Answer Keys

Chapter 11: Gradients, Derivatives and Differentiation Techniques

Tutorial 11.1

1. (a) $8x^7$ (b) $\frac{1}{2\sqrt{x}}$
   (c) $-\frac{5}{2\sqrt{x^7}}$ (d) $-\frac{1}{3\sqrt[3]{x^4}}$
2. (a) $12x$ (b) $\frac{9}{x^2}$
   (c) $\frac{21}{16}\sqrt{x}$ (d) 0
3. (a) $12x^3 - \frac{2}{x^2}$
   (b) $-\frac{16}{x^3}$
4. $2x$
5. (i) $\frac{1}{5}$
6. (a) 9 (b) $-\frac{1}{64}$
   (c) -11 (d) 0
7. $a = 56, b = -1$
8. $a = \frac{14}{3}, b = \frac{1}{3}$
9. $y = \frac{1}{2}x^2 - 4x + 5$

Tutorial 11.2

1. (a) $30(6x+1)^4$
   (b) $-72(2-9x)^7$
   (c) $6x\left(5+\frac{1}{4}x^2\right)^{11}$
   (d) $-\frac{3}{(3x-10)^2}$
   (e) $12(3x^6 - x - 8)^5(18x^5 - 1)$
   (f) $\frac{84x}{(9-2x^2)^4}$
   (g) $\frac{50}{3\sqrt[3]{(7+5x)^4}}$
   (h) $\frac{36}{5}\left(\sqrt{x^3} - \frac{1}{x}\right)^8\left(\frac{3}{2}\sqrt{x} + \frac{1}{x^2}\right)$
3. (i) $\frac{3}{4(4x+1)} + \frac{5}{4(4x+1)^2}$
   (ii) $-\frac{3}{(4x+1)^2} - \frac{10}{(4x+1)^3}$
4. (a) $\frac{21}{4}$ (b) 1
   (c) $-\frac{9}{2}$ (d) 0
5. $a = -3, b = 4$ or $a = 3, b = -4$

Tutorial 11.3

1. (a) $(5x+2)^7(45x+2)$
   (b) $18(10-3x)^3(4-5x)$
   (c) $(2x+1)^4(56x^2 - 4x + 69)$
   (d) $20x(2x^2+3)^8(8x^2 - 15)$
2. (a) $\frac{(x+1)^5(13x+1)}{2\sqrt{x}}$
   (b) $\frac{8-48x}{\sqrt{1-4x}}$
   (c) $\frac{63x-5}{2\sqrt{7x-2}}$
   (d) $\frac{9-20x\sqrt{x}-3x^2}{4\sqrt{9x-x^3}}$
3. (a) $3(x-3)^7(20x^2 - 9x - 1)$
   (b) $4(10x^2+7)^4\left(100x - 720 + \frac{56}{x^2}\right)$
4. $a = -1, b = 1$
5. $a = 4, m = 71$
6. (i) $(2-px)^6(-9px^2 + 4x - 35p)$

Tutorial 11.4

1. (a) $-\frac{3}{(10x-3)^2}$
   (b) $\frac{14-7x^2}{(2+x^2)^2}$
   (c) $\frac{44}{(4x+5)^2}$
   (d) $\frac{18x^2 - 108x^3}{(1-9x)^2}$
2. (a) $\frac{5x+70}{2\sqrt{(x+7)^3}}$
   (b) $-\frac{7x+4}{2\sqrt{x}(7x-4)^2}$
   (c) $\frac{3x(2-9\sqrt{x})}{(1-6\sqrt{x})^2}$
   (d) $\frac{90+32x}{3\sqrt[3]{(3+8x)^4}}$
3. $a = 3, b = 16$
4. (i) $-\frac{h^2x^2 + 14hx + h}{(hx^2-1)^2}$
   (ii) $h \leq 49$
5. (i) $(6x^2+1)(x-4)$
   (ii) $\frac{1}{x-4} - \frac{6x+24}{6x^2+1}$
   (iii) $-\frac{97}{16}$

Tutorial 11.5

1. (a) $\frac{1}{4}$; 0
   (b) $21x^2 + \frac{2}{x^2}$; $42x - \frac{4}{x^3}$
   (c) $-\frac{1}{2\sqrt{(x+8)^3}}$; $\frac{3}{4\sqrt{(x+8)^5}}$
   (d) $48x(5+3x^2)^7$;
   $240(5+3x^2)^6(9x^2+1)$
   (e) $(6x-1)^3(30x-1)$;
   $48(6x-1)^2(15x-1)$
   (f) $\frac{x^2+8x}{(x+4)^2}$; $\frac{32}{(x+4)^3}$
2. (i) $6x^3\left(\frac{1}{6}x^4 + 11\right)^8$
3. 2
4. $a = \frac{1}{4}$ and $b = \frac{10}{81}$
5. No
6. (i) $6x^5$; $30x^4$
   (iii) $\frac{d^2y}{dx^2} = 30\sqrt[3]{(y+27)^2}$
7. (a) True (b) True

Tutorial 11.6

1. $0 < x < \frac{4}{9}$
2. $t < 0$
3. (a) $x < -\frac{8}{3}$ or $x > 1$
   (b) $x > 0$
4. $-3 < x < 2$
5. (i) $x < -5$ or $x > 3$
   (ii) 81 or -175
6. $a = 6, b = -15$
8. (i) \$20 000

Quick Test 11

1. (a) $\frac{1}{4\sqrt{x}} + \frac{\sqrt{2}}{x^2}$
   (b) $90x^2 - 25x^4 - 2x$
2. (a) $\frac{144}{(4-3x)^7}$
   (b) $\frac{144}{(4-3x)^7}$
3. $\frac{1}{48}x^4 + 2x^2 + 7x - 9$
4. (i) $x \leq -3$ or $x \geq 3$
   (ii) 0.972 rad
   (iii) Yes

Chapter 12: Applications of Differentiation

Tutorial 12.1

1. $y = 208x - 400$
2. (i) -4 (ii) $y = -4x - 2$
3. $y = -\frac{3}{17}x - \frac{14}{17}$
4. (i) $-2 < k < 2$
   (ii) $y = -\frac{1}{18}x + \frac{77}{9}$
5. (i) $2x - \frac{2}{x^2}$
   (ii) $y = -\frac{9}{2}x - 6$
   (iii) $-\frac{1}{2}$
6. (i) 1.29 rad
   (ii) $y = \frac{2}{7}x + \frac{62}{7}$

Tutorial 12.2

1. -0.8 units/s
2. $\frac{17}{4}$
3. $\frac{5}{7}$
4. (ii) $\frac{p}{168}$ cm/s
5. (i) $-4\pi kr^2$ cm³/min
   (ii) $\frac{1}{16}I$ cm³/min
6. 0.72 m/s
7. 1.6 m/s

Tutorial 12.3

1. (i) $3x^2 + 2x - 5$
   (ii) $\left(-\frac{5}{3}, \frac{67}{27}\right)$, maximum point; $(1, -7)$, minimum point
2. (i) $18 + 30x - 12x^2$
   (ii) $\left(-\frac{1}{2}, \frac{9}{4}\right)$, minimum point; $(3, 88)$, maximum point
3. (i) $(2, 1)$
4. (ii) $\left(\frac{1}{3}, 27\right)$ (iii) $y = x^3$
5. (i) $\left(-\frac{7}{3}, \frac{1760}{27}\right)$, $(5, -132)$
   (ii) $\left(-\frac{7}{3}, \frac{1760}{27}\right)$, maximum point; $(5, -132)$, minimum point
6. (i) $(2, 5)$, $(-2, 5)$
   (ii) $(2, 5)$, minimum point; $(-2, 5)$, minimum point
7. (i) 3
   (ii) minimum point
8. $(\sqrt{3}, 3)$, minimum point; $(-\sqrt{3}, 3)$, minimum point
9. (i) $a = 1, b = 5$
   (ii) (a) Increasing
   (b) Decreasing
   (iii) Maximum point
   (iv) 0
10. (i) -6 or 2 (ii) -216
11. (i) $p = -\frac{3}{8}, q = 3$
    (iii) $(2, 3)$, maximum point

Tutorial 12.4

1. 961
2. (i) $\frac{49}{5k}$ (ii) $k < 0$
3. (ii) $6x - \frac{3}{5}x^2$
   (iii) 15
4. (ii) $x = \frac{1}{\sqrt{2}}$, $T = 0.805$
5. (i) $\frac{240 - 3x}{2\pi}$
   (iii) 49.9 (iv) Minimum
6. Height $= 5\sqrt{3}$ cm, radius $= 5\sqrt{6}$ cm

Quick Test 12

1. (i) $\left(\frac{1}{\sqrt{2}}, \sqrt{3} - 2\sqrt{2}\right)$, minimum point; $\left(-\frac{1}{\sqrt{2}}, \sqrt{3} + 2\sqrt{2}\right)$, maximum point
   (ii) $y = \frac{1}{6}x + \sqrt{3}$
2. $-\frac{5}{72}$ cm/s
3. (ii) $r = 2.44, h = 3.42$

Chapter 13: Differentiation of Trigonometric, Exponential and Logarithmic Functions and their Applications

Tutorial 13.1

1. (a) $4\cos x + 7\sin x$
   (b) $5 + \frac{1}{8}\sec^2 x$
   (c) $6(\tan x + 3\cos x)^5(\sec^2 x - 3\sin x)$
   (d) $-\frac{\cos x}{\sqrt{9 - 2\sin x}}$
   (e) $8x^2\sec^2 x + 16x\tan x$
   (f) $324x^3(\cos x - x\sin x)\cos^3 x$
   (g) $\frac{(x+2)\cos x - \sin x}{(x+2)^2}$
   (h) $\frac{6(\tan x - x - x\tan^2 x)}{(\tan x - x)^2}$
2. (a) $-3\sin(3x - 5)$
   (b) $\sec^2\frac{1}{2}x + \cos 4x$
   (c) $-6\cos\left(\frac{2\pi}{3} - 6x\right)$
   (d) $8x\cos x^2$
   (e) $-7\sec\left(\frac{\pi}{4} - 7x\right)\tan\left(\frac{\pi}{4} - 7x\right)$
   (f) $\frac{3\sin 2x - 6x\cos 2x}{\sin^2 2x}$
   (g) $16x\sec^2 4x + 4\tan 4x$
   (h) $-0.1\sin 0.1x\sec^2(\cos 0.1x)$
3. (a) $45\sin^8(5x - 2)\cos(5x - 2)$
   (b) $2.4\tan^2 0.4x\sec^2 0.4x$
   (c) $-\frac{2\cos^5 x\sin x}{\sqrt[3]{(\cos^6 x + 8)^2}}$
   (d) $-9\,\mathrm{cosec}^2 9x$
   (e) $-\frac{56\sin(1 - 7x)}{\cos^3(1 - 7x)}$
   (f) $-\frac{7\sec^2\frac{1}{2}x}{6\tan^8\frac{1}{2}x}$
   (g) $2x\cos x^2 - 2x^3\sin x^2$
   (h) $-\frac{\sin^2 x + 3\cos^2 x}{\sin^4 x}$
4. 16
5. -91
6. (i) 5 (ii) 0.25 units/s
7. (i) $\left(-\frac{\pi}{2}, 5 - \sqrt{2}\right)$
   (ii) Minimum point
8. (ii) 1.13

Tutorial 13.2

1. (a) $7e^x$ (b) $\frac{\sqrt{3}}{2}e^x$
   (c) $-3e^x(9 - e^x)^2$
   (d) $10e^x(e^x + 4e)^9$
   (e) $(x + 1)e^x$
   (f) $e^x(5\sec^2 5x + \tan 5x)$
   (g) $\frac{3e^x(x - 1)}{x^2}$
   (h) $\frac{1 - x}{4e^x}$
2. (a) $\frac{10}{9}e^{5x}$ (b) $\frac{1}{5}e^{x-7}$
   (c) $-4e^{-4x-8}$
   (d) $16(6e^{2x} + e^{-2x})^7(6e^{2x} - e^{-2x})$

(e) $(7\sec^2 x + 5\sin 5x)e^{7\tan x - \cos 5x}$

(f) $\left(\frac{3}{2}\sqrt{x^3} + 6x\right)e^{\sqrt{x}}$

(g) $2(2+x)e^{\frac{1}{2}x-1}$

(h) $e^{x+e^x}\cos\left(e^{e^x}\right)$

3. $p = \frac{1}{3}, q = -\frac{1}{4}$
4. 12
5. (i) $2e^{2x} - 2e^{-2x}$
   (ii) $\frac{1}{4e^8 - 4e^{-8}}$ units/s
6. Yes
8. 0
9. (i) 40 (ii) $20e^{12}(T+2)$
10. (i) $p = 80, k = 0.347$
    (ii) 1.23 g/h (iii) Leon

Tutorial 13.3

1. (a) $\frac{5}{x}$ (b) $\frac{128\ln x}{x}$
   (c) $\frac{\pi}{x}$ (d) $\frac{9}{x}(\ln x + 4)^8$
   (e) $e^x\left(\frac{1}{x} + \ln x\right)$
   (f) $3x(1 + 2\ln x)$
   (g) $1 - \frac{1}{2x}$ (h) $\frac{x+1-x\ln x}{x(x+1)^2}$
2. (a) $\frac{1}{x-5}$ (b) $\frac{30x^2}{2x^3+7}$
   (c) $\frac{9x}{4(1+9x^2)}$
   (d) $\frac{8(e^{-x}+1)}{x-e^{-x}}$
   (e) $-\cot x$ (f) $-\frac{\tan\sqrt[3]{x}}{3\sqrt[3]{x^2}}$
   (g) $2e^{4x}\left[\frac{3}{\sin 3x\cos 3x} + 4\ln(\tan 3x)\right]$
   (h) $\frac{2x\ln 2x - x}{(\ln 2x)^2}$
3. $f'(a)$
5. (ii) 0.0559 units/s
6. (i) 0 (iii) (e, e)
7. (i) $x > 1$
   (ii) $\frac{8x(\ln x)\cos 4x - 2\sin 4x}{x(\ln x)^2}$
8. (i) e
9. 3.39
10. Decrease throughout the next 10 years

Quick Test 13

1. (a) $e^{7x}(28\tan x - 7 + 4\sec^2 x)$
   (b) $\frac{10x^2 - (5x^2+9)\ln(5x^2+9)}{x^2(5x^2+9)}$
2. (ii) $y = -\frac{1}{(\ln 8)8^p}x + \frac{p}{(\ln 8)8^p} + 8^p$
3. −0.681 units/s
4. (ii) Minimum point

Chapter 14: Integration

Tutorial 14.1

1. (a) $\frac{1}{8}x^8 + c$
   (b) $-\frac{1}{2x^2} + c$
   (c) $\frac{4}{9}\sqrt[4]{x^9} + c$
   (d) $\frac{3}{2}\sqrt[3]{x^2} + c$
2. (a) $x^3 + c$
   (b) $-\frac{1}{2x} + c$
   (c) $\frac{1}{12}\sqrt{x^3} + c$
   (d) $0.4x + c$
3. (a) $3x^3 - 10x + c$
   (b) $\frac{3}{2}x^6 + \frac{51}{4}x^2 + c$
   (c) $32x - 6x^2 + 2x^4 - \frac{3}{5}x^5 + c$
   (d) $\frac{15}{7}\sqrt[3]{x^7} - \frac{15}{2}\sqrt[3]{x^4} + 6\sqrt[6]{x^5} + c$
4. (i) $27 - \frac{81}{x^3} + \frac{81}{x^6} - \frac{27}{x^9}$
   (ii) $27x + \frac{81}{2x^2} - \frac{81}{5x^5} + \frac{27}{8x^8} + c$
5. (a) $\frac{1}{36}(4x+7)^9 + c$
   (b) $-\frac{1}{12}\sqrt[3]{(2-9x)^4} + c$
   (c) $-\frac{1}{36(6x+1)^{10}} + c$
   (d) $\frac{1}{320(8-x)^5} + c$
   (e) $-\frac{9}{x-4} + c$
   (f) $\frac{1}{4}x + \frac{9}{8(2x+3)} + c$
6. 2
7. (i) $72x^3(5 + 2x^4)^8$
   (ii) $\frac{1}{72}(5+2x^4)^9 + c$
8. (i) $a = 6, b = -5, n = 7$
   (ii) $-3x^2(7-x)^8 + c$
9. (ii) $\frac{2x^2+5}{\sqrt{2x+1}} + \frac{5}{\sqrt{2x+1}} + c$
10. (i) $\frac{9}{4}x^2\left(\frac{x^3}{8} - 3\right)^5$
    (ii) $\frac{4}{9}\left(\frac{x^3}{8} - 3\right)^6 + c$
    (iii) $y = \frac{1}{288}x + \frac{9215}{144}$
11. (a) $f(x) = x^3, g(x) = x^3 + 4$
    (b) Degree of $\int f(x)\times g(x)\,dx$ = Degree of $f(x)$ + degree of $g(x) + 1$
12. (i) $-3x - 7$
    (ii) $y = -\frac{3}{2}x^2 - 7x - 6$
    (iii) 2
13. (ii) $y = x^3 - 2x^2 + 4x - 7$
14. (ii) $y = \frac{2}{81}x^3 + \frac{1}{27}x^2 - \frac{4}{9}x$
    (iii) $-\frac{44}{81}$
15. (i) $\frac{5}{3}x^3 + \frac{3}{2}x^2 - 8x + \frac{5}{6}$
    (ii) $\left(-1\frac{3}{5}, 10\frac{97}{150}\right)$, (1, −4)
    (iii) $\left(-1\frac{3}{5}, 10\frac{97}{150}\right)$, maximum point; (1, −4), minimum point
16. (ii) $-\frac{1}{3}, \frac{1}{2}$ or 4

Tutorial 14.2

1. (a) $8\sin x - 2\cos\frac{1}{2}x + c$
   (b) $25\cos\left(\frac{\pi}{4} - \frac{1}{5}x\right) + c$
   (c) $\frac{1}{3}\tan\left(3x - \frac{\pi}{6}\right) + c$
   (d) $\frac{45}{4}\tan 0.4x + c$
2. (a) $\frac{4}{7}\tan x + \frac{3}{7}x + c$
   (b) $\frac{1}{2}x + \frac{1}{36}\sin(18x+4) + c$
   (c) $-\frac{5}{12}\sin\frac{6}{5}x + \frac{1}{2}x - \frac{1}{2}\pi x^2 + c$
   (d) $\frac{1}{3}x^3 + \sqrt{2}x - 2\tan\frac{x}{\sqrt{2}} + c$
3. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$
4. (a) $y = \cos 2x + 1$
   (b) Amplitude = 1, period = π
5. $k = \frac{1}{2}$, $(2\pi, -4)$
6. $h = \frac{1}{2}t - \frac{1}{16}\sin(8t+2) + 8.06$
7. (ii) $-\cos x - \cot x + c$

8. (i) $3(x^2-1)\cos(x^3-x)$
   (ii) $-\frac{1}{3}$
9. (ii) $x^2-\tan 2x-\frac{1}{2}\cos 2x+c$
10. (i) $x\cos x+\sin x$
    (ii) $x\sin x+\cos x+c$
    (iii) $2x\cos x-x^2\sin x$; $2x\sin x+2\cos x-x^2\cos x+c$
11. $\tan\frac{x}{2}-2$

Tutorial 14.3

1. (a) $\frac{1}{10}e^{10x}+c$
   (b) $-\frac{2}{5}e^{8-x}+c$
   (c) $-\frac{7}{4e^{4x}}+c$
   (d) $\frac{8}{9}e^{9x+12}+c$
   (e) $-e^{-x}-\frac{4}{3}e^{-3x}-\frac{4}{5}e^{-5x}+c$
   (f) $\frac{12}{11}e^{\frac{11}{4}x-\frac{1}{2}}+c$
   (g) $\frac{1}{4}e^{4x}+e^{2x}-15x+c$
   (h) $-\frac{e}{7x^7}-\frac{1}{7}e^{1-7x}+c$
2. (a) $\frac{1}{9}\ln x+c$
   (b) $\ln(x-5)+c$
   (c) $-6\ln(x+4)+c$
   (d) $-7\ln(3-x)+c$
   (e) $\frac{2}{3}\ln(3x-11)+c$
   (f) $\frac{1}{2}\ln(1+8x)-\frac{9}{8x}+c$
   (g) $-\frac{3}{2x}-4\ln x-\frac{1}{2}x+c$
   (h) $\ln(x-1)+c$
3. (a) $x-5\ln(x+5)+c$
   (b) $\frac{1}{12}\ln(4x^3-1)+c$
   (c) $e^{3+\sin x}+c$
   (d) $e^{\tan x}+c$
4. (ii) $3e^x-4e^{-x}+6$
5. (i) $x^2+3x^2\ln x$
   (ii) $\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+c$
6. (i) $1+\frac{9}{5x-9}$
   (ii) $\frac{5x}{5x-9}+\ln(5x-9)$
   (iii) $\left(x-\frac{9}{5}\right)\ln(5x-9)-x+c$
7. (i) $-\operatorname{cosec}^2 x$
   (iii) $-\cot x-2\ln\left(\frac{\sin x}{1-\cos x}\right)+c$
8. (i) $4+\frac{3}{2x+1}$
   (ii) Increasing
   (iii) Decreasing
   (iv) $4x+\frac{3}{2}\ln(2x+1)+5$
9. (i) $\frac{1}{x}-\frac{2}{2x-1}+\frac{3}{(2x-1)^2}$
   (ii) $\ln x-\ln(2x-1)-\frac{3}{2(2x-1)}+c$
10. (a) $2\ln\frac{x-4}{3x+5}+c$
    (b) $x+\frac{1}{3}\ln(3x+5)+2\ln(x-4)+c$
11. (i) $A=7, B=4$
    (ii) $\frac{2x}{x^2+9}$
    (iii) $\frac{7}{3}\ln(3x-1)+2\ln(x^2+9)+c$
12. (i) $(x^2+16)(x-4)$
    (ii) $\frac{1}{4}\ln(x-4)-\frac{1}{8}\ln(x^2+16)+c$
    (iii) $x>4$

Quick Test 14

1. $\left(-\frac{10}{3},\frac{200}{27}\right)$, minimum point; (4, 106), maximum point
2. (ii) $-\frac{1}{x}-\frac{\ln x^2}{2x}+c$
   (iii) $-\frac{1}{x}-\frac{\ln x^2}{2x}+\frac{1}{2}$

Chapter 15: Applications of Integration

Tutorial 15.1

1. (a) 625 (b) $\frac{27}{2}$
   (c) 10 (d) −549
   (e) $\frac{1}{528}$ (f) $\frac{7}{72}$
   (g) $-\frac{63}{4}$ (h) $-\frac{85}{32}$
2. (a) $\frac{3}{2}$ (b) $\frac{1}{5}$
   (c) 36 (d) $-\frac{1}{8}$
   (e) $-\frac{\sqrt{3}+\pi}{6}$
   (f) $\frac{\pi}{2}-\frac{9}{8}\sqrt{3}$
   (g) $\frac{\pi}{4}$
   (h) $\frac{43\pi}{3}-\frac{9}{2}\sqrt{3}-30$
3. (a) $\frac{1}{3}(e^{11}-e^8)$
   (b) $5(e^9-e^7)$
   (c) $\frac{1}{6}e^{12}-\frac{1}{20}e^{-10}-\frac{7}{60}$
   (d) $16\sqrt{e}+\frac{2}{\sqrt{e}}-18$
   (e) $16-16e^{\frac{1}{2}}+15e+e^2$
   (f) $6e^{-\frac{1}{2}}-\frac{1}{2}e^{-2}-6e^{-\frac{3}{2}}-\frac{53}{2}$
   (g) $\frac{1}{6}e^3+\frac{1}{2}e^{1.5}-\frac{1}{24}e^{-1.5}-\frac{1}{4}$
   (h) $e^{0.5}-e^{-0.5}$
4. (a) $9\ln 3-4$
   (b) $\frac{5}{3}\ln\frac{5}{2}$ (c) $4\ln\frac{7}{3}$
   (d) $\frac{1}{5}\ln 3-\frac{14}{3}$
   (e) $\ln\frac{58}{7}$ (f) $\ln\sqrt{2}$
   (g) $\ln(\ln 2e)$
   (h) 0
5. (i) $\frac{5}{2}\sqrt{8x+1}+c$
   (ii) 5
6. $x^2+\frac{3}{x}+c$; $\frac{45}{4}$
7. (ii) $\frac{\pi}{4}+\frac{1}{2}$
8. $\frac{\pi}{16}$
9. (i) $\frac{3}{2}x+\frac{21}{4}$ (ii) $72\ln 3-6$
10. −1.79
11. (i) $\frac{8}{4x-1}-\frac{2}{x+2}$
    (ii) $2\ln(4x-1)-2\ln(x+2)+c$; 0.256
12. (i) $\frac{8}{x-1}-\frac{8}{x-2}+\frac{8}{(x-2)^2}$
13. (i) $\frac{5x}{\sqrt{5x^2-4}}$
    (ii) $\frac{1}{5}\sqrt{5x^2-4}+c$; $\frac{1}{5}(2\sqrt{19}-1)$
14. (i) $-\frac{16}{x^2}\left(1+\frac{1}{x}\right)^{15}$
    (ii) -2^{-20}
15. (i) $e^{4x}(4x+1)$
16. (ii) $p=\frac{10}{9}$, $q=-\frac{2}{9}$
17. (ii) $\sec x$

18. (a) 17 (b) 157
(c) $-\frac{3}{2}$ (d) 4

19. (a) Not possible to find
(b) $-h-k$
(c) $8k-3h$
(d) Not possible to find

Tutorial 15.2

1. (a) $\frac{32}{3}$ units^2
(b) 6 units^2
(c) 0.0859 units^2
(d) 1.01 units^2

2. (a) 24 units^2
(b) 7.41 units^2
(c) 0.879 units^2
(d) 0.183 units^2

3. (a) $\frac{10}{3}$ units^2
(b) 53.6 units^2
(c) 2.28 units^2
(d) 1.30 units^2

4. (i) 3 (ii) $-\frac{5}{11}$

5. (i) $A=2, B=-1$
(ii) $\frac{2}{3}e^{3x}+e^{-x}+c$; 270

7. (i) 1 (ii) -1
(iii) $\frac{3155}{162}$ units^2

8. $y=-2\cos 2x$

10. 0

11. (i) $C(3.5, 0)$
(ii) $\frac{155}{9}$ units^2

12. (ii) $\frac{27}{2}$ units^2

Tutorial 15.3

1. (i) $A(1, 3), B(2, 4)$
(ii) $\frac{7}{4}$ units^2

2. (ii) $\frac{56}{3}$ units^2
(iii) No

3. (ii) $P(-9, -3)$
(iii) $\frac{27}{2}$ units^2

4. (i) $x_A=\frac{5\pi}{8}$; $x_B=\frac{7\pi}{8}$
(ii) $\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{8}\pi\right)$ units^2

5. (i) $2-2x$ (ii) $Q(6, 0)$
(iii) $\frac{100}{3}$ units^2

6. (ii) $P(-5, 0), Q(-3, 0)$
(iii) $\frac{7}{12}$ units^2

7. (i) $S(3.5, 1)$ (ii) $\frac{1}{12}$ units^2

8. (i) $A(1.5, -1.25); C(3.5, -1.25)$
(ii) $\frac{2}{3}$ units^2

Quick Test 15

1. (i) $-\frac{5}{2}\sqrt{9-4x}+c$
(ii) 5

2. (ii) $\frac{2\sqrt{3}}{3}\cos 2x+\sin x$
(iii) $y=\frac{2}{3}x+\frac{\sqrt{3}}{6}-\frac{2\pi}{9}$

3. (ii) (a) $Q(e, 0)$
(b) $p=4, q=-\frac{1}{4}$

Chapter 16: Kinematics

Tutorial 16.1

1. (i) 27.3 m (ii) 4.80 m/s
(iii) 0.200 m/s^2

2. (ii) (a) $-12e^{-0.5t}+2t-9$
(b) $6e^{-0.5t}+2$
(iii) Yes

3. (i) t^2-2t+3
(ii) 8 m/s^2 (iv) $\frac{392}{3}$ m

4. (i) 16
(ii) Travelling away from O
(iii) 7.00 m

5. (i) $\frac{3}{2}$ (ii) 0.0797 m

6. (i) $s=t(t+1)(t-3)^2$

7. (ii) 1.15 (iv) 96 cm

Tutorial 16.2

1. (i) $-\frac{192}{(2t+1)^3}$
(ii) 13.5 m

2. (i) 18 m/s (ii) 92.3 m
(iii) 1.39 m/s^2

3. (i) 4.16 (ii) 19.7 cm

4. (i) $\frac{2}{3}$ or $\frac{5}{2}$ (ii) $\frac{93}{2}$ m
(iii) 74.8 m (iv) $0<t<\frac{19}{12}$

5. (i) 35
(ii) 33 s
(iii) 0.341 km
(iv) $500e^{20t}$

6. (i) 49 m/s (ii) 122.5 m

7. (ii) $3\sin t-4\cos t-8t+24$

Quick Test 16

1. (i) 4.46 m/s, -0.104 m/s^2
(iii) 310 m

2. (ii) $\frac{88}{3}$ cm

Secondary 4 Express (G3) Preliminary Examination Specimen Paper A Paper 1

1. 4 or -1

2. (i) $\frac{p}{\sqrt{p^2+1}}$ (ii) $\sqrt{p^2+1}$

3. $a=8, b=3$ or $a=-4, b=-6$

4. $y=\frac{10}{z-3}$

5. (ii) $-65\,625$

7. (i) $\frac{9}{5x-2}-\frac{3}{3x-1}$
(ii) $\frac{9}{5}\ln(5x-2)-\ln(3x-1)+c$

8. (a) $\sec 4x$ (b) $\frac{\pi}{2}$

9. (i) $a=4, b=36$
(ii) -5 or $\frac{-7\pm\sqrt{33}}{8}$

10. (a) $-\frac{8}{675}$ units/s, decrease
(b) $\frac{\pi}{2}$

11. (i) $4, -2$ (ii) $4, -4$
(iii) 720° (iv) 180°
(vi) 3

12. (i) $(x-2)(x^2+9)$
(ii) 2
(iii) $-\frac{23}{3}$

13. (i) (1, 1), $\left(0, \frac{1}{4}\right)$
(ii) 66 units^2

Secondary 4 Express (G3) Preliminary Examination Specimen Paper A Paper 2

1. $-\frac{1}{\sqrt{2}}<k<\frac{1}{\sqrt{2}}$

2. (i) $\frac{\pi}{6}$ (ii) $-\frac{\pi}{4}$
(iii) $\sqrt{3}+2$

3. $-1.32, 0, 1.32$

4. (i) $\frac{2x}{x^2+16}$
   (iii) $-\frac{1}{4} \leqslant k \leqslant \frac{1}{4}$
6. (a) −0.405 (b) $\frac{1}{16}$
   (c) $y = 4x - 2$
7. (ii) $A = 9.03, k = -5$
   (iii) 7.03 (iv) 1.4
   (v) 7.53
8. $\left(\frac{6}{5}\ln 6 - \frac{17}{12}\right)$ units^2
9. (i) No (ii) 0
   (iii) $\left[\frac{4}{5}(e^{20} - e^{10}) - 8\right]$ cm
10. (ii) $R = 10\sqrt{2}$; maximum perimeter = $(42 + 10\sqrt{2})$cm
    (iv) 1.30
11. (i) $\sqrt{69}$ units, (−4, 2)
    (ii) $Q(7, -1)$
    (iii) 5 units, (3, 2)
    (iv) (5, 0), C_2

Secondary 4 Express (G3) Preliminary Examination Specimen Paper B Paper 1

1. $161 - 72\sqrt{5}$
2. (i) $x > -\frac{1}{4}$ (ii) −4
3. $a = -5, b = 3$
4. (i) (a) $1 + 20x + 190x^2 + 1140x^3 + \ldots$
   (b) $1 - 20x + 190x^2 - 1140x^3 + \ldots$
   (ii) Less than
6. (a) $x \leqslant -1$ or $x \geqslant \frac{9}{5}$
   (b) $p < -\frac{251}{4}$
7. (ii) $e^{\sin x}(1 - \sin x - \sin^2 x)$
8. (i) $\frac{1}{7776}$ (ii) 1.63
9. (a) (i) $0° \leqslant \cos^{-1} x \leqslant 180°$
   (ii) $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$
   (b) (i) $a = -1, b = \frac{1}{4}, c = -2$
   (ii) (a) $q = p + \frac{\pi}{2}$
   (b) $p + r = \frac{\pi}{2}$
10. (ii) 0.0795; $1.57e^{0.0795x}$
    (iii) February
11. (i) 1, 3 or −5
    (iii) $a = 0, b = -40$
12. (i) 39 units^2 (ii) Yes

Secondary 4 Express (G3) Preliminary Examination Specimen Paper B Paper 2

1. (i) $\frac{\pi}{2} < A < \pi$
   (ii) $\frac{25\sqrt{3} - 42}{6}$
   (iii) $\frac{42 - 25\sqrt{3}}{6}$
2. (ii) Minimum point
3. (i) 48
   (ii) $(2x + 3)(2x + 1)(2x - 1)$
   (iii) $\frac{2}{2x+3} - \frac{5}{2x+1} + \frac{3}{2x-1}$
4. (i) 0.338
   (ii) Max value = 100, when $\theta = 0.876$; Min value = 39, when $\theta = 2.45$
5. (ii) −255°, −195°, −75°, −15°
   (iii) 8
6. (a) −6 or 3
   (b) $\frac{x-5}{12}$
7. (i) $(x + 3)^2 + (y + 7)^2 = 26$
   (ii) $(-3, \sqrt{26} - 7)$
8. (ii) $\frac{1}{112}$
   (iii) 0
9. (i) $9x^3 + e^{3x+1} - \frac{2}{3}$
   (ii) $\frac{9}{4}x^4 + \frac{1}{3}e^{3x+1} - \frac{2}{3}x - \frac{1}{12}$
10. (i) −5 cm/s