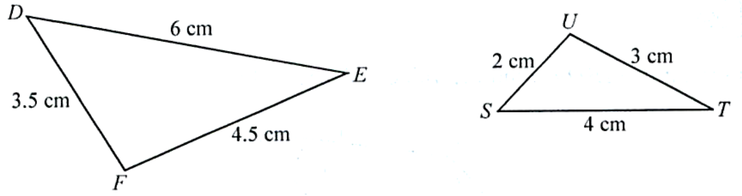
WORKED EXAMPLE 4

Are the triangles similar? Explain your answer.

(a)



(b)



Worked Solution:

(a) $\angle ABC = \angle BCA = \angle BAC = 60°$
$\angle PQR = \angle QRP = \angle QPR = 60°$
$\angle ABC = \angle PQR$
$\angle BCA = \angle QRP$
$\angle BAC = \angle QPR$
$\frac{AB}{PQ} = \frac{2.5}{5} = \frac{1}{2}$
$\frac{BC}{QR} = \frac{2.5}{5} = \frac{1}{2}$
$\frac{AC}{PR} = \frac{2.5}{5} = \frac{1}{2}$

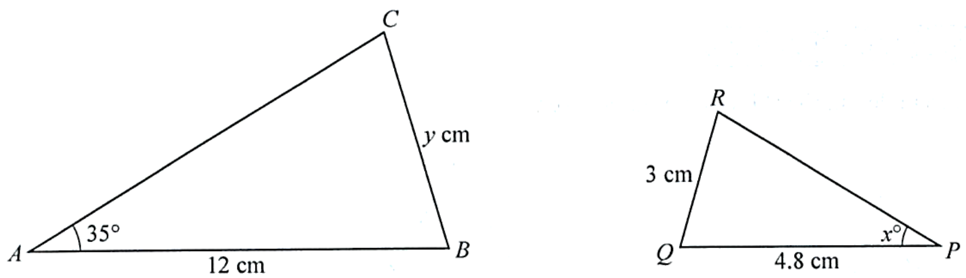
Hence $\triangle ABC$ is **similar** to $\triangle PQR$.

(b) $\frac{DE}{ST} = \frac{6}{4} = 1.5$
$\frac{FE}{UT} = \frac{4.5}{3} = 1.5$
$\frac{DF}{SU} = \frac{3.5}{2} = 1.75$

Since not all the ratios of the corresponding sides are equal, $\triangle DEF$ is **not similar** to $\triangle STU$.

WORKED EXAMPLE 5

$\triangle ABC$ is similar to $\triangle PQR$.



Find the values of x and y.

Worked Solution:

(a) $\angle QPR = \angle BAC$

$= 35°$

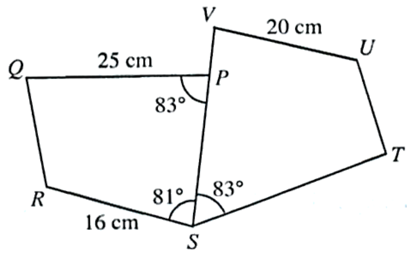
$x = \mathbf{35}$

(b) $\frac{y}{3} = \frac{12}{4.8}$

$y = \mathbf{7.5}$

WORKED EXAMPLE 6

Quadrilaterals *PQRS* and *STUV* are similar.



Find

(a) $\angle UVS$,

(b) the length of *ST*,

(c) the length of *QR* given that *UT* = 6 cm,

(d) the length of *VP* given that *VS* = 30 cm.

Worked Solution:

(a) $\angle UVS = \angle RSP$

$= \mathbf{81°}$

(b) $\frac{ST}{25} = \frac{20}{16}$

$ST = \frac{20}{16} \times 25$

$= \mathbf{31.25\ cm}$

(c) $\frac{QR}{6} = \frac{16}{20}$

$QR = \frac{16}{20} \times 6$

$= \mathbf{4.8\ cm}$

(d) $\frac{PS}{30} = \frac{16}{20}$

$PS = \frac{16}{20} \times 30$

$= 24$ cm

$VP = VS - PS$

$= 30 - 24$

$= \mathbf{6\ cm}$

9.3 MAP SCALE

1. When the linear scale on a map is 1 : x, 1 unit on the map represents x units on the actual ground.

WORKED EXAMPLE 7

On a map of scale 1 : 500 000, the distance between two towns is 18 cm. Find the actual distance between the two towns in kilometres.

Worked Solution:

1 : 500 000
1 cm : 500 000 cm
1 cm : 5000 m
1 cm : 5 km

1 cm $\longrightarrow$ 5 km
18 cm $\longrightarrow$ 5 × 18
= 90 km

The actual distance between the two towns is **90 km**.

WORKED EXAMPLE 8

Using the scale of 1 : 250 000, what is the length on the map if the actual length of an expressway is 20 km?

Worked Solution:

1 : 250 000
1 cm : 250 000 cm
1 cm : 2500 m
1 cm : 2.5 km

2.5 km $\longrightarrow$ 1 cm

1 km $\longrightarrow$ $\frac{1}{2.5}$ cm

20 km $\longrightarrow$ $\frac{1}{2.5} \times 20$

$= 8$ cm

The length on the map is **8 cm**.

WORKED EXAMPLE 9

A 60-metre bridge is represented as 4 cm on a map using a scale of 1 : n. Find the value of n.

Worked Solution:

4 cm : 60 m
4 cm : 6000 cm
4 : 6000
4 ÷ 4 : 6000 ÷ 4
1 : 1500

The value of n is **1500**.

2. When the linear scale on a map is $1 : x$, the area scale of the map is $1 : x^2$.

WORKED EXAMPLE 10

A vacant plot of land of size 1.26 km² will be developed into a theme park. What is this area, in cm², when represented on a map of scale 1 : 200 000?

Worked Solution:

$$\begin{array}{ll} 1 & : 200\,000 \\ 1\text{ cm} & : 200\,000\text{ cm} \\ 1\text{ cm} & : 2\text{ km} \\ (1\text{ cm})^2 & : (2\text{ km})^2 \\ 1\text{ cm}^2 & : 4\text{ km}^2 \end{array}$$

> ***Note:***
> *To convert from linear scale to area scale, you need to square both sides of the scale.*

$$\begin{aligned} 4\text{ km}^2 &\longrightarrow 1\text{ cm}^2 \\ 1\text{ km}^2 &\longrightarrow \frac{1}{4}\text{ cm}^2 \\ 1.26\text{ km}^2 &\longrightarrow \frac{1}{4} \times 1.26 \\ &\quad = 0.315\text{ cm}^2 \end{aligned}$$

The area represented on the map is **0.315 cm²**.

WORKED EXAMPLE 11

The map area of a plot of land is 2 cm². The actual area of the land is 162 km². The scale of the map is $1 : n$. Calculate n.

Worked Solution:

$$\begin{array}{ll} 2\text{ cm}^2 & : 162\text{ km}^2 \\ 1\text{ cm}^2 & : 162 \div 2\text{ km}^2 \\ 1\text{ cm}^2 & : 81\text{ km}^2 \\ \sqrt{1\text{ cm}^2} & : \sqrt{81\text{ km}^2} \\ 1\text{ cm} & : 9\text{ km} \\ 1\text{ cm} & : 9000\text{ m} \\ 1\text{ cm} & : 900\,000\text{ cm} \\ 1 & : 900\,000 \end{array}$$

> ***Note:***
> *To convert area scale to linear scale, you need to square root both sides of the scale.*

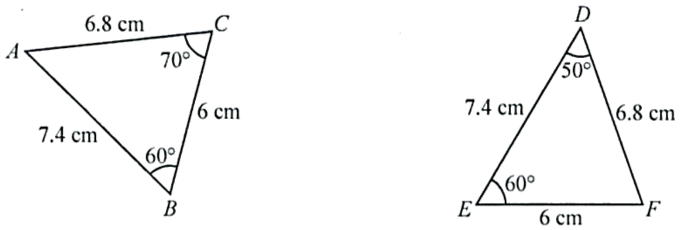
The value of n is **900 000**.

> ***Student's common mistake:***
> '1 : 900 000' means 1 cm on the map represents 900 000 cm on actual ground. It is incorrect to write the scale as '1 : 9'.

PRACTICE QUESTIONS

1. Are the triangles congruent? Explain your answer.

(a)



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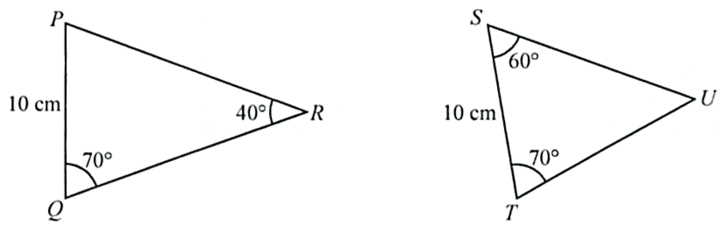
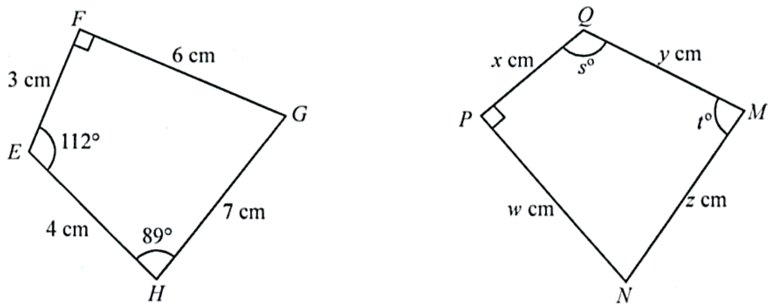
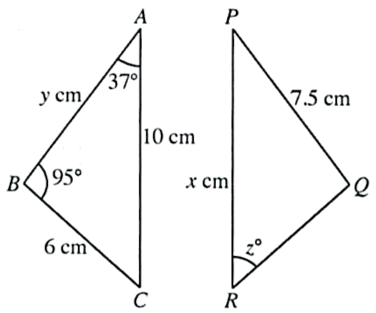


2. Quadrilateral *EFGH* is congruent to quadrilateral *QPNM*. Find the value of each unknown.



3. It is given that $\triangle ABC$ is congruent to $\triangle PQR$. Find the values of x, y, and z.

(a)



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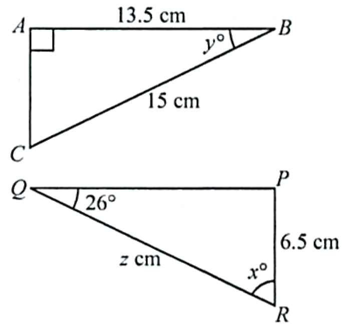
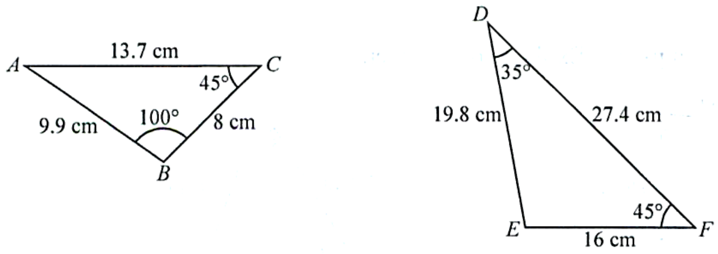


4. Are the triangles similar? Explain your answer.

(a)



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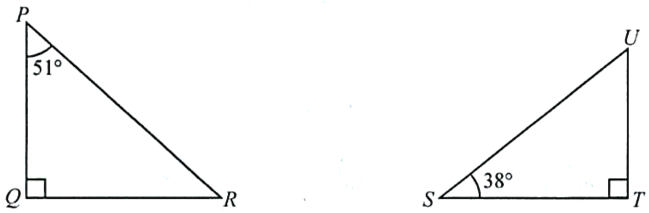
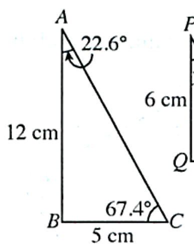


5. $\triangle ABC$ is similar to $\triangle PQR$. Find the values of x, y and z.

(a)



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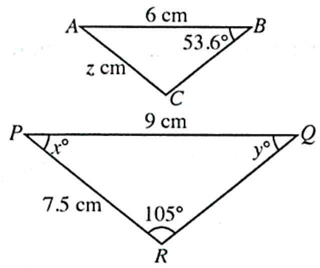

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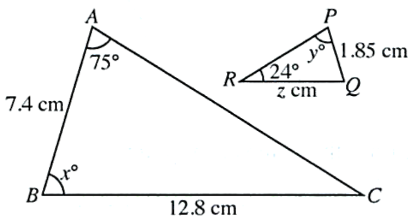

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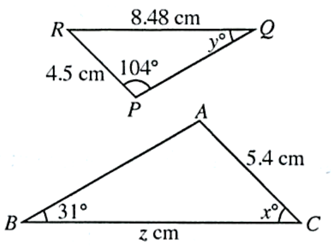

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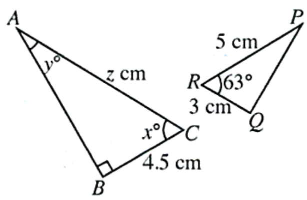
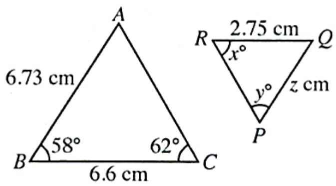
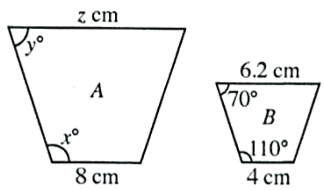


(f)



6. Quadrilaterals A and B are similar. Find the values of x, y and z.

(a)



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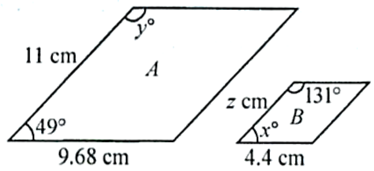
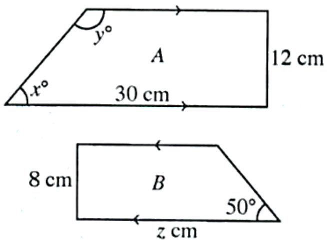
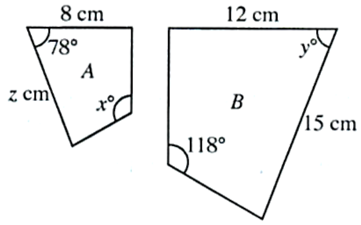


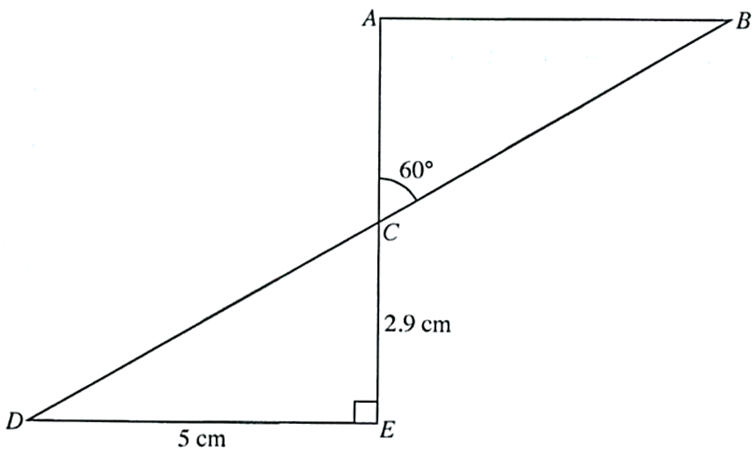
(c)



(d)



7. In the diagram, $\triangle ABC$ is congruent to $\triangle EDC$. It is given that $\angle ACB = 60°$, $CE = 2.9$ cm and $DE = 5$ cm.

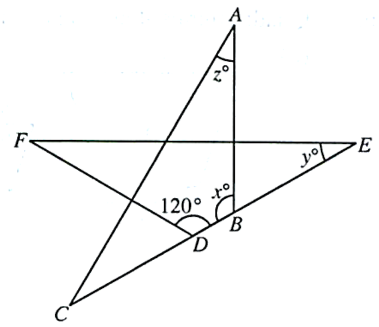


(a) Find the length of AC.

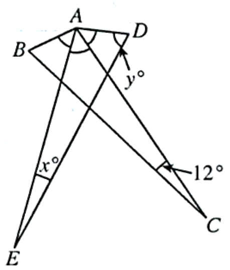
(b) Find $\angle ABC$.

(c) State the relationship between line AB and line DE.

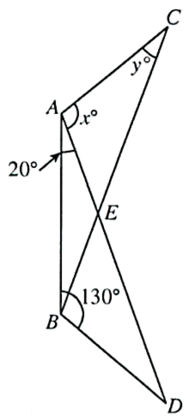
8. In the diagram, $\triangle ABC$ is congruent to $\triangle EDF$.
It is given that $\angle EDF = 120°$ and $ED = DF$.
Find the values of x, y and z.



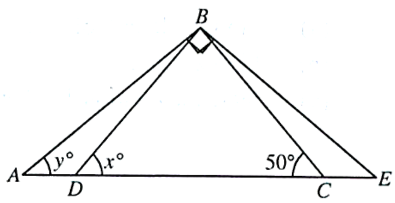
9. In the diagram, $\triangle ABC$ is congruent to $\triangle ADE$. $\angle BAD = 150°$,
$\angle ACB = 12°$ and $\angle BAE = \angle EAC = \angle DAC$.
Find the values of x and y.



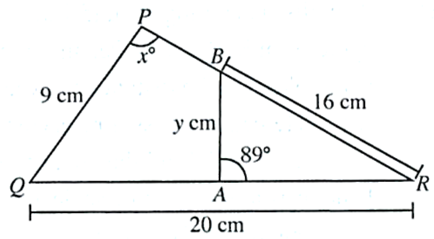
10. In the diagram, $\triangle BAC$ is congruent to $\triangle ABD$.
Given that $\angle BAD = 20°$ and $\angle ABD = 130°$,
find the values of x and y.



11. The triangles ABC and EBD are right-angled triangles
and they are congruent to one another.
Given that $\angle BCA = 50°$, find the values of x and y.



12. In the diagram, $\triangle RPQ$ and $\triangle RAB$ are similar.
Find the values of x and y.



13. Jane stands under a lamppost and the light casts a long shadow of her on the ground. Jane is 1.6 m tall. She stands 2 m from the lamppost. The length of her shadow is 2.5 m. Find the height of the lamppost.

14. Raymond travels 145 km on a straight road. He is using a road map of scale 1 : 250 000. Find the map distance, in centimetres, that he has covered.

15. The distance between two cliffs shown on a 1 : 50 000 map is 3 cm. Find the actual distance, in kilometres, between the two cliffs.

16. After walking for two hours, Betty traced her distance covered on a road map. The road map she used had a scale of 1 : 150 000 and the distance she covered was 5.5 cm. What was the actual distance, in kilometres, she covered?

17. The distance between two police stations is 45 km. This distance, when represented on a map, is 3 cm. Find the scale of the map.

18. Using the scale of 1 : 250 000, what is the ground area of a nature reserve if the map area is 2.3 cm^2?

19. When printed on a map of scale 1 : 60 000, the area of a secondary school is 4.4 cm^2 on the map. What is the actual area of the school in km^2?

20. 12 cm^2 on the map is equivalent to 432 km^2 on the actual ground if the scale of 1 : n is used. What is the value of n?

21. A cyclist is cycling around Malaysia. He is using a road map of scale 1 : 750 000. He plans to cycle from Malacca to Penang which is 510 km apart.

(a) What is the distance between the two cities on the map?

(b) He came across a reservoir while cycling on the road. The actual area of the reservoir is 2.25 km^2. Find the area of the reservoir, in cm^2, on the map.

If he uses a new map of scale 1 : n, the distance between Malacca and Penang is 40.8 cm when represented on this new map.

(c) Find the value of n.

(d) Find the area of the reservoir, in cm^2, on the new map.